LONGITUDINAL DYNAMIC STABILITY
OF
RTAF 5 AIRCRAFT

BY

FLIGHT OFFICER YUTHANA TRANGARN

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Ae	effective aspect ratio
AR	aspect ratio
A_π	a commonly used area for a component which is the reference area for C_{D_π}
a.c.	aerodynamic center of wing
a_o	lift-curve slope at zero lift
a_v	lift-curve slope of vertical tail
BHP	engine brake horsepower
b	wing span
b_a	aileron span
b_v	vertical tail span
C	wing force parallel to the airplane reference line
C_C	coefficient of wing force parallel to the airplane reference line $\frac{C}{\frac{1}{2}\rho U^2 S}$
C_D	drag coefficient $\frac{D}{\frac{1}{2}\rho U^2 S}$
C_{D_f}	zero lift drag, found in C_D section
C_{D_q}	$\frac{\partial C_D}{\partial \frac{qc}{2U}}$
C_{D_o}	three-dimensional drag coefficient found in lateral stability derivatives
C_{d_o}	two-dimensional parasite drag coefficient

C_{D_u} $\frac{U}{2} \frac{\partial C_D}{\partial u}$

C_{D_α} $\frac{\partial C_D}{\partial \alpha}$

$C_{D_{\dot{\alpha}}}$ $\frac{\partial C_D}{\partial \frac{\dot{\alpha} c}{2U}}$

$C_{D_{\delta_E}}$ $\frac{\partial C_D}{\partial \delta_E}$

C_{D_π} component drag coefficient based on the area A_π

C_{h_o} elevator hinge-moment coefficient when $\alpha_t = 0^o$ and $\delta_e = 0^o$

C_{h_δ} variation of control-surface hinge-moment coefficient with deflection $\frac{\partial C_h}{\partial \delta}$

C_L lift coefficient $(L/\frac{1}{2}\rho U^2 S)$

C_{L_u} $\frac{U}{2} \frac{\partial C_L}{\partial u}$

C_{L_α} $\frac{\partial C_L}{\partial \alpha}$

$\frac{dC_L}{d\alpha}_\infty$ wing lift curve slope with infinite aspect ratio

$C_{L_{\dot{\alpha}}}$ $\frac{\partial C_L}{\partial \frac{\dot{\alpha} c}{2U}}$

$C_{L_{\delta_E}}$ $\frac{\partial C_L}{\partial \delta_E}$

C_ℓ section lift coefficient or rolling moment coefficient $(L/\frac{1}{2}\rho U^2 Sb)$

$C_{\ell_{max}}$ maximum section lift coefficient

C_{ℓ_p} $\dfrac{\partial C_\ell}{\partial \frac{pb}{2U}}$

C_{ℓ_r} $\dfrac{\partial C_\ell}{\partial \frac{rb}{2U}}$

$C_{\ell_{\alpha_t}}$ section lift-curve slope of the horizontal tail

C_{ℓ_β} $\dfrac{\partial C_\ell}{\partial \beta}$

C_{ℓ_δ} change in rolling moment coefficient due to change in aileron deflection

$C_{\ell_{\delta_A}}$ $\dfrac{\partial C_\ell}{\partial \delta_A}$

$C_{\ell_{\delta_E}}$ section lift-curve slope due to elevator deflection (this is not to be confused with rolling moment coefficient)

$C_{\ell_{\delta_R}}$ $\dfrac{\partial C_\ell}{\partial \delta_R}$

C_m pitching-moment coefficient $(M/\frac{1}{2}\rho U^2 Sc)$

C_{m_T} pitching-moment coefficient due to thrust force

$C_{m_{a.c.}}$ airfoil section pitching moment about the aerodynamic center

C_{m_q} $\frac{\partial C_m}{\partial \frac{qc}{2U}}$

C_{m_α} $\frac{\partial C_m}{\partial \alpha}$

$C_{m_{\dot{\alpha}}}$ $\frac{\partial C_m}{\partial \frac{\dot{\alpha}c}{2U}}$

$C_{m_{\delta_E}}$ $\frac{\partial C_m}{\partial \delta_E}$

C_N coefficient of wing force normal to the airplane reference line

C_n yawing-moment coefficient $(N/\frac{1}{2}\rho U^2 Sb)$

C_{n_p} $\frac{\partial C_n}{\partial \frac{pb}{2U}}$

C_{n_r} $\frac{\partial C_n}{\partial \frac{rb}{2U}}$

C_{n_β} $\frac{\partial C_n}{\partial \beta}$

$C_{n_{\delta_A}}$ $\frac{\partial C_n}{\partial \delta_A}$

$C_{n_{\delta_R}}$ $\frac{\partial C_n}{\partial \delta_R}$

C_T thrust coefficient $(T/\frac{1}{2}\rho U^2 S)$

C_{T_u}	$\frac{U}{2}\frac{\partial C_T}{\partial u}$
C_{y_p}	$\frac{\partial C_y}{\partial \frac{pb}{2U}}$
C_{y_r}	$\frac{\partial C_y}{\partial \frac{rb}{2U}}$
C_{y_β}	$\frac{\partial C_y}{\partial \beta}$
$C_{y_{\delta_A}}$	$\frac{\partial C_y}{\partial \delta_A}$
$C_{y_{\delta_R}}$	$\frac{\partial C_y}{\partial \delta_R}$
c	mean aerodynamic chord
c_a	aileron chord
c_e	elevator chord
c_f	flap chord
c.g.	airplane center of gravity
D	drag force
e	base of natural system of logarithms or Oswald's span efficiency factor
e_1	induced-angle span efficiency factor
F_A	stick force due to aileron actuation
F_s	stick force

f	the equivalent parasite area discussed in the C_D section
G	ratio of elevator displacement to the product of stick length and stick angular displacement
g	acceleration due to gravity (32.2 ft/sec^2)
HP	horsepower
h_t	height of the horizontal tail a.c. above the c.g. (positive for a.c. above c.g.)
I_{xx}	moment of inertia about the x-axis
I_{yy}	moment of inertia about the y-axis
I_{zz}	moment of inertia about the z-axis
I_{xz}	product of inertia
i	incidence angle
k_x	radius of gyration about the x-axis
k_y	radius of gyration about the y-axis
k_z	radius of gyration about the z-axis
L	lift or rolling moment
ℓ_b	length of fuselage or body
ℓ_t	length from c.g. to tail quarter chord
ℓ'_t	length from wing quarter chord to tail quarter chord
$\frac{\ell_t S_t}{c S_W}$	tail volume factor
ℓ_V	length from c.g. to vertical tail aerodynamic center
M	pitching-moment about the c.g.
$M_{fus\,nac}$	pitching-moment about the c.g. due to the fuselage and nacelles

$M_{y_{aero}}$	aerodynamic moments about the y-axis (pitching moment)
m	mass in slugs
N	wing force normal to the airplane reference line or yawing moment
n	normal acceleration in g's
p	rolling velocity
$\dot{p}$	rate of change of rolling velocity
$\frac{pb}{2U}$	helix roll angle
q	dynamic pressure ($\frac{1}{2}\rho U^2$) or pitching velocity
$\dot{q}$	rate of change of pitching velocity
q_t	dynamic pressure at the horizontal tail
q_V	dynamic pressure at the vertical tail
RPM	propeller revolutions per minute
R_N	Reynolds Number
r	yawing velocity
$\dot{r}$	rate of change of yawing velocity
S	wing area
SRP	seat reference point
S_R	rudder area
S_e	elevator area
S_f	flap area
S_h	horizontal tail area
S_s	body side area
s	distance in half chords which is used as a non-dimensionalizing $\frac{2Ut}{c}$

T	thrust
TDPF	tail damping power factor
TDR	tail damping ratio
T_u	$\frac{1}{m}\frac{\partial T}{\partial u}$
t	time or airfoil thickness
t	time
Δt	lag time of the downwash at the tail plane
U	airplane velocity
U_S	stall speed
U_{Trim}	trim speed
W	airplane weight
w_f	maximum width of the fuselage or nacelles
x'	distance from c.g. to wing quarter chord (positive for c.g. ahead or quarter chord)
$\bar{x}$	longitudinal distance rearward from c.g. to wing aerodynamic center
x_a	distance parallel to relative wind from the wing a.c. to the c.g. (positive for a.c. ahead of the c.g.)
y_i	distance from body centerline to inboard edge of aileron
z_T	perpendicular distance from thrust line to c.g. (positive for thrust line below the c.g.)
z_a	vertical distance from wing a.c. to c.g. (positive for a.c. above c.g.)
z_v	distance from the center of pressure of the vertical tail to the aircraft center-line (positive for vertical tail above the x-axis)

z_w distance from body centerline to quarter-chord point of exposed wing root chord (positive for the quarter-chord point below the body centerline)

Γ dihedral angle

Λ wing sweep angle

Ω resultant angular velocity

α angle of attack

$\dot{\alpha}$ rate of change of angle of attack

α_i induced angle of attack

α_o angle of attack for zero lift

α_s stall angle of attack

$\frac{d\alpha_t}{d\delta_E}$ elevator efficiency factor $(dC_L/d\delta_E)/(dC_L/d\alpha_t)$

β sideslip angle

γ flight path angle from the horizontal

δ correction factor for induced drag

δ_A aileron deflection

δ_E elevator deflection

δ_R rudder deflection

δ_a aileron deflection

δ_e elevator deflection

δ_{e_o} elevator angle at zero aircraft lift

δ_f flap deflection

δ_t trim tab deflection

ε downwash angle

$\frac{d\varepsilon}{d\alpha}$	change in downwash angle due to change in angle of attack
ζ_d	Dutch Roll damping ratio
$\zeta_{s.p.}$	short period damping ratio
η	efficiency factor for tail, q_t/q, or the propeller efficiency
θ	pitch angle
$\dot{\theta}$	rate of change of pitch angle
λ	taper ratio (tip chord/foot chord)
μ	airplane relative density factor ($m/\rho Sb$)
π	3.1416
ρ	density
σ	sidewash angle
τ	correction factor for induced angle or elevator effectiveness factor $\frac{d\alpha_t}{d\delta_E}$
τ_R	time constant for roll mode
ϕ	roll angle
$\ddot{\phi}$	second derivative with respect to time of the roll angle
$\frac{\phi_{osc}}{\phi_{av}}$	a measure of the ratio of the oscillatory component of bank angle to the average component of bank angle following a rudder-pedal-free impulse aileron control demand
ψ	yaw angle
ψ_β	phase angle of Dutch Roll component of sideslip
ω_{n_d}	Dutch Roll natural frequency

$\omega_{n_{s.p.}}$ short period natural frequency

Subscripts

a.c.	aerodynamic center
c.g.	center of gravity
c/4	wing quarter chord
h	horizontal
t	tail
v	vertical tail
w	wing

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Abstract

The theoretical investigation of longitudinal dynamic stability of the RTAF-5 aircraft is presented. The solution of motion problem is developed by transforming a set of linearized equations derived from equations of motion into transfer functions. The stability characteristics and the parameters of two oscillations so-called "short period" mode and "phugoid" mode are calculated by using LOPROG computer program. The effect of variation in the moment of inertia about the y-axis, I_{yy}, on stability behaviour is also studied. Results obtained from the calculation are analysed by comparing with MIL-F-8785 B and those indicate that the response of longitudinal motion of the RTAF-5 at selected range satisfies the requirements which established in that military specification and is influenced by the change in I_{yy} values when the motion is of short period mode.

Chapter I

Introduction

In recent years, it has been evident that the development of any kinds of aircrafts has greatly increased. The essential of this development is to increase the efficiency and effectiveness of modern aircrafts to meet such a requirements in a competative market. However, an increase in either efficiency or effectiveness of an aircraft leads to an increase in its cost and this affects the development of air power of the Royal Thai Air Force of which the budget is constraint. Thus, the Royal Thai Air Force has sometimes had to build some aircrafts itself so that not only the intended plans can be accomplished but also its engineers can gain their knowledge and experience in the future.

RTAF-5 is the latest aircraft which was designed by the Royal Thai Air Force in 1975. It is of high wing and twin booms, with two seats in tandem. Its primary mission is for advanced pilot training and it can also be adaptable for forward air controller mission. The distinct characteristic of RTAF-5, in comparison with other aircrafts, is an installation of the engine at the rear end of the fuselage which is so-called pusher type. The essential benefits of the pusher type are those of the significant accommodation of a cockpit due to the aerodynamics smoothness of the aircraft nose and no disturbed airflow over the wings caused by the slipstream due to rotating of propellers.

Because of many inherent problems occuring during its development and construction, a prototype of the RTAF-5 was just flown for the first time on 5 October 1984. Subsequently, it has been flown for collecting some basic aerodynamic data, handling qualities and performance testing, in order to develop and improve its efficiency as good as possible. Since this experimental investigation is being conducted by the Royal Thai Air Force, this study is, thus, proposed to adapt a mathematical model for analysing numerically the longitudinal dynamic stability of the RTAF-5.

In the following sections, a description, three views drawing and dimensional data will be presented respectively. Some of these data will be used as input data in the calculation of associated parameters, which are damping ratio and natural frequency, of each mode of motion.

In chapter 2, the derivation of the equations of motion will be presented. These equations follow from Newton's second law (1). The several assumptions used to form the basis for the derivation will be also presented throughout this chapter. Two independent sets of equations, which describe the longitudinal and lateral motions, developed from the equations of motion with the assumptions made will be given.

Chapter 3 concerns the further development of a solution of longitudinal dynamic stability. The linearized equations of motion obtained in chapter 2 will be transformed into the transfer functions. These functions will be subsequently rearranged into the form of quadratics to yield the required solution.

Chapter 4 and 5 present the results of an investigation of the response of longitudinal motion of the RTAF-5, conclusion and discussion respectively. The investigation is emphasized on an effect of changing the values of I_{yy}, ranging from 1000 to 6000 slug-ft^2, on each mode of motion. In calculating the associated parameters, a computer program is developed. This program is generally based upon the expressions obtained in previous chapters and upon the typically aerodynamic data presented in text books (2-7) and published reports (8,9). The results of the investigation will be analysed by comparing with the handling qualities requirements of MIL-F-8785B(10).

Description of the RTAF-5

Type	:	two-seat advanced trainer
Wings	:	Cantilever shoulder-wing monoplane, constant-chord wing without dihedral or sweep.
Fuselage	:	Short pod-type fuselage of conventional aluminium semimonocoque construction.
Tail Unit	:	Cantilever all-metal structure carried on twin booms of semi-monocoque construction.
Landing gear	:	Retractable tricycle type, with single wheel on each unit.
Power Plant	:	one turboprop.

DIMENSION AND GENERAL DATA

WING	
SPAN (OVER ALL)	376.00 IN.
CENTER SECTION	
AIRFOLL SECTION	NACA 632A415
CHORD (CONSTANT)	65.00 IN.
SPAN	148.00 IN.
INCIDENCE	3 DEG.
DIHEDRAL	0 DEG.
SWEEP OF LEADING EDGE	0 DEG.
OUTER PANELS	
AIRFOLL SECTION	
WING STA. 74.00	632A415
WING STA. 188.00	631A412
CHORD	
WING STA. 74.00	65.00 IN.
WING STA. 188.00	56.50 IN.
SPAN	228.00 IN.
INCIDENCE	
WING STA. 74.00	3 DEG.
WING STA. 188.00	0 DEG.
DIHEDRAL	3 DEG.

SWEEP OF LEADING EDGE	1.5 DEG.
WASHOUT	3 DEG.
ASPECT RATIO	6.2
TAPER RATIO	0.87
<u>AILERON</u>	
SPAN	107.25 IN.
CHORD	
WING STA. 122.125	21.494 IN.
WINC STA. 175.75	20.095 IN.
<u>HIGH LIFT DEVICE</u>	
TYPE	SINGLE SLOT FLAPS
INBOARD	
AIRFOLL SECTION	NACA 6321
SPAN	63.05 IN.
CHORD	
WING STA. 20.25	19.50 IN.
WING STA. 52.00	19.50 IN.
OUTBOARD	
AIRFOLL SECTION	NACA 6318
SPAN	93.76 IN.
CHORD	
WING STA. 75.00	19.478 IN.
WINC STA. 121.875	18.429 IN.
<u>HORIZONTAL TAIL</u> (ABOVE ℄ BOOM 51.36 IN.)	
AIRFOLL SECTION	NACA 0012

SPAN	127.00 IN.
CHORD	41.50 IN.
INCIDENCE (25/CHORDLINE)	2 DEG.
SWEEP OF LEADING EDGE	0 DEG.
DIHEDRAL	0 DEG.
ASPECT RATIO	3.06
SPAN	127.00 IN.
CHORD (AFT. HINGELINE)	26.98 IN.

VERTICAL TAIL

AIRFOLL SECTION	NACA 0012
SPAN (ABOVE C BOOM)	48.06 IN.
CHORD	39.00 IN.
SWEEP OF LEADING EDGE	23 DEG.
ASPECT RATIO	

RUDDER

SPAN (ABOVE C BOOM)	44.36 IN.
CHORD (AFT. HIGH LINE)	10.92 IN.

FUSELAGE

LENGTH	221.082 IN.
MAXIMUM WIDTH	37.00 IN.
HEIGHT	120.00 IN.
LENGTH (OVERALL)	368.00 IN.

LOCATION OF THRUST LINE

HORIZONTALLY FROM FUSELACE STA. 0	217.30 IN.
VERTICALLY FROM C BOOM	10.25 IN.
(W_L + FUSELACE STA. 217.35)	

INCLINATION OF THRUST LINE TO FUSELAGE	
PROPELLER DIAMETER	96.00 IN.
WHEEL BASE	139.20 IN.
WHEEL TRACK	130.00 IN.

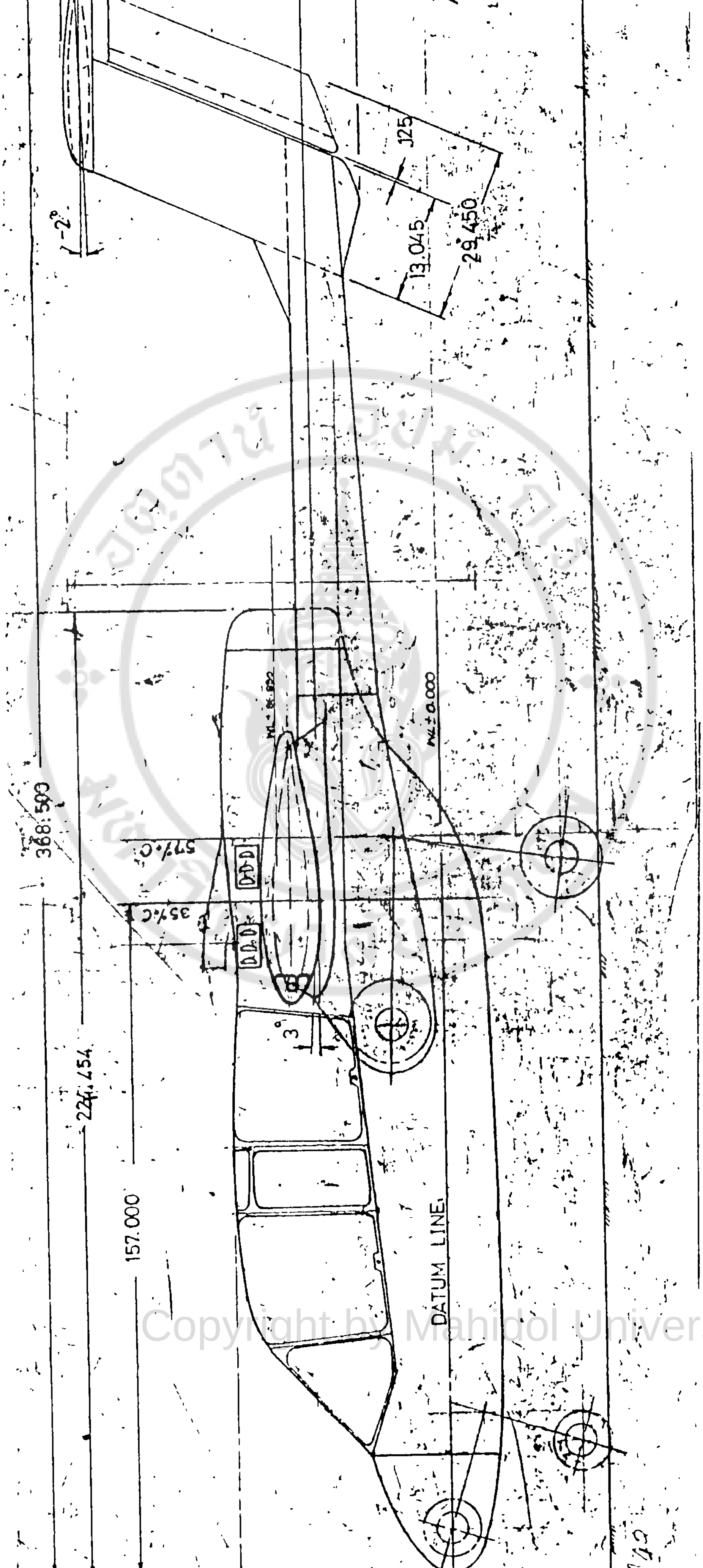
CONTROL SURFACE AND CONTROL MOVEMENT

AILERON	
TRAILLING EDGE UP	25 DEG.
TRAILLING EDGE DOWN	20 DEG.
FLAPS	
TRAILLING EDGE DOWN	40 DEG.
RUDDER	
INBOARD	17±2 DEG.
OUTBOARD	22±2 DEG.
HORIZONTAL STABILIZER	
TRAILLING EDGE UP	25±2 DEG.
TRAILLING EDGE DOWN	15±2 DEG.

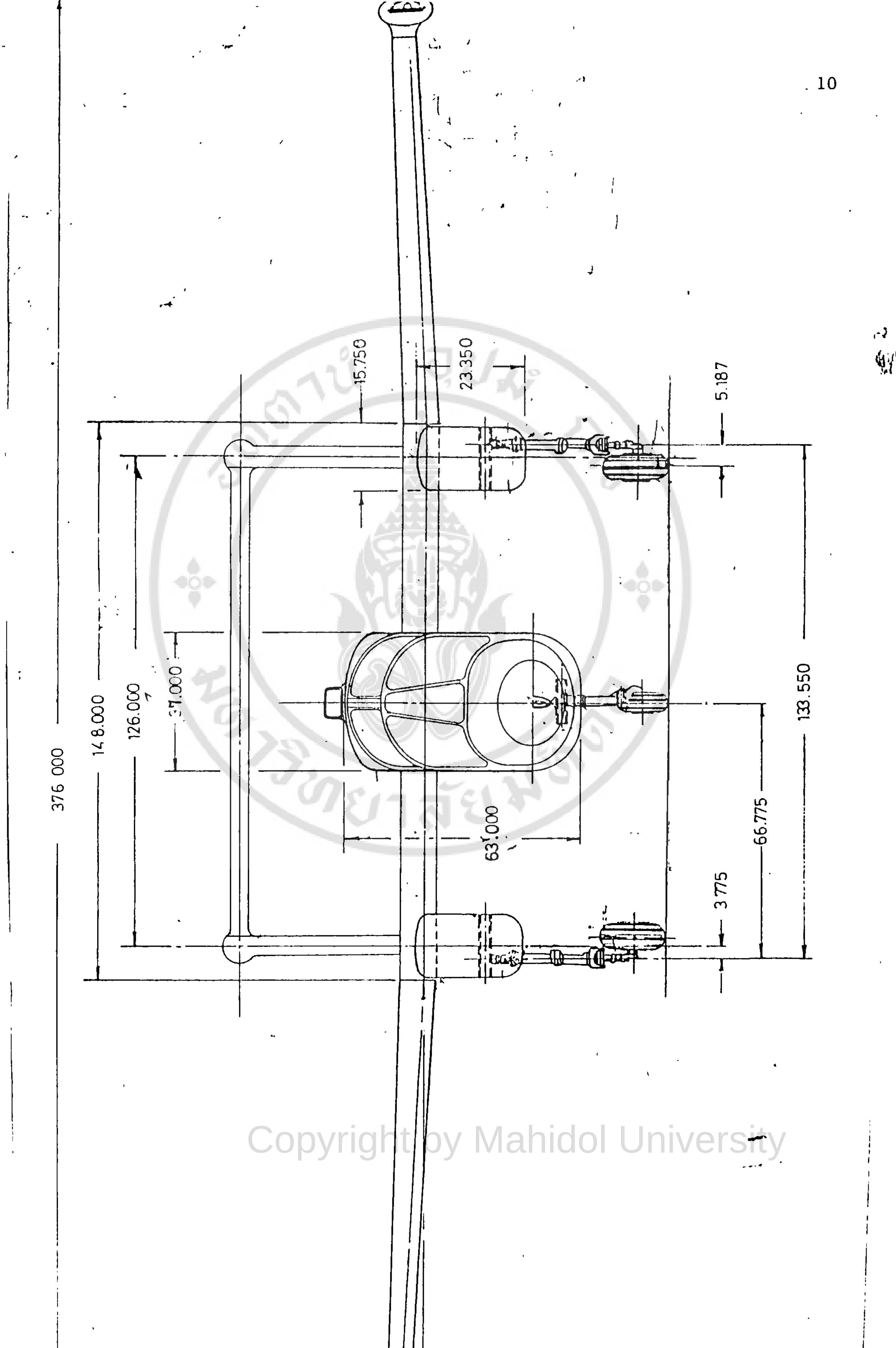
AREAS

WING	163 FT^2
CENTER SECTION	67.79 FT^2
OUTER SECTION	95.20 FT^2
FLAPS	22.50 FT^2
INBOARD	9.95 FT^2
OUTBOARD	12.55 FT^2

AILERON	15.48	FT^2
HORIZONTAL TAIL	35.45	FT^2
STABILIZER	21.71	FT^2
ELEVATOR	13.45	FT^2
VERTICAL TAIL	34.42	FT^2
FIN	22.12	FT^2
RUDDER	12.30	FT^2



-2°
125
13.045
29.450
368.500
224.454
157.000
35%C
57%C
3°
WL ± 0.000
DATUM LINE



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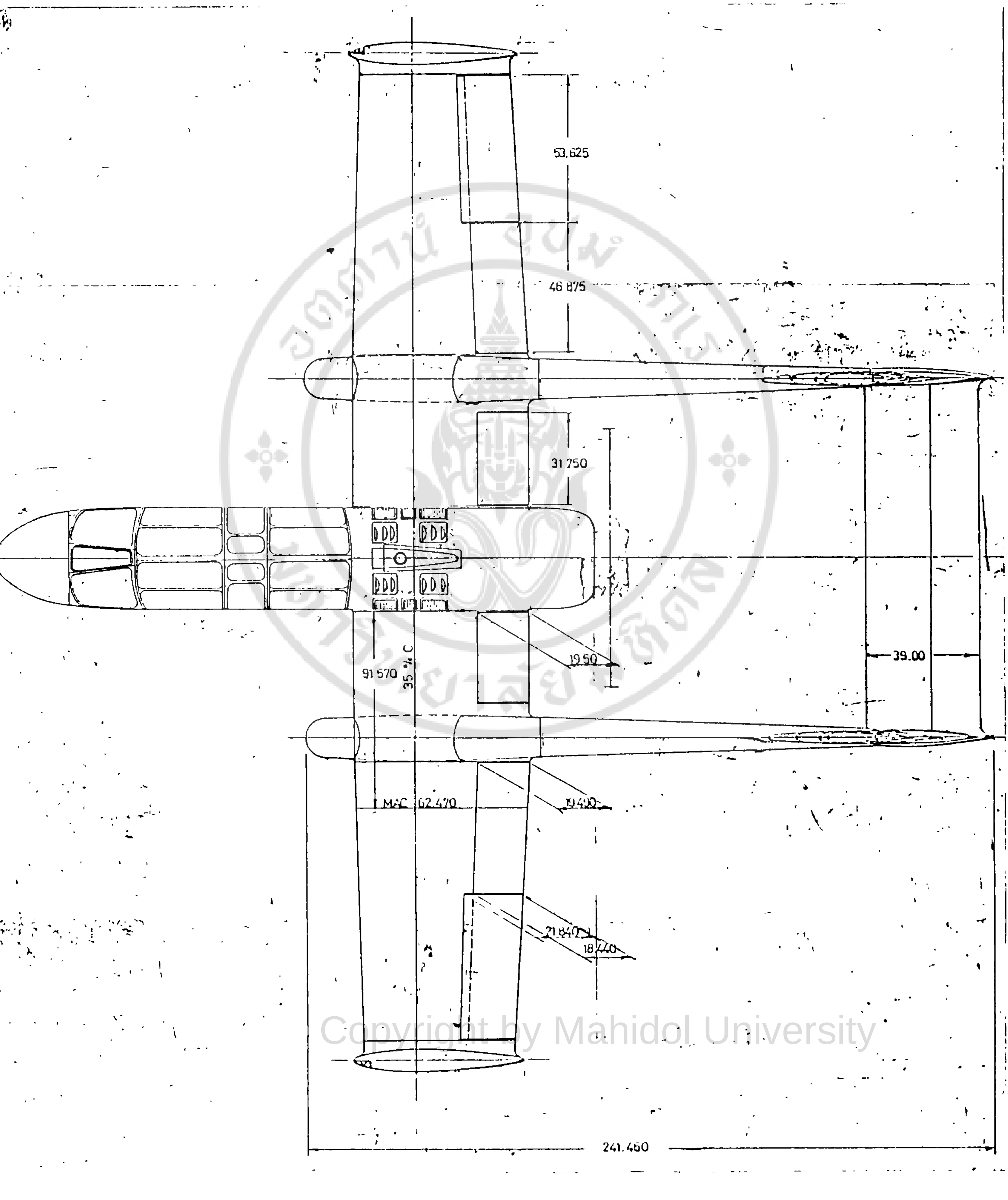
3 775
63.000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46 875
31 750
19.50
91 570
35 % C
39.00
MAC 62.470
19.400
21.840
18.440
241.460

Chapter 2

Equations of Motion

Motion of an aircraft can be described by a set of equations. The derivation of these equations usually follows from Newton's second law under various assumptions depending on a type of aircrafts and flight conditions.

In this chapter, it is proposed to develop the mathematical formulation of equations of motion emphasizing for stability analysis of a typically general aircraft. The assumptions used to form the basis for the derivation will be presented throughout the formulation.

Formulation of Problem.

Table I, with the aid of fig. 1, defines the direction of the axes with respect to the aircraft, as well as the nomenclature needed to apply Newton's second law of motion. The first two assumptions, which specify the nature of the body being studied and the atmosphere in which it is set, are as follows.

Assumption I The airframe is assumed to be a rigid body; thus, the distances between any specified points in the body are invarient.

Assumption II The earth is assumed to be fixed in space, and the earth's atmosphere is assumed to be fixed with respect to the earth.

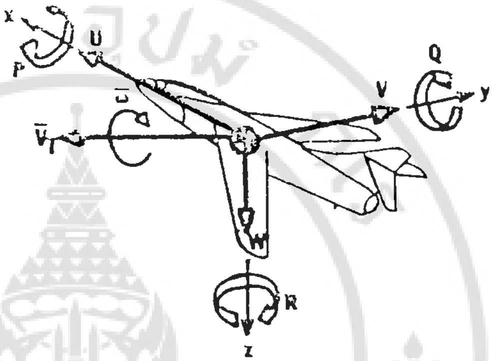

Figure 1 Direction of the axes with respect to the aircraft.

Table I Direction of the axes with respect to the aircraft and nomenclature needed to apply Newton's law.

Axis	Linear Velocity Along Axis	Angular Velocity Along Axis	Summation of Moments About Axis	Summation of Forces Along Axis	Displacements About Axis	Moments of Momentum About Axis	Moment of Inertia
X	U	P Rolling Vel.	ΣL	ΣF_x	Φ	h_x	I_{xx}
Y	V	Q Pitching Vel.	ΣM	ΣF_y	θ	h_y	I_{yy}
Z	W	R Yawing Vel.	ΣN	ΣF_z	ψ	h_z	I_{zz}

The mathematical statements of Newton's second law of motion, which states that the rate of change of momentum of a body is proportional to the net force applied to the body and that the rate of change of the moment of momentum is proportional to the net torque applied to the body, can be written as

$$\Sigma F_x = \frac{d}{dt}(mU) \quad , \quad \Sigma F_y = \frac{d}{dt}(mV) \quad ,$$

$$\Sigma F_z = \frac{d}{dt}(mW) \quad , \qquad (1)$$

and

$$\Sigma L = \frac{dh_x}{dt} \quad , \quad \Sigma M = \frac{dh_y}{dt} \quad ,$$

$$\Sigma N = \frac{dh_z}{dt} \qquad (2)$$

Assumption III The mass of the aircraft is assumed to remain constant for the duration for any particular dynamic analysis. This assumption permits the mass of the aircraft to be written outside the differentiation sign in Equation (1).

The moments of momentum referred to in equation (2) can be expanded by using an element of mass of the aircraft dm which is rotating with the angular velocity $\bar{\omega}$ ($\bar{\omega}$ = Pi + Qj + Rk). This element of mass is at the point (x, y, z) measured relative to the center of gravity, c.g., of the aircraft. The motion of the element of mass can be approximated by six linear velocity components (Py, Pz, Qx, Qz, Rx, and Ry), as seen in Figure 2.

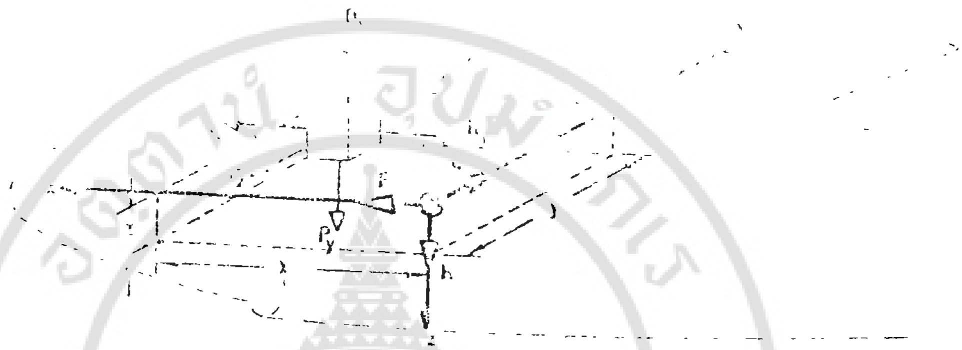

Fig.2 : Linear velocity components of an element of mass caused by an angular velocity $\bar{\omega}$ having components P, Q, and R.

The x, y, and z components of the moment of momentum are calculated by summing the moments of these velocity components about each axis and multiplying by mass dm. For example,

$$dh_x = y(Py)dm + z(Pz)dm - z(Rx)dm - y(Qx)dm.$$

Thus, if the moments are taken about the x, y, and z axes, the set of equations is obtained as follows,

$$dh_x = (y^2 + z^2)P\ dm - zx\ R\ dm - yx\ Q\ dm,$$

$$dh_y = (z^2 + x^2)Q\ dm - xy\ P\ dm - yz\ R\ dm,$$

$$dh_z = (x^2 + y^2)R\ dm - xz\ P\ dm - zy\ Q\ dm. \quad (3)$$

For a finite mass, the components of the moment of momentum are the integrals of equations (3). If we define the moments of inertia about x, y and z axes as

$$I_{xx} = \int (y^2 + z^2)\, dm ,$$

$$I_{yy} = \int (x^2 + z^2)\, dm , \text{ and}$$

$$I_{zz} = \int (x^2 + y^2)\, dm \text{ respectively} ,$$

and the products of inertia as $I_{xy} = \int xy\, dm$, $I_{yz} = \int yz\, dm$ and $I_{xz} = \int xz\, dm$, the integral relations become :

$$h_x = PI_{xx} - QI_{xy} - RI_{xz}$$

$$h_y = QI_{yy} - RI_{yz} - PI_{xy}$$

$$h_z = RI_{zz} - PI_{xz} - QI_{yz} \qquad (4)$$

By differentiating equations (4) with respect to time, the equations of motion relative to inertial axes become :

$$\Sigma F_x = m\frac{dU}{dt} , \quad \Sigma F_y = m\frac{dV}{dt} , \quad \Sigma F_z = m\frac{dW}{dt} ,$$

$$\Sigma L = \frac{dh_x}{dt} = \dot{P}I_{xx} + P\dot{I}_{xx} - \dot{Q}I_{xy} - Q\dot{I}_{xy} - \dot{R}I_{xz} - R\dot{I}_{xz} ,$$

$$\Sigma M = \frac{dh_y}{dt} = \dot{Q}I_{yy} + Q\dot{I}_{yy} - \dot{R}I_{yz} - R\dot{I}_{yz} - \dot{P}I_{xy} - P\dot{I}_{xy} , \text{ and}$$

$$\Sigma N = \frac{dh_z}{dt} = \dot{R}I_{zz} + R\dot{I}_{zz} - \dot{P}I_{xz} - P\dot{I}_{xz} - \dot{Q}I_{yz} - Q\dot{I}_{yz} . \qquad (5)$$

In interpreting flight measurements, it is desirable to change the fixed axes system to an Eulerian axis system, i.e., a right-hand system of orthogonal coordinate axes which has its origin at the center of gravity of the aircraft and its orientation fixed with respect to the aircraft. Velocities of the aircraft measured relative to these axes are absolute velocities, since the Eulerian axes are considered to be fixed in space. Also, moment and products of inertia in the Eulerian axis system are independent of time, since these axes are fixed in the aircraft, thus, dI/dt = 0. Since the Eulerian axis system moves with respect to inertial space, the absolute acceleration (measured in the Eulerian system) can be written :

$$\bar{A}_{abs} = d\bar{V}_T / dt + \bar{\omega} \times \bar{V}_T$$

If U, V, and W are components of $\bar{V}_T$ and P, Q and R are the components of $\bar{\omega}$, then

$$a_x = \dot{U} + QW - RV \qquad a_y = \dot{V} + RU - PW$$

$$a_z = \dot{W} + PV - QU \tag{6}$$

In a similar manner, the change in the amount of momentum can be written :

$$d\bar{h}_{abs} / dt = d\bar{h} / dt + \bar{\omega} \times \bar{h}$$

where

$$\bar{\omega} \times \bar{h} = \begin{vmatrix} i & j & k \\ P & Q & R \\ h_x & h_y & h_z \end{vmatrix}$$

Thus,

$$\left.\frac{dh_x}{dt}\right|_{abs} = \frac{dh_x}{dt} + h_z Q - h_y R$$

$$\left.\frac{dh_y}{dt}\right|_{abs} = \frac{dh_y}{dt} + h_x R - h_z P$$

$$\left.\frac{dh_z}{dt}\right|_{abs} = \frac{dh_z}{dt} + h_y P - h_x Q \qquad (7)$$

Assumption IV The xz plane is assumed to be a plane of symmetry.

Using Assumption IV and the orientation convention of Figure 1, the equations of motion can be written :

$$\Sigma F_x = m(\dot{U} + QW - RV)$$

$$\Sigma F_y = m(\dot{V} + RU - RW)$$

$$\Sigma F_z = m(\dot{W} + PV - QU)$$

$$\Sigma L = \dot{P}I_{xx} - \dot{R}I_{xz} + OR(I_{zz} - I_{yy}) - PQI_{xz}$$

$$\Sigma M = \dot{Q}I_{yy} + PR(I_{xx} - I_{zz}) - R^2 I_{xz} + P^2 I_{xz}$$

$$\Sigma N = \dot{R}I_{zz} - \dot{P}I_{xz} + PQ(I_{yy} - I_{xx}) + QRI_{xz} \qquad (8)$$

Eulerian angles are those angles through which one axis system must be rotated to superimpose it upon another having an initial angular displacement from the first. Because the angles are not orthogonal, the order of rotation is important, if the indicated operations are to yield correct results. The sequence of these angular changes are yaw, pitch, and roll. To carry out this superposition, one first yaws through a positive angle ψ in accordance with a right-hand system so that :

$$\bar{X}_1 = \bar{X} \cos \psi + \bar{Y} \sin \psi$$

$$\bar{Y}_1 = \bar{Y} \cos \psi - \bar{X} \sin \psi$$

$$\bar{Z}_1 = \bar{Z} \qquad (9)$$

(see Figure 3).

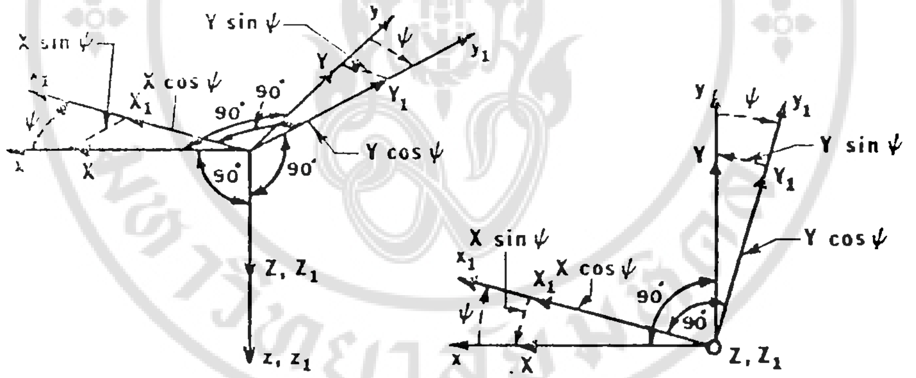

Figure 3. Yaw through a positive angle ψ in accordance with a right-hand system.

The next rotation (Figure 4) is a positive pitch through the angle θ, which gives :

$$\bar{X}_2 = \bar{X}_1 \cos \theta - \bar{Z}_1 \sin \theta$$

$$\bar{Y}_2 = \bar{Y}_1$$

$$\bar{Z}_2 = \bar{Z}_1 \cos \theta + \bar{X}_1 \sin \theta \qquad (10)$$

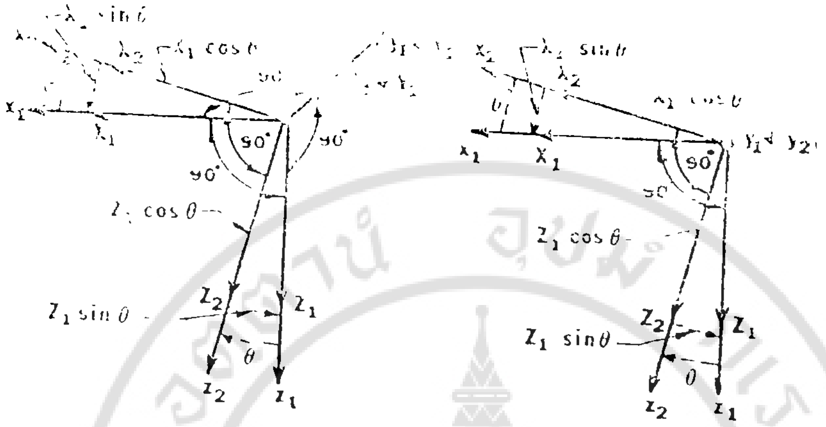

Figure 4. Positive pitch through the angle θ.

The final rotation (Figure 5) is through the roll angle ϕ, which gives :

$$\bar{X}_3 = \bar{X}_2$$

$$\bar{Y}_3 = \bar{Y}_2 \cos\phi + \bar{Z}_2 \sin\phi$$

$$\bar{Z}_3 = \bar{Z}_2 \cos\phi - \bar{Y}_2 \sin\phi \qquad (11)$$

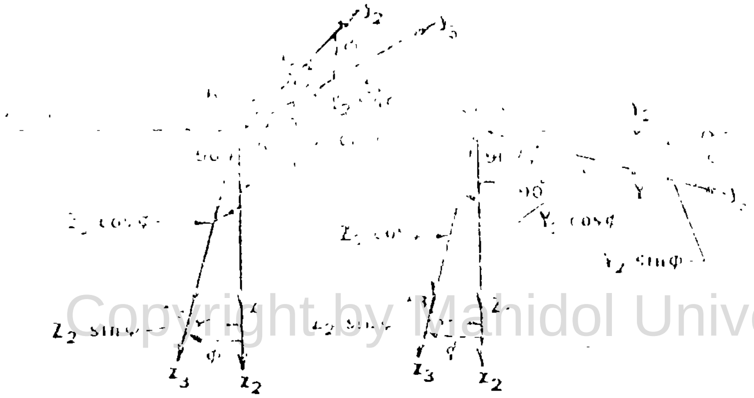

Figure 5. Rotation through the roll angle ϕ.

Substituting Equations (9) and (10) into Equations (11) yield the transformation needed to convert the initial axis system to the final system :

$$\bar{X}_3 = \bar{X} \cos\theta \cos\psi + \bar{Y} \cos\theta \sin\psi - \bar{Z} \sin\theta$$

$$\bar{Y}_3 = \bar{X}(\cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi)$$
$$+ \bar{Y}(\cos\psi \cos\phi + \sin\psi \sin\theta \sin\phi)$$
$$+ \bar{Z}(\cos\theta \sin\phi)$$

$$\bar{Z}_3 = \bar{X}(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi)$$
$$+ \bar{Y}(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi)$$
$$+ \bar{Z}(\cos\theta \cos\phi) \qquad (12)$$

The analysis of flight motions is concerned primarily with the vehicle's behavior in response to disturbances from initial conditions. It is convenient, therefore, to chose as the initial conditions flight behavior for which the velocities and accelerations are well known and in which the aircraft spends most of its flight time. Equilibrium (unaccelerated) flight is flight along a straight path during which the linear velocity vector measured relative to a fixed space is invarient, and the angular velocity is zero. "Steady flight" is flight during which the linear and angular velocities in the Eulerian reference frame remain constant. Hence, equilibrium flight and flight with constant angular velocity are both forms of steady flight.

Always acting on the aircraft even during periods of steady flight is gravity. Since it is unidirectional, it provides an orientation to the motion. The components of gravity acting along the steady flight aircraft axes relative to inertial space can be determined from Figure 6 by direct resolution of the gravity force along the x_o, y_o, and z_o (steady flight) axes.

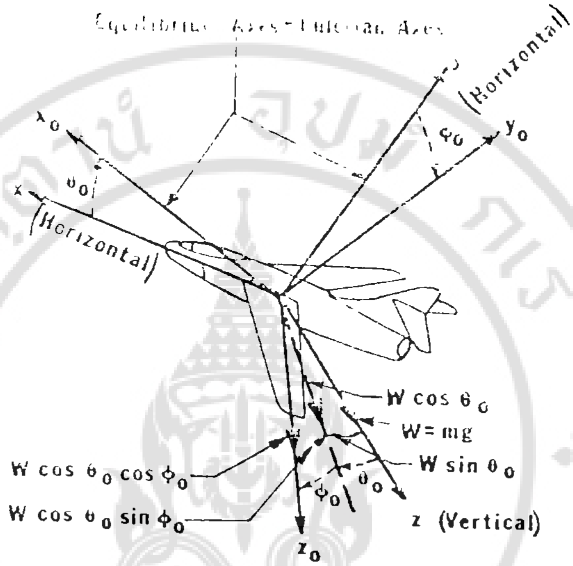

Figure 6. Gravity acting on an aircraft in steady flight with initial angles θ_o and ϕ_o with respect to the gravity vector.

Thus,

$$X_o = - W \sin \theta_o$$

$$Y_o = W \cos \theta_o \sin \phi_o$$

$$Z_o = W \cos \theta_o \cos \phi_o \qquad (13)$$

The components of gravity, acting along the disturbed Eulerian axes, are then :

$$X_3 = (-W \sin \theta_o) \cos \theta \cos \psi + (W \cos \theta_o \sin \phi_o) \cos \theta \sin \psi$$

$$- (W \cos \theta_o \cos \phi_o) \sin \theta$$

$$Y_3 = (-W \sin \theta_o)(\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi)$$

$$+ (W \cos \theta_o \sin \phi_o)(\cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi)$$

$$+ (W \cos \theta_o \cos \phi_o)(\cos \theta \sin \phi)$$

$$Z_3 = (-W \sin \theta_o)(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi)$$

$$+ (W \cos \theta_o \sin \phi_o)(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi)$$

$$+ (W \cos \theta_o \cos \phi_o)(\cos \theta \cos \phi) \qquad (14)$$

The right-hand sides of Equations (8) express the aircraft acceleration in terms of the linear and angular velocities. The left-hand sides of Equations (8) represent the unbalanced forces (thrust forces, aerodynamic forces, and gravity forces) which produce the aircraft motion. The gravity forces have already been expanded and transformed to the Eulerian axes (13); ideally, the same procedure could be applied to the thrust and aerodynamic forces. Because of the difficulty in expressing the aerodynamic and thrust forces explicitly in terms of the linear and angular velocities, it is customary to represent these forces by Taylor series expansions, taking a sufficient number of terms to insure adequate accuracy for the maneuver being considered.

Because of this special requirement, it is helpful to separate the aerodynamic and thrust forces from the gravity forces, thus :

$$\Sigma F_x = \Sigma F_x' + X_3$$

$$\Sigma F_y = \Sigma F_y' + Y_3$$

$$\Sigma F_z = \Sigma F_z' + Z_3 \qquad (15)$$

Where the primed quantities are the summations of the aerodynamic and thrust forces, and X_3, Y_3, and Z_3 are the gravity components drived in Equations (14). It should be noted that, if the instant under consideration occurs during the steady flight condition, then $\theta = \phi = \psi = 0$, and the components X_3, Y_3, and Z_3 reduce to Equations (13). The force relations from Equations (8), with the gravity terms transposed to the right side, are re-written as

$$\Sigma F_x' = m(\dot{U} + QW - RV) - X_3 \ ,$$

$$\Sigma F_y' = m(\dot{V} + RU - PW) - Y_3 \ ,$$

$$\Sigma F_z' = m(\dot{W} + PV - QU) - Z_3 \ . \qquad (16)$$

By substituting Equations (14) in Equations (16), Equations (8) may be written as

$$\Sigma F_x' = m(\dot{U} + QW - RV) + (W \sin \theta_o) \cos \theta \cos \psi$$
$$- (W \cos \theta_o \sin \phi_o) \cos \theta \sin \psi + (W \cos \theta_o \cos \phi_o) \sin \theta,$$

$$\Sigma F_y' = m(\dot{V} + RU - PW) + (W \sin \theta_o)(\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi)$$

$$- (W \cos \theta_o \sin \phi_o)(\cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi)$$

$$- (W \cos \theta_o \cos \phi_o)(\cos \theta \sin \phi),$$

$$\Sigma F_z' = m(\dot{W} + PV - QU) + (W \sin \theta_o)(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi)$$

$$- (W \cos \theta_o \sin \phi_o)(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi)$$

$$- (W \cos \theta_o \cos \phi_o)(\cos \theta \cos \phi),$$

$$\Sigma L = \dot{P}I_{xx} - \dot{R}I_{xz} (I_{zz} - I_{yy}) - PQI_{xz},$$

$$\Sigma M = \dot{Q}I_{yy} + PR (I_{xx} - I_{zz}) - R^2 I_{xz} + p^2 I_{xz},$$

$$\Sigma N = \dot{R}I_{zz} - \dot{P}I_{xz} + PQ (I_{yy} - I_{xx}) + QRI_{xz}, \qquad (17)$$

These equations are then complete, except for the external forces and moments on the left side, which include aerodynamic and thrust forces as well as moments resulting from control surface deflections.

Definitions of the Eulerian axis system and the Eulerian angles, as discussed above, show that the rates of change of the Eulerian angles $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ are not orthogonal. The fixed axis angular velocities when written in terms of the rates of change of the Eulerian angles become :

$$P = \dot{\phi} - \dot{\psi} \sin \theta$$

$$Q = \dot{\theta} \cos \phi + \dot{\psi} \sin \phi \cos \theta$$

$$R = \dot{\psi} \cos \phi \cos \theta - \dot{\theta} \sin \phi \qquad (18)$$

Equations (17) match the aerodynamic and thrust forces acting on an aircraft to the gravity and resulting inertia forces. These equations are non-linear, since (1) they can contain products of the dependent variables and (2) the dependent variables appear as transcendental fuctions.

The airframe motion can always be considered the result of disturbances to the airframe from some steady flight condition. Accordingly, each of the total instantaneous velocity components of the airframe can be written as the sum of a velocity component during the steady flight condition and a change in velocity caused by the disturbance :

$$U = U_o + u$$

$$V = V_o + v$$

$$W = W_o + w$$

$$P = P_o + p$$

$$Q = Q_o + q$$

$$R = R_o + r \qquad (19)$$

It should be noted that the zero subscripts of Equations (19) indicate the steady flight velocities, and the lower case letters represent the disturbance velocities. By substituting Equations (19) into Equations (17) and noting that derivatives with respect to time of the steady state conditions are zero, Equations (17) become :

$$\Sigma F'_x = m[\dot{u} + Q_oW_o + W_oq + Q_ow + wq$$

$$- R_oV_o - R_ov - V_or - vr$$

$$+ (g \sin \theta_o) \cos \theta \cos \psi - (g \cos \theta_o \sin \phi_o) \cos \theta \sin \psi$$

$$+ (g \cos \theta_o \cos \phi_o) \sin \theta]$$

$$\Sigma F'_y = m[\dot{v} + U_oR_o + U_or + R_ou + ru - P_oW_o - P_ow - W_op$$

$$- wp + (g \sin \theta_o)(\cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi)$$

$$- (g \cos \theta_o \sin \phi_o)(\cos \psi \cos \phi + \sin \psi \sin \theta \sin \phi)$$

$$- (g \cos \theta_o \cos \phi_o)(\cos \theta \sin \phi)]$$

$$\Sigma F'_z = m[\dot{w} + P_oV_o + P_ov + V_op + pv - Q_oU_o - Q_ou - U_oq - qu$$

$$+ (g \sin \theta_o)(\cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi)$$

$$- (g \cos \theta_o \sin \phi_o)(\sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi)$$

$$- g (\cos \theta_o \cos \phi_o)(\cos \theta \cos \phi)]$$

$$\Sigma L = \dot{p}I_{xx} - \dot{r}I_{xz} + (Q_oR_o + Q_or + R_oq + qr)(I_{zz} - I_{yy})$$

$$- (P_oQ_o + P_oq + Q_op + pq) I_{xz}$$

$$\Sigma M = \dot{q}I_{yy} + (P_oR_o + P_or + R_op + pr)(I_{xx} - I_{zz})$$

$$- (R_o^2 + 2R_or + r^2) I_{xz} + (P_o^2 + 2P_op + p^2) I_{xz}$$

$$\Sigma N = \dot{r}I_{zz} - \dot{p}I_{xz} + (P_oQ_o + P_oq + Q_op + pq)(I_{yy} - I_{xx})$$

$$+ (Q_oR_o + Q_or + R_oq + qr)\ I_{xz} \qquad (20)$$

Assumption V The disturbances from the steady flight condition are assumed to be small enough so that the products and squares of the changes in velocities are negligible in comparison to the changes themselves. Also, the disturbance angles are assumed to be small enough so that the sines of these angles may be set equal to the angles and the cosines set equal to one. Products of these angles are also approximately zero and can be neglected. Since the disturbances are small, the change in air density encountered by the aircraft during any disturbance can be considered zero.

By applying Assumption V to equations (20), we get

$$\Sigma F_x' = m[\dot{u} + Q_oW_o + W_oq + Q_ow - R_oV_o - R_ov - V_or$$

$$+ g \sin\theta_o - (g\cos\theta_o \sin\phi_o)\ \psi + (g\cos\theta_o\cos\phi_o)\theta]$$

$$\Sigma F_y' = m[\dot{v} + U_oR_o + U_or + R_ou - P_oW_o - P_ow - W_op$$

$$- (g\sin\theta_o)\ \psi - g\cos\theta_o\sin\phi_o - (g\cos\theta_o\cos\phi_o)\phi]$$

$$\Sigma F_z' = m[\dot{w} + P_oV_o + P_ov + V_op - Q_oU_o - Q_ou - U_oq$$

$$+ (g\sin\theta_o)\ \theta + (g\cos\theta_o\sin\phi_o)\ \phi \cdot (g\cos\theta_o\cos\phi_o)]$$

$$\Sigma L = \dot{p}I_{xx} - \dot{r}I_{xz} + (Q_oR_o + Q_or + R_oq)(I_{zz} - I_{yy})$$
$$- (P_oQ_o + P_oq + Q_op)\ I_{xz}$$
$$\Sigma M = \dot{q}I_{yy} + (P_oR_o + P_or + R_op)(I_{xx} - I_{zz})$$
$$- (R_o^2 + 2R_or)\ I_{xz} + (P_o^2 + 2P_op)\ I_{xz}$$
$$\Sigma N = \dot{r}I_{zz} - \dot{p}I_{xz} + (P_oQ_o + P_oq + Q_op)(I_{yy} - I_{xx})$$
$$+ (Q_oR_o + Q_or + R_oq)\ I_{xz} \qquad (21)$$

Equations (21) limit the applicability of the analysis to so-called small perturbations. In the strictly mathematical sense, Equations (21) are applicable only to infinitesimal disturbances. However, experience has shown that quite accurate results can be obtained by applying these equations to disturbances of finite, non-zero magnitude. An additional application of Assumption V is the reduction of Equations (18) to :

$$P = \dot{\phi} - \dot{\psi}\theta$$
$$Q = \dot{\theta} + \dot{\psi}\phi$$
$$R = \dot{\psi} - \dot{\theta}\phi \qquad (22)$$

If the products of perturbations are neglected, the above equations are reduced to :

$$P = \dot{\phi}$$
$$Q = \dot{\theta}$$
$$R = \dot{\psi} \qquad (23)$$

Equations (23) shown that, within the limits of small perturbation theory, the instantaneous angular velocities, P, Q, and R may be set equal to the rates of change of the Eulerian angles

Assumption VI During the steady flight condition, the aircraft is assumed to be flying with wings level and with all components of velocity zero except U_o and W_o.

Thus, $V_o = P_o = Q_o = R_o = \theta_o = \psi_o = 0.$

Assumption VI reduces the equations of motion to :

$$\Sigma F_x' = m[\dot{u} + W_o q + g \sin \theta_o + g\theta \cos \theta_o]$$

$$\Sigma F_y' = m[\dot{v} + U_o r - W_o p - g\psi \sin \theta_o - g\phi \cos \theta_o]$$

$$\Sigma F_z' = m[\dot{w} \quad U_o q + g\theta \sin \theta_o - g \cos \theta_o]$$

$$\Sigma L = \dot{p} I_{xx} - \dot{r} I_{xz}$$

$$\Sigma M = \dot{q} I_{yy}$$

$$\Sigma N = \dot{r} I_{zz} - \dot{p} I_{xz} \qquad (24)$$

The aerodynamic forces and moments are then expressed in coefficient form as

$$L = C_L \tfrac{1}{2}\rho V^2 S = \text{Lift}$$

$$D = C_D \tfrac{1}{2}\rho V^2 S = \text{Drag}$$

$$X = C_X \tfrac{1}{2}\rho V^2 S = \text{Aerodynamic Force Along x-Axis}$$

$$Y = C_y \tfrac{1}{2}\rho V^2 S = \text{Aerodynamic Force Along y-Axis}$$

$$Z = C_z \tfrac{1}{2}\rho V^2 S = \text{Aerodynamic Force Along z-Axis}$$

$$L = C_l \tfrac{1}{2}\rho V^2 Sb = \text{Rolling Moment}$$

$$M = C_m \tfrac{1}{2}\rho V^2 Sc = \text{Pitching Moment}$$

$$N = C_n \tfrac{1}{2}\rho V^2 Sb = \text{Yawing Moment} \qquad (25)$$

where

S = Wing Area

c = Mean Aerodynamic Chord = The wing chord which has the average characteristics of all chords in the wing.

b = Wing span

The lift and drag are the forces acting normal and parallel respectively to the flight path.

As noted previously, each of the forces and moments can be expressed as a function of the variables by expanding the forces and moments in a Taylor series. The series has the form :

$$F = F_o + (\partial F/\partial\alpha)_o\alpha + (\partial F/\partial\beta)_o\beta + (\partial F/\partial\delta)_o\delta + \ldots \qquad (26)$$

where α, β, and δ are variables, and the subscript zero indicates that the quantities are evaluated at the steady flight condition. In Equation (26), terms of the order $(\partial^2F/\partial\alpha^2)(\alpha^2/2!)$ and all higher order terms are omitted in accordance with Assumption V. Before expanding each of the forces and moments in the above form, a simplification can be made. Because the xz plane is a plane of symmetry, the rate of change of the X and Z forces and the moment M, with respect to the disturbance velocities p, r, and v, is zero. Thus, the forces and moments acting on a disturbed aircraft can be expressed :

$$X = X_o + \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial \dot{u}} \dot{u} + \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial \dot{q}} \dot{q} + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \dot{w}} \dot{w} + \frac{\partial X}{\partial \delta_E} \delta_E$$

$$+ \frac{\partial X}{\partial \dot{\delta}_E} \dot{\delta}_E + \frac{\partial X}{\partial \ddot{\delta}_E} \ddot{\delta}_E + \frac{\partial X}{\partial \delta_F} \delta_F + \frac{\partial X}{\partial \dot{\delta}_F} \dot{\delta}_F$$

$$Y = Y_o + \frac{\partial Y}{\partial r} r + \frac{\partial Y}{\partial \dot{r}} \dot{r} + \frac{\partial Y}{\partial v} v + \frac{\partial Y}{\partial \dot{v}} \dot{v} + \frac{\partial Y}{\partial p} p + \frac{\partial Y}{\partial \dot{p}} \dot{p} + \frac{\partial Y}{\partial \delta_A} \delta_A$$

$$+ \frac{\partial Y}{\partial \dot{\delta}_A} \dot{\delta} + \frac{\partial Y}{\partial \ddot{\delta}_A} \ddot{\delta}_A + \frac{\partial Y}{\partial \delta_R} \delta_R + \frac{\partial Y}{\partial \dot{\delta}_R} \dot{\delta}_R + \frac{\partial Y}{\partial \ddot{\delta}_R} \ddot{\delta}_R$$

$$Z = Z_o + \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial \dot{u}} \dot{u} + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial \dot{q}} \dot{q} + \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \dot{w}} \dot{w} + \frac{\partial Z}{\partial \delta_E} \delta_E$$

$$+ \frac{\partial Z}{\partial \dot{\delta}_E} \dot{\delta}_E + \frac{\partial Z}{\partial \ddot{\delta}_E} \ddot{\delta}_E + \frac{\partial Z}{\partial \delta_F} \delta_F + \frac{\partial Z}{\partial \dot{\delta}_F} \dot{\delta}_F$$

$$L = L_o + \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial \dot{r}} \dot{r} + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial \dot{v}} \dot{v} + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \dot{p}} \dot{p} + \frac{\partial L}{\partial \delta_A} \delta_A$$

$$+ \frac{\partial L}{\partial \dot{\delta}_A} \dot{\delta}_A + \frac{\partial L}{\partial \ddot{\delta}_A} \ddot{\delta}_A + \frac{\partial L}{\partial \delta_R} \delta_R + \frac{\partial L}{\partial \dot{\delta}_R} \dot{\delta}_R + \frac{\partial L}{\partial \ddot{\delta}_R} \ddot{\delta}_R$$

$$M = M_o + \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial \dot{u}} \dot{u} + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \dot{q}} \dot{q} + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w} + \frac{\partial M}{\partial \delta_E} \delta_E$$

$$+ \frac{\partial M}{\partial \dot{\delta}_E} \dot{\delta}_E + \frac{\partial M}{\partial \ddot{\delta}_E} \ddot{\delta}_E + \frac{\partial M}{\partial \delta_F} \delta_F + \frac{\partial M}{\partial \dot{\delta}_F} \dot{\delta}_F$$

$$N = N_o + \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial \dot{v}} \dot{v} + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial \dot{r}} \dot{r} + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial \dot{p}} \dot{p} + \frac{\partial N}{\partial \delta_R} \delta_R$$

$$+ \frac{\partial N}{\partial \dot{\delta}_R} \dot{\delta}_R + \frac{\partial N}{\partial \ddot{\delta}_R} \ddot{\delta}_R + \frac{\partial N}{\partial \delta_A} \delta_A + \frac{\partial N}{\partial \dot{\delta}_A} \dot{\delta}_A + \frac{\partial N}{\partial \ddot{\delta}_A} \ddot{\delta}_A \qquad (27)$$

where

δ_E = Angle of deflection of elevator

δ_F = Angle of deflection of flaps

δ_A = Angle of deflection of ailerons

δ_R = Angle of deflection of rudder

The thrust force previously mentioned in Equations (15) can be introduced into the equations of motion in such the same way as the gravity force was introduced. The thrust is considered to be a function of the power plant revolutions per minute and the forward speed of the aircraft. With the power plant located in the plane of symmetry, the thrust contributes to the X and Z forces and to the moment M. With the aid of Figure 7, it is evident that, by setting the steady flight thrust equal to T_o the equations for the steady flight condition become :

$$X_o = T_o \cos \xi$$

$$Z_o = - T_o \sin \xi$$

$$M_o = T_o z_j \qquad (28)$$

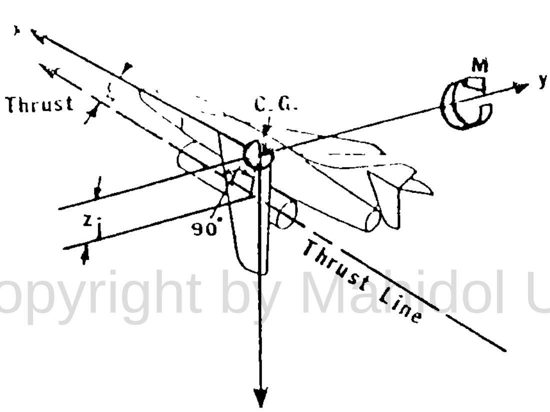

Figure 7 Thrust force relationship to X and Z forces and moment M.

where

ξ = angle between x-axis and thrust line

z_j = perpendicular distance from c.g. to thrust line

Since the Eulerian axes remain fixed with reference to the aircraft during a disturbance, the thrust components relative to the disturbed axes become :

$$X = T_1 \cos \xi$$

$$Z = - T_1 \sin \xi$$

$$M = T_1 z_j \qquad (29)$$

where

T_1 (the thrust during the disturbance) $= T_o + \Delta T$

If a Taylor series expansion is assumed, then :

$$\Delta T = \frac{\partial T}{\partial u} u + \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM}$$

Thus,

$$X = T_o \cos \xi + (\cos \xi) \frac{\partial T}{\partial u} u + (\cos \xi) \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM}$$

$$Z = - T_o \sin \xi - (\sin \xi) \frac{\partial T}{\partial u} u - (\sin \xi) \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM}$$

$$M = T_o z_j + z_j \frac{\partial T}{\partial u} u + z_j \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM} \qquad (30)$$

The individual contributions to the equations of motion have now been examined in some detail, giving perhaps some insight into the basis of the complete equations of motion of the airframe. Before continuing, however, it should be noted that the equations for steady flight can be found by substituting the steady flight values of the aerodynamic, weight and thrust forces and moments into Equations (24) and setting the disturbance terms equal to zero :

$$X_o - W \sin \theta_o + T_o \cos \xi = 0$$

$$Y_o + 0 + 0 = 0$$

$$Z_o + W \cos \theta_o - T_o \sin \xi = 0$$

$$L_o + 0 + 0 = 0$$

$$M_o + 0 + T_o z_j = 0$$

$$N_o + 0 + 0 = 0 \qquad (31)$$

The equations of motion for the disturbed aircraft are then found by substituting the disturbed values of the forces and moments into Equations (24) :

$$m[\dot{u} + W_o q + \boxed{g \sin \theta_o} + g\theta \cos \theta_o] = X_o + \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial \dot{u}} \dot{u}$$

$$+ \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial \dot{q}} \dot{q} + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \dot{w}} \dot{w} + \frac{\partial X}{\partial \delta_E} \delta_E + \frac{\partial X}{\partial \dot{\delta}_E} \dot{\delta}_E + \frac{\partial X}{\partial \ddot{\delta}_E} \ddot{\delta}_E + \frac{\partial X}{\partial \delta_F} \delta_F$$

$$+ \frac{\partial X}{\partial \dot{\delta}_F} \dot{\delta}_F + \boxed{T_o \cos \xi} + (\cos \xi) \frac{\partial T}{\partial u} u + (\cos \xi) \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM}$$

$$m[\dot{v} + U_o r - W_o p - g\psi \sin \theta_o - (g \cos \theta_o)\phi] = \boxed{Y_o} + \frac{\partial Y}{\partial r} r + \frac{\partial Y}{\partial \dot{r}} \dot{r}$$

$$+ \frac{\partial Y}{\partial v} v + \frac{\partial Y}{\partial \dot{v}} \dot{v} + \frac{\partial Y}{\partial p} p + \frac{\partial Y}{\partial \dot{p}} \dot{p} + \frac{\partial Y}{\partial \delta_A} \delta_A + \frac{\partial Y}{\partial \dot{\delta}_A} \dot{\delta}_A + \frac{\partial Y}{\partial \ddot{\delta}_A} \ddot{\delta}_A$$

$$+ \frac{\partial Y}{\partial \delta_R} \delta_R + \frac{\partial Y}{\partial \dot{\delta}_R} \dot{\delta}_R + \frac{\partial Y}{\partial \ddot{\delta}_R} \ddot{\delta}_R$$

$$m[\dot{w} - U_o q + g\theta \sin \theta_o - \boxed{g \cos \theta_o}\,] = \boxed{Z_o} + \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial \dot{u}} \dot{u} + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial \dot{q}} \dot{q}$$

$$+ \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \dot{w}} \dot{w} + \frac{\partial Z}{\partial \delta_E} \delta_E + \frac{\partial Z}{\partial \dot{\delta}_E} \dot{\delta}_E + \frac{\partial Z}{\partial \ddot{\delta}_E} \ddot{\delta}_E + \frac{\partial Z}{\partial \delta_F} \delta_F + \frac{\partial Z}{\partial \dot{\delta}_F} \dot{\delta}_F$$

$$- \boxed{T_o \sin \xi} - (\sin \xi) \frac{\partial T}{\partial u} u - (\sin \xi) \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM}$$

$$\dot{p} I_{xx} - \dot{r} I_{xz} = \boxed{L_o} + \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial \dot{r}} \dot{r} + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial \dot{v}} \dot{v} + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \dot{p}} \dot{p}$$

$$+ \frac{\partial L}{\partial \delta_A} \delta_A + \frac{\partial L}{\partial \dot{\delta}_A} \dot{\delta}_A + \frac{\partial L}{\partial \ddot{\delta}_A} \ddot{\delta}_A + \frac{\partial L}{\partial \delta_R} \delta_R + \frac{\partial L}{\partial \dot{\delta}_R} \dot{\delta}_R + \frac{\partial L}{\partial \ddot{\delta}_R} \ddot{\delta}_R$$

$$\dot{q} I_{yy} = \boxed{M_o} + \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial \dot{u}} \dot{u} + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \dot{q}} \dot{q} + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w} + \frac{\partial M}{\partial \delta_E} \delta_E + \frac{\partial M}{\partial \dot{\delta}_E} \dot{\delta}_E$$

$$+ \frac{\partial M}{\partial \ddot{\delta}_E} \ddot{\delta}_E + \frac{\partial M}{\partial \delta_F} \delta_F + \frac{\partial M}{\partial \dot{\delta}_F} \dot{\delta}_F + \boxed{T_o z_j} + z_j \frac{\partial T}{\partial u} u + z_j \frac{\partial T}{\partial \delta_{RPM}} \delta_{RPM}$$

$$\dot{r} I_{zz} - \dot{p} I_{xz} = \boxed{N_o} + \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial \dot{v}} \dot{v} + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial \dot{r}} \dot{r} + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial \dot{p}} \dot{p} + \frac{\partial N}{\partial \delta_R} \delta_R$$

$$+ \frac{\partial N}{\partial \dot{\delta}_R} \dot{\delta}_R + \frac{\partial N}{\partial \ddot{\delta}_R} \ddot{\delta}_R + \frac{\partial N}{\partial \delta_A} \delta_A + \frac{\partial N}{\partial \dot{\delta}_A} \dot{\delta}_A + \frac{\partial N}{\partial \ddot{\delta}_A} \ddot{\delta}_A \qquad (23)$$

The quantities in boxes disappear because of the steady flight conditions of Equations (31). Dividing the force equations by the mass m and the moment equations by the appropriate moments of inertia yields terms of the form :

$$\frac{1}{m}\ \frac{\partial X}{\partial u}\ u \quad \text{and} \quad \frac{1}{I_{xx}}\ \frac{\partial L}{\partial r}\ r$$

Replacing $(1/m)(\partial X/\partial u)$ by X_u and $(1/I_{xx})(\partial L/\partial r)$ by L_r simplifies the notation. These quantities are called either "dimensional stability derivatives" or simply "stability derviatives." By eliminating those terms whose sum, in accordance with Equations (32), is zero because of the steady flight conditions, and by using the previous notation, Equations (32) are reduced to the form :

$$\dot{u} + W_o q + g\theta \cos\theta_o = X_u u + X_{\dot{u}}\dot{u} + X_q q + X_{\dot{q}}\dot{q} + X_w w + X_{\dot{w}}\dot{w} + X_{\delta_E}\delta_E + X_{\dot{\delta}_E}\dot{\delta}_E$$
$$+ X_{\ddot{\delta}_E}\ddot{\delta}_E + X_{\delta_F}\delta_F + X_{\dot{\delta}_F}\dot{\delta}_F + \cos\xi\ T_u u + \cos\xi\ T_{\delta_{RPM}}\delta_{RPM}$$

$$\dot{v} + U_o r - W_o p - g\psi \sin\theta_o - g\phi \cos\theta_o = Y_r r + Y_{\dot{r}}\dot{r} + Y_v v + Y_{\dot{v}}\dot{v} + Y_p p + Y_{\dot{p}}\dot{p}$$
$$+ Y_{\delta_A}\delta_A + Y_{\dot{\delta}_A}\dot{\delta}_A + Y_{\ddot{\delta}_A}\ddot{\delta}_A + Y_{\delta_R}\delta_R + Y_{\dot{\delta}_R}\dot{\delta}_R + Y_{\ddot{\delta}_R}\ddot{\delta}_R$$

$$\dot{w} - U_o q + g\theta \sin\theta_o = Z_u u + Z_{\dot{u}}\dot{u} + Z_q q + Z_{\dot{q}}\dot{q} + Z_w w + Z_{\dot{w}}\dot{w} + Z_{\delta_E}\delta_E + Z_{\dot{\delta}_E}\dot{\delta}_E$$
$$+ Z_{\ddot{\delta}_E}\ddot{\delta}_E + Z_{\delta_F}\delta_F + Z_{\dot{\delta}_F}\dot{\delta}_F - (\sin\xi)\ T_u u - (\sin\xi)\ T_{\delta_{RPM}}\delta_{RPM}$$

$$\dot{p} - \dot{r}\frac{I_{xz}}{I_{xx}} = L_r r + L_{\dot{r}}\dot{r} + L_v v + L_{\dot{v}}\dot{v} + L_p p + L_{\dot{p}}\dot{p} + L_{\delta_A}\delta_A + L_{\dot{\delta}_A}\dot{\delta}_A + L_{\ddot{\delta}_A}\ddot{\delta}_A$$
$$+ L_{\delta_R}\delta_R + L_{\dot{\delta}_R}\dot{\delta}_R + L_{\ddot{\delta}_R}\ddot{\delta}_R$$

$$\dot{q} = M_u u + M_{\dot{u}}\dot{u} + M_q q + M_{\dot{q}}\dot{q} + M_w w + M_{\dot{w}}\dot{w} + M_{\delta_E}\delta_E + M_{\dot{\delta}_E}\dot{\delta}_E + M_{\ddot{\delta}_E}\ddot{\delta}_E$$

$$+ M_{\delta_F}\delta_F + M_{\dot{\delta}_F}\dot{\delta}_F + \frac{z_{jm}}{I_{yy}} T_u u + \frac{z_{jm}}{I_{yy}} T_{\delta_{RPM}}\delta_{RPM}$$

$$\dot{r} - \dot{p}\frac{I_{xz}}{I_{zz}} = N_v v + N_{\dot{v}}\dot{v} + N_r r + N_{\dot{r}}\dot{r} + N_p p + N_{\dot{p}}\dot{p} + N_{\delta_R}\delta_R + N_{\dot{\delta}_R}\dot{\delta}_R$$

$$+ N_{\ddot{\delta}_R}\ddot{\delta}_R + N_{\delta_A}\delta_A + N_{\dot{\delta}_A}\dot{\delta}_A + N_{\ddot{\delta}_A}\ddot{\delta}_A \quad (33)$$

Assumption VII The flow is assumed to be quasi-steady.

Because of Assumption VII, all derivatives with respect to the rates of change of velocities are omitted, with the exception of those involving $\dot{w}$, which are retained to account for the effect on the horizontal tail of the downwash from the wing. It should also be pointed out that the change in angle of attack can be approximated by $\Delta\alpha = w/U$.

The only restrictions thus far imposed on the orientation of the Eulerian axes with respect to the aircraft that the y axis be a principal axis and that the origin be located at the center of gravity of the aircraft. When the x axis is oriented so that it is a principal axis, the Eulerian axes are referred to as principal axes, but when the x axis in the aircraft is parallel to the relative wind during steady flight, the Eulerian axes are referred to as stability axes. When the airframe is disturbed from the steady flight condition, the Eulerian axes rotate with the airframe and do not change direction with respect to the aircraft. Consequently, the disturbed x axis may or may not be parallel to the relative wind while the aircraft is in the disturbed flight condition. It should be noted that for small angles of attack the moments of inertia about the stability axes are approximately equal to those about the body axes; however, at low speeds (high angles of attack) the moments of inertia about the stability axes can differ significantly from those about the body axes. The use of the stability axes eliminates the terms containing W_o from Equations (33) by eliminating the following quantities : (1) all terms containing W_o, which disappears because of the direction of the stability axes; (2) all aerodynamic partial derivatives with respect to rates of change of velocities, except those with respect to W; and (3) all aerodynamic partial derivatives with respect to rates of change of control surface deflections. Equations (33) then reduce to Equations (34) and (35). To avoid confusion with body axis coordinate systems, where θ_o is the inclination of the x axis with respect to the horizon, the angles between the horizontal and the x stability axis is called γ_o. It will be recognized that because of the way the stability axis system is defined, γ_o is indeed the angle of the flight path with respect to the earth.

$$\dot{u} + g\theta \cos \gamma_o = T_u (\cos \xi)\, u + T_{\delta_{RPM}} \delta_{RPM} \cos \xi + X_u u + X_q q + X_w w$$

$$+ X_{\dot{w}} \dot{w} + X_{\delta_E} \delta_E + X_{\delta_F} \delta_F$$

$$\dot{w} - U_o q + g\theta \sin \gamma_o = - T_u (\sin \xi) u - T_{\delta_{RPM}} \delta_{RPM} \sin \xi + Z_u u$$

$$+ Z_q q + Z_w w + Z_{\dot{w}} \dot{w} + Z_{\delta_E} \delta_E + Z_{\delta_F} \delta_F$$

$$\dot{q} = \frac{z_{jm}}{I_{yy}} T_u u + \frac{z_{jm}}{I_{yy}} T_{\delta_{RPM}} \delta_{RPM} + M_u u + M_q q + M_w w + M_{\dot{w}} \dot{w} + M_{\delta_E} \delta_E + M_{\delta_F} \delta_F \quad (34)$$

$$\dot{v} + U_o r - g\psi \sin \gamma_o - g\phi \cos \gamma_o = Y_r r + Y_v v + Y_p p + Y_{\delta_A} \delta_A + Y_{\delta_R} \delta_R$$

$$\dot{p} - \frac{I_{xz}}{I_{xx}} \dot{r} = L_r r + L_v v + L_p p + L_{\delta_A} \delta_A + L_{\delta_R} \delta_R$$

$$\dot{r} - \frac{I_{xz}}{I_{zz}} \dot{p} = N_r r + N_v v + N_p p + N_{\delta_A} \delta_A + N_{\delta_R} \delta_R \quad (35)$$

An examination of these equations shows that Equations (34) are functions of the variables u, θ, and w; whereas, Equations (35) are functions of the variables v, r, and p. As a result of the assumptions made in this analysis, the equations of motion can be treated as two independent sets of three equations with Equations (34) describing the lingitudinal or x-z plane motions and Equations (35) the lateral motions.

Chapter 3

Longitudinal dynamic Stability

Normally, an aircraft exhibits five characteristic dynamic modes of motion. These are two longitudinal and three lareral-directional modes. The two longitudinal modes are the short period and the phugoid; the three lateral-directional modes are the Dutch roll, the spiral, and the roll mode.

Since this study is confined to the analysis of longitudinal stability, only the motion modes concerning with this analysis, such as the short period and the phugoid modes, will be considered. The short period mode is normally defined as one of short period with relatively heavy damping and is characterized primarily by variations in angle of attack and pitch angle with very little change in forward speed. The phugoid-mode is defined as another of long period with very light damping and is characterized mainly by variations in speed and pitch angle with angle of attack nearly constant.

In specifying the mode of motion, it is necessary to know the associated parameters, such as the period and damping. Since these parameters cannot be obtained directly from the linearized equations of motion; i.e. equations (34) and (35), the further development is needed. In this chapter, the solution required for solving the parameters of longitudinal motion is derived by transforming the linearized equations of motion into the transfer functions and by rearranging these functions into quadratic forms respectively.

Derivation of the transfer functions.

As previously mentioned in chapter 2 that equations (34) describe the longitudinal or x - z plane motions and equations (35) concern the lateral motions. In this study, only the equations (34) will therefore be used and its conversion into the transfer functions can be obtained as follows.

The angle of attack and the angle of sideslip are first written in the approximate form as

$$\Delta\alpha \simeq \frac{W}{U_o} \quad \text{and} \quad \beta \simeq \frac{V}{U_o}$$

The angle between the equilibrium flight path and the disturbed flight path is defined as Ξ. It should be pointed out that only when Ξ equals zero is the sideslip angle β equal to the negative of the yaw angle.

The longitudinal equations can be written by generally rearranging Equations (34), as shown below :

$$\dot{u} - X_u u - (T_u \cos \xi)u - X_q q + g\theta \cos \gamma_o - X_{\dot{w}}\dot{w} - X_w w$$

$$= X_{\delta_E}\delta_E + T_{\delta_{RPM}}\delta_{RPM} + X_{\delta_F}\delta_F - Z_u u + (T_u \sin \xi)\, u - U_o q - Z_q q$$

$$+ g\theta \sin \gamma_o + \dot{w} - Z_{\dot{w}}\dot{w} - Z_w w = Z_{\delta_E}\delta_E - (T_{\delta_{RPM}} \sin \xi)\delta_{RPM} + Z_{\delta_F}\delta_F$$

$$- M_u u - \frac{z_j m}{I_{yy}} T_u u + \dot{q} - M_q q - M_{\dot{w}}\dot{w} - M_w w = M_{\delta_E}\delta_E + \frac{z_j m}{I_{yy}} T_{\delta_{RPM}}\delta_{RPM} \qquad (36)$$

The right sides of above equations are the control forces and represent the means by which either the human pilot or an autopilot can control the motion of the airframe. The thrust and the control surface inputs are the forcing functions which determine the resultant motion of the airframe. Since the airframe equations of motion are linear equations, the principle of superposition may be used to obtain a solution. For instance, the response to simultaneous application of elevator and rudder deflections can be determined by calculating the response to each of these deflections seperately and then adding the results to complete the solution.

The transfer functions are obtained by applying the method of laplace transforms. If equations (36) are transformed into the laplacian domain, they become :

$$[s - (X_u + A')]\, u(s) - (sX_{\dot{w}} + X_w)\, w(s) - (sX_q - g\cos\gamma_o)\theta(s)$$

$$= X_{\delta_E}\delta_E(s) + B'\delta_{RPM}(s) + X_{\delta_F}\delta_F(s)$$

$$- (Z_u - C')u(s) + [s(1 - Z_{\dot{w}}) - Z_w]\, w(s) - [s(U_o + Z_q)$$

$$- g\sin\gamma_o]\, \theta(s) = Z_{\delta_E}\delta_E(s) - D'\delta_{RPM}(s) + Z_{\delta_F}\delta_F(s)$$

$$- (M_u + E')u(s) - (sM_{\dot{w}} + M_w)w(s) + (S^2 - M_q s)\theta(s)$$

$$= M_{\delta_E}\delta_E(s) + F'\delta_{RPM}(s) + M_{\delta_F}\delta_F(s) \qquad (37)$$

where

$$A' = T_u \cos\xi$$

$$B' = T_{\delta_{RPM}} \cos\xi$$

$$C' = T_u \sin\xi$$

$$D' = T_{\delta_{RPM}} \sin \xi$$

$$E' = \frac{z_j m}{I_{yy}} T_u$$

$$F' = \frac{z_j m}{I_{yy}} T_{\delta_{RPM}}$$

It should be noted that p, q, and r were replaced by $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$ respectively. This is permitted because of the small perturbation approximation. Using Equations (37) and Cramer's rule for solving equations by determinants, the longitudinal transfer function for $U(s)/\delta_E(s)$ with $\delta_F = 0$ and $\delta_{RPM} = 0$ can be written as

$$\frac{u(s)}{\delta_E(s)} = \frac{\begin{vmatrix} X_{\delta_E} & -(sX_{\dot{w}} + X_w) & -(sX_q - g\cos\gamma_o) \\ Z_{\delta_E} & [s(1 - Z_{\dot{w}}) - Z_w] & -[s(U_o + Z_q) - g\sin\gamma_o] \\ M_{\delta_E} & -(sM_{\dot{w}} + M_w) & (s^2 - M_q s) \end{vmatrix}}{\begin{vmatrix} [s - (X_u + A')] & -(sX_{\dot{w}} + X_w) & -(sX_q - g\cos\gamma_o) \\ -(Z_u - C') & [s(1 - Z_{\dot{w}}) - Z_w] & -[s(U_o + Z_q) - g\sin\gamma_o] \\ -(M_u + E') & -(sM_{\dot{w}} + M_w) & (s^2 - M_q s) \end{vmatrix}}$$

$$= \frac{\text{Numerator}}{\text{Denominator}} \qquad (38)$$

The denominator determinant is the system characteristic equation denoted by D_1. The expansion of D_1 gives :

$$D_1 = As^4 + Bs^3 + Cs^2 + Ds + E \tag{39}$$

where

$$A = 1 - Z_{\dot{w}}$$

$$B = -(1 - Z_{\dot{w}})\left[(X_u + A') + M_q\right] - Z_w - M_{\dot{w}}(U_o + Z_q) - X_{\dot{w}}(Z_u - C')$$

$$C = (X_u + A')\left[M_q(1 - Z_{\dot{w}}) + Z_w + M_{\dot{w}}(U_o + Z_q)\right] - (M_u + E')\left[X_{\dot{w}}(U_o + Z_q) + X_q(1 - Z_{\dot{w}})\right] + M_q Z_w + (Z_u - C')\left[M_q X_{\dot{w}} - X_w) - X_q M_{\dot{w}}\right] + M_{\dot{w}}\, g \sin\gamma_o - M_w (U_o + Z_q)$$

$$D = g \sin\gamma_o\left[(M_u + E) X_{\dot{w}} + M_w - M_{\dot{w}}(X_u + A')\right] + g \cos\gamma_o\left[(Z_u - C')M_{\dot{w}} + (M_u + E')(1 - Z_{\dot{w}})\right] + (M_u + E')\left[- X_w(U_o + Z_q) + Z_w X_q\right] + (Z_u - C')\left[X_w M_q - X_q M_w\right] + (X_u + A')\left[M_w(U_o + Z_q) - M_q Z_w\right]$$

$$E = g \cos\gamma_o\left[M_w(Z_u - C') - Z_w(M_u + E')\right] + g \sin\gamma_o\left[(M_u + E') X_w - (X_u + A') M_w\right]$$

The numerator determinant is also expanded as

$$N_u/\delta_E = A_u s^3 + B_u s^2 + C_u s + D_u \tag{40}$$

where

$$A_u = X_{\delta_E}(1 - Z_{\dot{w}}) + Z_{\delta_E} X_{\dot{w}}$$

$$B_u = -X_{\delta_E}\left[(1 - Z_{\dot{w}})M_q + Z_w + M_{\dot{w}}(U_o + Z_q)\right]$$

$$+ Z_{\delta_E}\left[X_q M_{\dot{w}} - X_{\dot{w}} M_q + X_w\right]$$

$$+ M_{\delta_E}\left[X_{\dot{w}}(U_o + Z_q) + (1 - Z_{\dot{w}})X_q\right]$$

$$C_u = X_{\delta_E}\left[M_q Z_w + M_{\dot{w}}\, g \sin\gamma_o - M_w (U_o + Z_q)\right]$$

$$+ Z_{\delta_E}\left[X_q M_w - M_{\dot{w}}\, g\cos\gamma_o - X_w M_q\right]$$

$$+ M_{\delta_E}\left[-X_{\dot{w}}\, g \sin\gamma_o + X_w (U_o + Z_q) - (1 - Z_{\dot{w}})\, g\cos\gamma_o - Z_w X_q\right]$$

$$D_u = X_{\delta_E}(M_w\, g\sin\gamma_o) - Z_{\delta_E}(M_w\, g\cos\gamma_o)$$

$$+ M_{\delta_E}(Z_w\, g\cos\gamma_o - X_w\, g\sin\gamma_o)\ .$$

By factoring higher order equation (39) using Lin's method, the two quadratics can be obtained as

$$D_1 = (s^2 + 2\xi_p \omega_{np} s + \omega_{np}^2)(s^2 + 2\xi_{sp}\omega_{nsp} s + \omega_{nsp}^2) = 0 \qquad (41)$$

where

ξ_p and ξ_{sp} = damping ratios of phugoid and short period respectively,

ω_{np} and ω_{nsp} = natural frequencies of phugoid and short period respectively.

The corresponding roots of equation (39) can be determined by using the quadratic formula. The type of dynamic response can be indicated by these roots as

$$S_{1,2} = Re_1 \pm j\, Im_1 \qquad \text{Phugoid Roots}$$

$$S_{3,4} = Re_2 \pm j\, Im_2 \qquad \text{Short period Roots}$$

where $Im_2 > Im_1$ (42)

Chapter 4

Results and discussion

In this chapter, the response of longitudinal motion of the RTAF-5 is investigated. The investigation is emphasized on an effect of changing the value of I_{yy} since it is one of the most important parameters which affect significantly the longitudinal motion. In calculating all the parameters associated to the motion, a computer program, based upon the expressions obtained in previous chapters and upon typically aerodynamic data in references, is developed. This program is written in basic language and is provided data files for storing input data in order that some of which can be conveniently varied (Appendix A). The results of the investigation are presented in graphical forms and are analysed by comparing with the handling qualities requirements of MIL-F-8785B(10).

4.1 Results

Fundamentally, the change in flight conditions and aircraft configurations will affect significantly and slightly the associated parameters of short period and of phugoid respectively. This trend can be shown apparently by the following expressions of short period and phugoid approximations, of which the derivations are presented in Ref. (11),

(a) short period approximation,

$$\xi = f\left[\left(\sqrt{\frac{\rho}{I_{yy}}}\right)\left(\frac{-c_{Mq}}{\sqrt{-c_{M\alpha}}}\right)\right] ,$$

$$\omega_n = f \left[U_o \left(\sqrt{\frac{\rho}{I_{yy}}}\right) \left(\sqrt{- c_{M\alpha}}\right)\right] ,$$

(b) phugoid approximation,

$$\xi = \left(\frac{1}{\sqrt{2}}\right) \left(\frac{C_D}{C_L}\right) , \text{ and}$$

$$\omega_n = 45.5 / U_o$$

It is seen from the above expressions that only the short period damping ratio and natural frequency are inversely proportional to I_{yy}. This means that these short period associated parameters may be directly affected if the I_{yy} is varied when other parameters are assumed to be constant. For the phugoid mode, its damping ratio and natural frequency are not a function of I_{yy}, thus any effect of I_{yy} may not occur. To investigate the effect of changing the value of I_{yy}, let us consider the response of longitudinal motion of the RTAF-5 at various values of I_{yy} ranging from 1000 to 6000 slug-ft^2 during cruise. It should be noted that the selected range is feasible possibility of variation in I_{yy} for this aircraft.

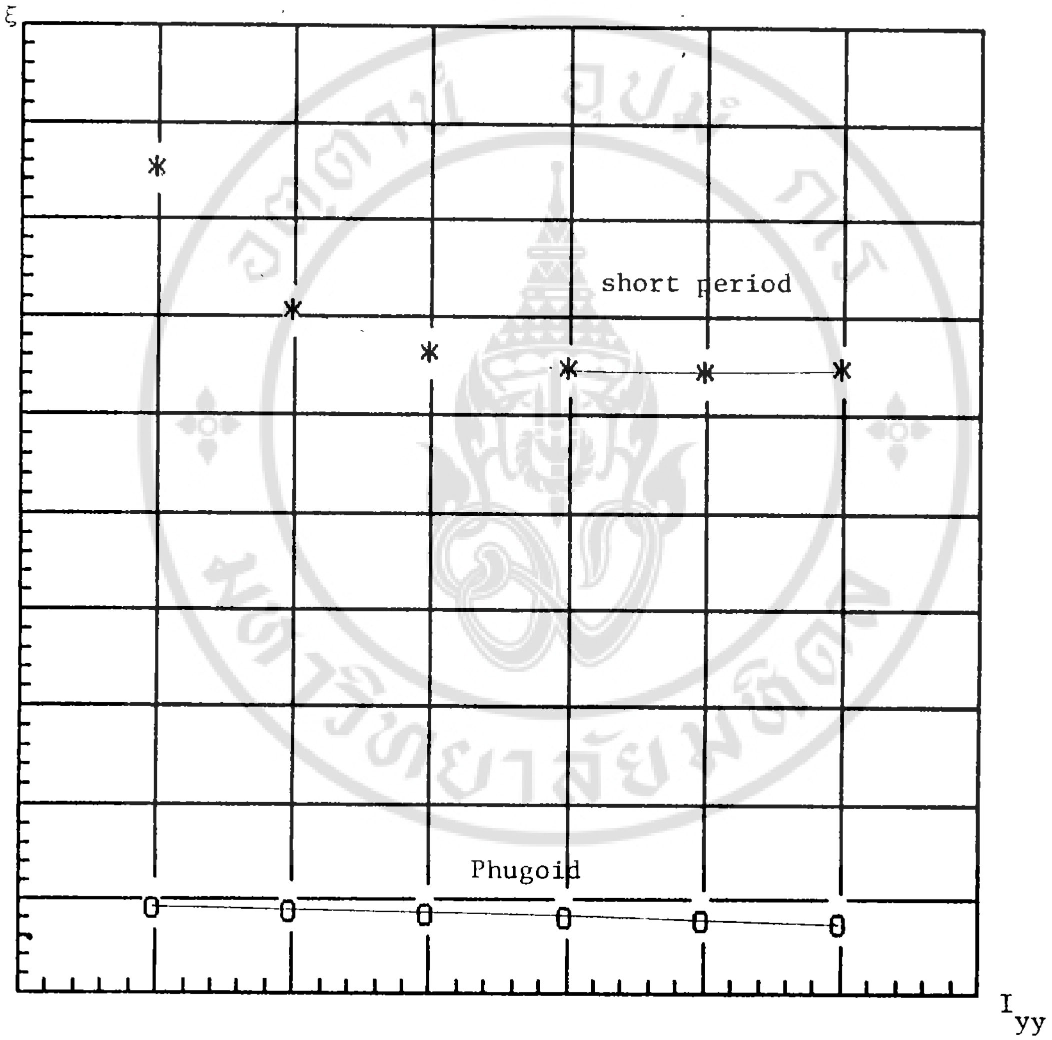

Figure 8 Short period and phugoid damping ratios for I_{yy} variation.

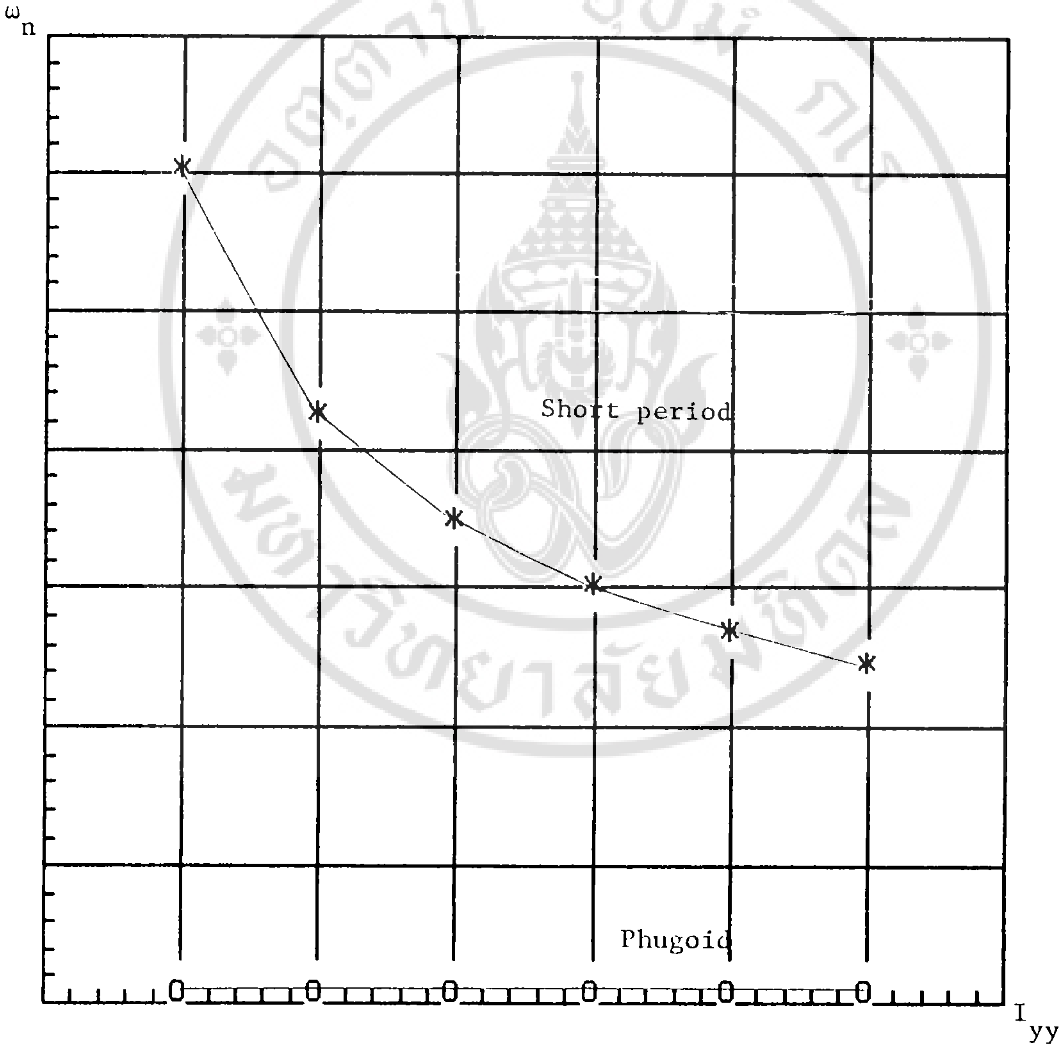

Figure 9 Short period and phugoid natural frequencies for I_{yy} variation.

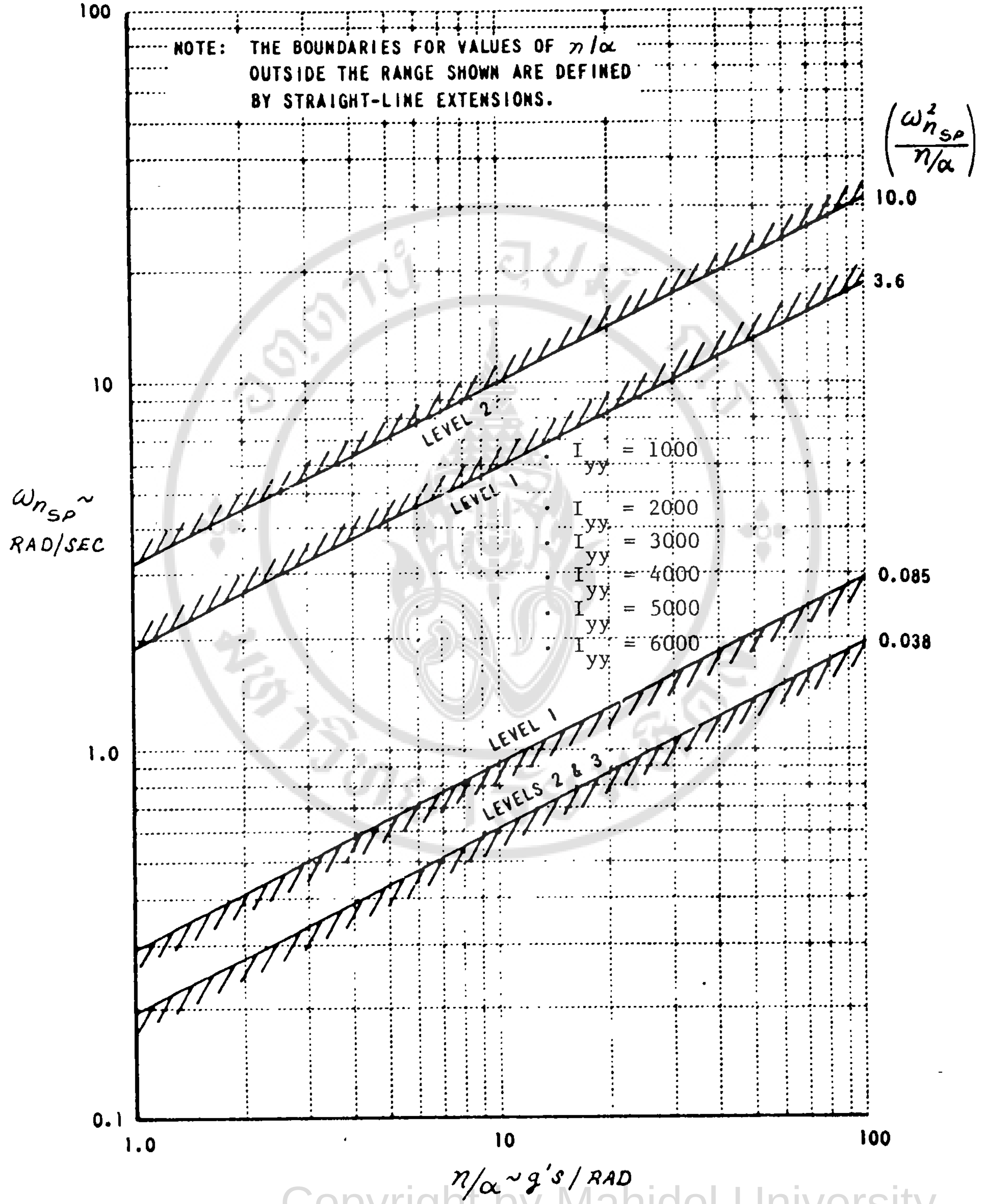

Figure 10 Comparison between calculated frequency and required frequency of short period mode-category B flight phases.

Table 2 comparison between calculated and required damping ratios of short period mode-category B Flight Phases.

I_{yy} (slug-ft^2)	$\xi_{cal.}$	$\xi_{req.}$ (10)
1000	0.853612	
2000	0.707077	
3000	0.661869	0.3 - 2.00
4000	0.646445	
5000	0.643733	
6000	0.647478	

Table 3 comparison between calculated and required damping ratios of phugoid-flying qualities level 1.

I_{yy} (Slug - ft^2)	$\xi_{cal.}$	$\xi_{req.}$ (10)
1000	0.0972027	
2000	0.0935936	
3000	0.0899827	at least 0.04
4000	0.0863687	
5000	0.0827562	
6000	0.0791423	

4.2 Discussion.

It is apparent from fig 8 and 9 that the damping ratios and natural frequencies of each mode of motion, especially short period, decrease as I_{yy} value increases. The decrease of these associated parameters will result more stable to the response of longitudinal motion of the RTAF-5 during its criuse condition. However, because the investigation of this response is conducted at the given range of I_{yy} values, it is therefore necessary to make further determination to check whether the response tentatively meets the handling qualities requirements of MIL-F-8785 B or not. It may be noted that these requirements are established in the military specification flying qualities of pilot airplane, which forms one of the bases for the determination by the procuring activity of airplane acceptability and serves as design requirements and as criteria for use in stability and control calculations, analysis of wind-tunnel test results, flying qualities simulation test, and flight testing and evaluation.

According to some excepts of MIL-F-8785 B presented in Appendix D, the classification of the RTAF-5 and its flight condition can be grouped into class I primary trainer and category B flight phase. In addition to the classification, the level of flying qualities desired in this investigation is level 1 which is defined as flying qualities clearly adequate for the mission flight phase.

From tables 2 and 3 , it is seen that all the calculated damping ratios of each mode are within the limits of damping ratios established in MIL-F-8785 B. It is also evident from fig. 10 that the short period natural frequency at any I_{yy} value locates inside the limits of requirement. Hence, the response of longitudinal motion of the RTAF-5 at the possibly selected range of I_{yy} values satisfies all the requirements of MIL-F-8785 B.

Chapter 5

Conclusions and Recommendations

In this study the equations of longitudinal symmetric motion were derived and the equations could be reduced to a set of linear differential equations with constant coefficients, provided that the disturbances were small. There are two main uses of these equations :

(i) to determine whether the aircraft is stable for small disturbances, and

(ii) to determine the motion (or response) of the aircraft following some small disturbances.

The computer program named "LOPROG" which was written in BASIC language had been used to investigated the stability, damping and frequency of the various modes of RTAF-5 aircraft's motion. The RTAF-5 aircraft configuration and flight condition datas were input to the program and the output were compared with the MIL-F-8785 B. Owing to insufficient of technical data available from the manufacturer such as wind tunnel data, weight and inertia, etc.; so the estinated data were used in this study. It has been shown that for the possible range of the pitch inertia, I_{yy}, the RTAF-5 during cruise condition is longitudinally stable. The natural frequency and damping ratio of the short period and phugoid modes were adequate and quite satisfactory for the flying qualities level I requirement.

It is recommended that the dynamic motion of thr RTAF-5 should be investigated again with the more accurate input data for the computer program. For those who will use the LOPROG program to investigate the longitudinal dynamic stability of the aircraft, care should be taken for the input data and for more accurate results the stability derivatives which are collected from wind tunnel testing should be used instead of those computed from the program itself. Finally, it is hopefully that this study would be useful enough for the aeronautical engineers to investigate the aircraft longitudinal stability during the design phase.

Appendix A

LOPROG computer program

LOPROG is a mechanization of the methods for computing longitudinal stability derivatives and for substituting these values into longitudinal transfer functions, finding the roots of the characteristic equation, and solving for the associated parameters of short period and phugoid modes. LOPROG provides data files for storing input data which are entered from computer keyboard. If the data file has been already existed, the input data in this file could be called into the program or they could be changed their values by using the associated program named CORFIL (Appendix B). The variables used in LOPROG as well as in CORFIL are defined in the following section.

The output produced by LOPROG for the RTAF-5 at some sample value of I_{yy} is shown in Appendix C. Listed first are the pertinent aircraft characteristics, most of which are entered in. This should be consulted to check that the desired input data actually entered the program. Next are the calculated dimensionless stability derivative values and the dimensional stability derivative values. Then the coefficients of the denominator polynomial of the transfer functions are displayed. Finally, the roots of the denominator polynomial and the associated parameters of short period and phugoid modes are written.

```
LIST
10 REM PROGRAM LOPROG
20 DIM NUS(5),NAS(5),NTHS(5),DS(6),ROOTR(10),ROOTI(10),C(5),KC(4),OUF1(3)
30 DIM OUF2(3),RR1(2),RR2(2),RI1(2),RI2(2),CC(4),RR(3),COEFI(6),RI(3),T2(5)
40 DIM XI(3),COF(3),RE(2),RIM(2),ROOT(1),COEF(2),KKK(21),NWS(5),T1(14)
50 PRINT "*****************************************************************"
60 PRINT "*                                                               *"
70 PRINT "*              AIRPLANE LONGITUDINAL STABILITY                  *"
80 PRINT "*                                                               *"
90 PRINT "*                                                               *"
100 PRINT "*****************************************************************"
110 PRINT
120 PRINT : PRINT "ENTER NAME OF A/C MODEL "
130 A$=INKEY$
140 INPUT A$
150 C$=INKEY$
160 C$=A$+".LOP"
170 PRINT
180 PRINT "IS THIS YOUR NEW MODEL ? (Y/N)" : B$=INKEY$ : INPUT B$
190 IF B$ = "Y"  THEN 240
200 ON ERROR GOTO 3760
210 OPEN C$ FOR INPUT AS #1
220 INPUT #1,RHO,U,MS,IYY,THRUST,ZJ,GCOSGM,GSINGM,COSXZ,SINXZ,S,ST,B,BT,EFF,CHE,
TR,TRT,CL2DW,ITAIL,CLA2DW,CLA2DT,CDPIE,CDA2DW,ALPHA,IWING,LT,LT1,XA,ZA,LB,WFUS,X
FUS,XCG,ZT,SELEV
230 GOTO 430
240 OPEN C$ FOR OUTPUT AS #1
250 PRINT "NOTE :- SEE USER MANUAL FOR DEFINITION OF THE FOLLOWING VARIABLES"
260 PRINT : PRINT "RHO,U,MS,IYY,THRUST,ZJ = "; : INPUT RHO,U,MS,IYY,THRUST,ZJ
270 PRINT #1,RHO;U;MS;IYY;THRUST;ZJ
280 PRINT : PRINT "GCOSGM,GSINGM,COSXZ,SINXZ,S,ST = ";
290 INPUT GCOSGM,GSINGM,COSXZ,SINXZ,S,ST
300 PRINT #1,GCOSGM;GSINGM;COSXZ;SINXZ;S;ST
310 PRINT : PRINT "B,BT,EFF,CHE,TR,TRT = "; : INPUT B,BT,EFF,CHE,TR,TRT
320 PRINT #1,B,BT;EFF;CHE;TR;TRT
330 PRINT : PRINT "CL2DW,ITAIL,CLA2DW,CLA2DT,CDPIE,CDA2DW - ";
340 INPUT CL2DW,ITAIL,CLA2DW,CLA2DT,CDPIE,CDA2DW
350 PRINT #1,CL2DW;ITAIL;CLA2DW;CLA2DT;CDPIE;CDA2DW
360 PRINT : PRINT "ALPHA,IWING,LT,LT1,XA,ZA - ";
370 INPUT ALPHA,IWING,LT,LT1,XA,ZA
380 PRINT #1,ALPHA;IWING;LT;LT1;XA;ZA
390 PRINT : PRINT "LB,WFUS,XFUS,XCG,ZT,SELEV - ";
400 INPUT LB,WFUS,XFUS,XCG,ZT,SELEV
410 PRINT #1,LB;WFUS;XFUS;XCG;ZT;SELEV
420 CLOSE #1
430 CLS
440 AR=B^2/S
```

```
460 CH=S/B
470 CHT=ST/BT
480 IF TR > 0 OR TR < 1 THEN 500
490 PRINT" ***** TR = ";TR;" IS OUTSIDE DESIREABLE RANGE OF 0.0 TO 1.0 WHEN CALC
ULATING TAU OR DELTA *****"
500 IF (AR < 3 AND TR = 1) OR (AR > 12 AND TR = 1) THEN PRINT " ***** AR = ";AR;
" IS OUTSIDE DESIRABLE RANGE OF 3.0 TO 12.0 FOR TR=1.0 WHEN CALCULATING TAU1 OR
DELTA1 ******"
510 IF TR=1 THEN 550
520 TAU=6526*TR^13-33936.5*TR^12+65708.3*TR^11-40275.9*TR^10-55022.8*TR^9+134075
!*TR^8-128452!*TR^7+71152.4*TR^6-24169.6*TR^5+4910.63*TR^4-541.19*TR^3+27.1966*T
R^2-1.31155*TR+.170212
530 E1=1/(1+TAU)
540 GOTO 570
550 TAU1=2.97115E-05*AR^4-8.11747E-04*AR^3+.0071717*AR^2-2.98853E-03*AR+1.07739
560 E1=1/TAU1
570 CLAW=(57.3*CLA2DW)/(1+CLA2DW*57.3/(3.1416*E1*AR))
580 IF TRT > 0 OR TRT < 1 THEN 600
590 PRINT" ***** TRT = ";TRT;" IS OUTSIDE DESIRABLE RANGE OF 0.0 TO 1.0 WHEN CAL
CULATING TAUT *****"
600 IF (ART < 3 AND TRT = 1) OR (ART > 12 AND TRT = 1 ) THEN PRINT " ***** ART =
 ";ART;" IS OUTSIDE DESIRABLE RANGE OF 3.0 TO 12.0 FOR TRT = 1.0 WHEN CALCULATIN
G TAU1T *****"
610 TAUT=6526*TRT^13-33936.5*TRT^12+65708.3*TRT^11-40275.9*TRT^10-55022.8*TRT^9+
134075!*TRT^8-128452!*TRT^7+71152.4*TRT^6-24169.6*TRT^5+4910.63*TRT^4-541.19*TRT
^3+27.1966*TRT^2-1.31155*TRT+.170212
620 E1T=1/(1+TAUT)
630 GOTO 660
640 TAU1T=2.97115E-05*ART^4-8.11747E-04*ART^3+.0071717*ART^2-2.98853E-03*ART+1.0
7739
650 E1T=1/TAU1T
660 IF TR = 1 THEN 700
670 DELTA=2.48637*TR^6-9.29906*TR^5+14.0692*TR^4-11.1199*TR^3+5.01493*TR^2-1.242
62*TR+.141122
680 E=1/(1+DELTA)
690 GOTO 720
700 DELTA1=-1.43113E-05*AR^3+1.56924E-05*AR^2+.0113969*AR+.988661
710 E=1/DELTA1
720 CLW=CL2DW/(1+2/AR)
730 DW=20*CLW*(((1/TR)^.3)/(AR^.725))*(3*CH/LT1)^.25
740 ALPHAT=ALPHA-IWING+ITAIL-DW
750 SRATIO=SELEV/ST
760 IF SRATIO > 0 OR SRATIO < .7 THEN 780
770 PRINT" ***** SRATIO = ";SRATIO;" IS OUTSIDE DESIRABLE RANGE OF 0.0 TO 0.7 WH
EN CALCULATING DADDE ***** "
780 DADDE=21.7549*SRATIO^5-46.4744*SRATIO^4+36.9347*SRATIO^3-14.259*SRATIO^2+3.7
0551*SRATIO-5.7815E-05
790 IF SRATIO > .7 THEN DADDE=SRATIO
800 CLAT=(57.3*CLA2DT)/(1+CLA2DT*57.3/(3.1416*E1T*ART))
810 CLT=CLW*XA*S/(LT*ST*EFF)
820 ALPHAR=CLT/(CLAT/57.3)
830 DALPHA=ALPHAR-ALPHAT
840 DELTAE=DALPHA/DADDE
850 CL=CLW+CLT*(ST/S)*EFF
860 CD=CDPIE+CL^2/(3.1416*E*AR)
870 CM=(THRUST/(.5*RHO*U^2*S))*(ZT/CH)
880 CT=THRUST/(.5*RHO*U^2*S)
890 CLAW=(57.3*CLA2DW)/(1+CLA2DW*57.3/(3.1416*E1*AR))
900 CLAT=(57.3*CLA2DT)/(1+CLA2DT*57.3/(3.1416*E1T*ART))
910 DEDA=20*(CLAW/57.3)*(((1/TR)^.3)/AR^.725)*(3*CH/LT1)^.25
920 CLA=CLAW
930 CDA=CDA2D+2*CL*CLA/(3.1416*E*AR)
940 CMAW=CLA*((1+(2*CL/(3.1416*E*AR))*((ALPHA-IWING)/57.3)+CD/CLA)*(XA/CH)+(2*CL
/(3.1416*E*AR)-(ALPHA-IWING)/57.3-CL/CLA)*(ZA/CH))
950 CMAT=CLAT*(1-DEDA)*(ST/S)*(LT/CH)*EFF
```

```
960 XLB=XF83/LB
970 IF XLB > .08 OR XLB < .65 THEN 990
980 PRINT" ***** XLB = ";XLB;" IS OUTSIDE DESIRABLE RANGE OF 0.1 TO 0.65 WHEN CA
LCULATING KF ***** "
990 KF=-19.6558*XLB^4+50.3465*XLB^3-26.5479*XLB^2+7.47299*XLB-.464064
1000 CMAFUS=KF*WFUS*WFUS*LB/(S*CH)
1010 CMA=CMAW+CMAFUS-CMAT
1020 CLDA=2*CLAT*DEDA*(LT1/CH)*(ST/S)*EFF
1030 CDDA=0
1040 CMDA=-2*CLAT*DEDA*(LT/CH)*(LT1/CH)*(ST/S)*EFF
1050 CLQ=2*(XCG/CH)*CLA+2*(LT/CH)*CLAT*(ST/S)*EFF
1060 CDQ=0
1070 CMQ=-2*(XCG/CH^2)*DABS(XCG)*CLA-2*(LT/CH)^2*CLAT*(ST/S)*EFF
1080 CHECHT=CHE/CHT
1090 IF CHECHT > 0 OR CHECHT < 1 THEN 1110
1100 PRINT" ***** CHE/CHT = ";CHECHT;" IS OUTSIDE DESIRABLE RANGE OF 0.0 TO 1.0
WHEN CALCULATING CLDE2D ***** "
1110 CLDE2D=-.0580767*(CHE/CHT)^4+.146101*(CHE/CHT)^3-.169823*(CHE/CHT)^2+.19408
4*(CHE/CHT)-2.02833E-03
1120 IF ART > 0 OR ART < 10 THEN 1140
1130 PRINT" ***** ART = ";ART;" IS OUTSIDE DESIRABLE RANGE OF 0.0 TO 10.0 WHEN C
ALCULATING CLDE ***** "
1140 AD2D=4.65842*(CHE/CHT)^4-10.9873*(CHE/CHT)^3+9.47521*(CHE/CHT)^2-4.09969*(C
HE/CHT)-.0432959
1150 IF AD2D > = -.1 THEN 1270
1160 IF AD2D < -.1 AND AD2D > = -.2 THEN 1290
1170 IF AD2D < -.2 AND AD2D > = -.3 THEN 1330
1180 IF AD2D < -.3 AND AD2D > = -.4 THEN 1370
1190 IF AD2D < -.4 AND AD2D > = -.5 THEN 1410
1200 IF AD2D < -.5 AND AD2D > = -.6 THEN 1450
1210 IF AD2D < -.6 AND AD2D > = -.7 THEN 1490
1220 IF AD2D < -.7 AND AD2D > = -.8 THEN 1530
1230 AD1=2.55224E-05*ART^4-8.0707E-04*ART^3+9.35254E-03*ART^2-.0487466*ART+1.120
06
1240 AD2=1
1250 AD=AD1+(AD2-AD1)*(AD2D-(-.8))/(-.2)
1260 GOTO 1560
1270 AD=-5.80764E-05*ART^5+2.03746E-03*ART^4-.0284903*ART^3+.203783*ART^2-.80271
5*ART+2.77199
1280 GOTO 1560
1290 AD1=-5.80764E-05*ART^5+2.03746E-03*ART^4-.0284903*ART^3+.203783*ART^2-.8027
15*ART+2.77199
1300 AD2=-8.58209E-04*ART^3+.0224724*ART^2-.206709*ART+1.81719
1310 AD=AD1+(AD2-AD1)*(AD2D-(-.1))/(-.1)
1320 GOTO 1560
1330 AD1=-8.58209E-04*ART^3+.0224724*ART^2-.206709*ART+1.81719
1340 AD2=1.32994E-04*ART^4-3.94386E-03*ART^3+.0450323*ART^2-.249244*ART+1.69816
1350 AD=AD1+(AD2-AD1)*(AD2D-(-.2))/(-.1)
1360 GOTO 1560
1370 AD1=1.32994E-04*ART^4-3.94386E-03*ART^3+.0450323*ART^2-.245244*ART+1.69816
1380 AD2=-7.24351E-06*ART^6+2.35239E-04*ART^5-2.85365E-03*ART^4+.0150882*ART^3-.
0214656*ART^2-.108685*ART+1.47429
1390 AD=AD1+(AD2-AD1)*(AD2D-(-.3))/(-.1)
1400 GOTO 1560
1410 AD1=-7.24351E-06*ART^6+2.35239E-04*ART^5-2.85365E-03*ART^4+.0150882*ART^3-.
0214656*ART^2-.108685*ART+1.47429
1420 AD2=-1.1681E-05*ART^5+4.30597E-04*ART^4-6.21814E-03*ART^3+.0456723*ART^2-.1
64917*ART+1.41226
1430 AD=AD1+(AD2-AD1)*(AD2D-(-.4))/(-.1)
1440 GOTO 1560
1450 AD1=-1.1681E-05*ART^5+4.30597E-04*ART^4-6.21814E-03*ART^3+.0456723*ART^2-.1
64917*ART+1.41226
1460 AD2=-7.3708E-06*ART^5+3.07952E-04*ART^4-4.89224E-03*ART^3+.0380384*ART^2-.1
54526*ART+1.32116
1470 AD=AD1+(AD2-AD1)*(AD2D-(-.5))/(-.1)
1480 GOTO 1560
```

```
1490 AD1=-7.3708E-06*ART^5+3.07952E-04*ART^4-4.89224E-03*ART^3+.0380384*ART^2-.1
54526*ART+1.32116
1500 AD2=2.43494E-06*ART^4-2.55415E-04*ART^3+.0057768*ART^2-.0487234*ART+1.16569
1510 AD=AD1+(AD2-AD1)*(AD2D-(-.6))/(-.1)
1520 GOTO 1560
1530 AD1=2.43494E-06*ART^4-2.55415E-04*ART^3+.0057768*ART^2-.0487234*ART+1.16569
1540 AD2=2.55244E-05*ART^4-8.0707E-04*ART^3+9.35254E-03*ART^2-.0487466*ART+1.120
06
1550 AD=AD1+(AD2-AD1)*(AD2D-(-.7))/(-.1)
1560 CLDE=CLDE2D*(CLAT/(CLA2DT))*AD*(ST/S)*EFF
1570 CLIN=CLDE
1580 CDDE=0
1590 CDIN=CDDE
1600 CMDE=(-LT/CH)*CLDE
1610 CMIN=CMDE
1620 CLU=0
1630 CDU=0
1640 CMU=0
1650 CTU=0
1660 CTRPM=0
1670 CLS
1680 PRINT"                 PERTINENT AIRPLANE CHARACTERISTICS               "
1690 PRINT"                 ----------------------------------               "
1700 PRINT
1710 PRINT "                 AIRCRAFT MODEL : "; : PRINT A$
1720 PRINT
1730 PRINT "DENSITY (SLUGS/FT**3)              = ";RHO
1740 PRINT "VELOCITY (FT/SEC)                  = "; U
1750 PRINT "MASS (SLUGS)                       = ";MS
1760 PRINT "IYY (SLUG-FT**2)                   = ";IYY
1770 PRINT "THRUST (POUNDS)                    = ";THRUST
1780 PRINT "ZJ (FT)                            = ";ZJ
1790 PRINT "G*COS(GAMMA) (FT/SEC/SEC)          = ";GCOSGM
1800 PRINT "G*SIN(GAMMA) (FT/SEC/SEC)          = ";GSINGM
1810 PRINT "COS(XZ)                            = ";COSXZ
1820 PRINT "SIN(XZ)                            = ";SINXZ
1830 PRINT
1840 PRINT "WING AREA (FT**2)                  = ";S
1850 PRINT "HORZ. TAIL AREA (FT**2)            = ";ST
1860 PRINT "WING SPAN (FT)                     = ";B
1870 PRINT "HORZ. TAIL SPAN (FT)               = ";BT
1880 PRINT "WING CHORD (FT)                    = ";CH
1890 PRINT "HORZ. TAIL CHORD                   = ";CHT
1900 PRINT "WING ASPECT RATIO                  = ";AR
1910 PRINT "HORZ. TAIL ASPECT RATIO            = ";ART
1920 PRINT "WING TAPER RATIO                   = ";TR
1930 PRINT "HORZ. TAIL TAPER RATIO             = ";TRT
1940 PRINT "WING ALPHA (DEGREES)               = ";ALPHA
1950 PRINT "TAIL ALPHA (DEGREES)               = ";ALPHAT
1960 PRINT "IWING (DEGREES)                    = ";IWING
1970 PRINT "ITAIL (DEGREES)                    = ";ITAIL
1980 PRINT "DOWNWASH ANGLE (DEGREES)           = ";DW
1990 PRINT "DOWNWASH/ALPHA                     = ";DEDA
2000 PRINT "ELEVATOR ANGLE (DEGREES)           = ";DELTAE
2010 PRINT "ELEVATOR AREA (FT**2)              = ";SELEV
2020 PRINT "TAIL EFFICIENCY                    = ";EFF
2030 PRINT "ELEVATOR CHORD (FT)                = ";CHE
2040 PRINT "2-D WING CLA                       = ";CLA2DW
2050 PRINT "2-D TAIL CLA                       = ";CLA2DT
2060 PRINT "CDPIE                              = ";CDPIE
2070 PRINT "2-D WING CDA                       = ";CDA2D
2080 PRINT "2-D WING CL                        = ";CL2DW
2090 PRINT
2100 PRINT "                        DISTANCES                    "
2110 PRINT
2120 PRINT "LENGTH OF FUSELAGE                 = ";LB
```

```
2130 PRINT "WIDTH OF FUSELAGE                        = ";WFUS
2140 PRINT "C.G. TO TAIL QUARTER-CHORD (FT)          = ";LT
2150 PRINT "WING TO TAIL QUARTER-CHORD (FT)          = ";LT1
2160 PRINT "C.G. TO WING A.C.(CHORDWISE) (FT)        = ";XA
2170 PRINT "C.G. TO WING A.C.(VERTICAL) (FT)         = ";ZA
2180 PRINT "NOSE TO WING QUARTER-CHORD (FT)          = ";XFUS
2190 PRINT "C.G. TO WING QUARTER-CHORD (FT)          = ";XCG
2200 PRINT "C.G. TO THRUST AXIS (FT)                 = ";ZT
2210 PRINT
2220 PRINT "     LONGITUDINAL STABILITY DERIVATIVES        "
2230 PRINT
2240 PRINT USING "CL   = ###.####          ";CL;  : PRINT USING "CLA  = ###.####
     ";CLA;  : PRINT USING "CLDA  = ###.####          ";CLDA
2250 PRINT USING "CLQ  = ###.####          ";CLQ;  : PRINT USING "CLDE = ###.####
     ";CLDE; : PRINT USING "CLU   = ###.####          ";CLU
2260 PRINT USING "CT   = ###.####          ";CT;   : PRINT USING "CD   = ###.####
     ";CD;   : PRINT USING "CDA   = ###.####          ";CDA
2270 PRINT USING "CDDA = ###.####          ";CDDA;: PRINT USING "CDQ  = ###.####
     ";CDQ;  : PRINT USING "CDDE  = ###.####          ";CDDE
2280 PRINT USING "CDU  = ###.####          ";CDU;  : PRINT USING "CTU  = ###.####
     ";CTU;  : PRINT USING "CM    = ###.####          ";CM
2290 PRINT USING "CMA  = ###.####          ";CMA;  : PRINT USING"CMDA = ###.####
     ";CMDA; : PRINT USING "CMQ   = ###.####          ";CMQ
2300 PRINT USING "CMDE = ###.####          ";CMDE;: PRINT USING"CMU  = ###.####
     ";CMU;  : PRINT USING "CTRPM = ###.####          ";CTRPM
2310 REM ******************************************************
2320 REM CALCULATION OF DIMENSIONAL STABILITY DERIVATIVES
2330 REM ******************************************************
2340 D=RHO*U*S/MS
2350 DC=RHO*U*S*CH/IYY
2360 REM ******************************************************
2370 REM DIMENSIONAL STABILITY DERIVATIVES
2380 REM ******************************************************
2390 XU=D*(-(CDU+CD))
2400 ZU=D*(-(CLU+CL))
2410 MU=DC*(CMU+CM)
2420 TU=D*(CTU+CT)
2430 XW=(D/2)*(CL-CDA)
2440 ZW=(D/2)*(-(CLA+CD))
2450 MW=(DC/2)*CMA
2460 ZDW=-RHO*S*CH*CLDA/(4*MS)
2470 XDW=-RHO*S*CH*CDDA/(4*MS)
2480 MDW=RHO*S*CH*CH*CMDA/(4*IYY)
2490 XQ=-RHO*U*S*CH*CDQ/(4*MS)
2500 ZQ=-RHO*U*S*CH*CLQ/(4*MS)
2510 MQ=DC*CH*CMQ/4
2520 TRPM=30*CH*D*CTRPM
2530 XIN=-D*CDIN*U/2
2540 ZIN=-D*CLIN*U/2
2550 MIN=DC*CMIN*U/2
2560 PRINT
2570 PRINT "            DIMENSIONAL STABILITY DERIVATIVES      "
2580 PRINT
2590 PRINT USING "XU   = ###.####          ";XU;   : PRINT USING "ZU   = ###.####
     ";ZU;   : PRINT USING "MU   = ###.####          ";MU
2600 PRINT USING "TU   = ###.####          ";TU;   : PRINT USING "XW   = ###.####
     ";XW;   : PRINT USING "ZW   = ###.####          ";ZW
2610 PRINT USING "MW   = ###.####          ";MW;   : PRINT USING "XDW  = ###.####
     ";XDW;  : PRINT USING "ZDW  = ###.####          ";ZDW
2620 PRINT USING "MDW  = ###.####          ";MDW;  : PRINT USING "XQ   = ###.####
     ";XQ;   : PRINT USING "ZQ   = ###.####          ";ZQ
2630 PRINT USING "MQ   = ###.####          ";MQ;   : PRINT USING "TRPM = ###.####
     ";TRPM;: PRINT USING "XIN  = ###.####          ";XIN
2640 PRINT USING "ZIN  = ###.####          ";ZIN;  : PRINT USING "MIN  = ###.####
     ";MIN
2650 REM ******************************************************
```

```
2660 REM DENOMINATOR COEFFICIENTS
2670 REM ***********************************************
2680 A1=TU*COSXZ+XU
2690 A2=SINXZ*TU-ZU
2700 A3=ZJ*MS*TU/IYY+MU
2710 A4=1-ZDW
2720 A5=ZQ+U
2730 DS(5)=A4
2740 DS(4)=-A4*(MQ+A1)-ZW-MDW*A5+XDW*A2
2750 DS(3)=A1*(MQ*A4+ZW+MDW*A5)-A3*(XDW*A5+XQ*A4)+MQ*ZW-A2*(MQ*XDW-XW-XQ*MDW)+MD
W*GSINGM-MW*A5
2760 DS(2)=GSINGM*(XDW*A3+MW-MDW*A1)+GCOSGM*(-A2*MDW+A3*A4)+A3*(-XW*A5+ZW*XQ)-A2
*(-XQ*MW+XW*MQ)+A1*(-MQ*ZW+MW*A5)
2770 DS(1)=GCOSGM*(MW*(-A2)-ZW*A3)+GSINGM*(XW*A3-MW*A1)
2780 PRINT
2790 PRINT "  POLYNOMIAL COEFFICIENTS FOR THE DENOMINATOR "
2800 PRINT "  ---------------------------------------- "
2810 FOR I=1 TO 5
2820 PRINT "DS(";6-I;") = "; DS(6-I)
2830 NEXT I
2840 ON ERROR GOTO 3230
2850 REM ROOTS OF POLYNOMIAL ORDER 4
2860 P=DS(4)/DS(5)
2870 Q=DS(3)/DS(5)
2880 R=DS(2)/DS(5)
2890 S-DS(1)/DS(5)
2900 KC(4)=1
2910 KC(3)=-.5*Q
2920 KC(2)=.25*(P*R-4*S)
2930 KC(1)=.125*(4*Q*S-P*P*S-R*R)
2940 HKC=(3*KC(4)*KC(2)-KC(3)*KC(3))/9*KC(4)*KC(4)
2950 GKC=(2*KC(3)*KC(3)*KC(3)-9*KC(4)*KC(3)*KC(2)+27*KC(4)*KC(4)*KC(1))/(27*KC(4
)*KC(4)*KC(4))
2960 RAD=GKC*GKC+4*HKC*HKC*HKC
2970 IF RAD < 0 THEN 3240
2980 UK3=((-GKC+DSQR(GKC*GKC+4*HKC*HKC*HKC))/2)
2990 RTUKC=(DABS(UK3))^.3333333333333334#
3000 UKC=SGN(UK3)*ABS(RTUKC)
3010 VKC=-HKC/UKC
3020 KROOT=UKC+VKC-KC(3)/(3*KC(4))
3030 A=SQR(2*KROOT+P*P*.25-Q)
3040 B=(KROOT*P-R)/(2*A)
3050 QUF1(3)=1
3060 QUF2(3)=1
3070 QUF1(2)=.5*P-A
3080 QUF2(2)=.5*P+A
3090 QUF1(1)=KROOT-B
3100 QUF2(1)=KROOT+B
3110 GOSUB 3300
3120 ROOTR(1)=RR1(1)
3130 ROOTR(2)=RR1(2)
3140 ROOTI(1)=RI1(1)
3150 ROOTI(2)=RI1(2)
3160 GOSUB 3420
3170 ROOTR(3)=RR2(1)
3180 ROOTR(4)=RR2(2)
3190 ROOTI(3)=RI2(1)
3200 ROOTI(4)=RI2(2)
3210 GOSUB 3540
3220 GOSUB 3610
3230 END
3240 THETA1=(-GKC/(2*SQR(-HKC*HKC*HKC)))
3250 THETA2=SQR(1-THETA1*THETA1)
3260 THETA=ATN(THETA2/THETA1)
3270 TH3=THETA/3
3280 KROOT=2*COS(TH3)*SQR(-HKC)-KC(3)/(3*KC(4))
```

```
3290 GOTO 3030
3300 DIS=QUF1(2)*QUF1(2)-4*QUF1(3)*QUF1(1)
3310 IF DIS < 0 THEN 3370
3320 RR1(1)=(-QUF1(2)+SQR(DIS))/(2*QUF1(3))
3330 RR1(2)=(-QUF1(2)-SQR(DIS))/(2*QUF1(3))
3340 RI1(1)=0
3350 RI1(2)=0
3360 GOTO 3410
3370 RR1(1)=-QUF1(2)/(2*QUF1(3))
3380 RR1(2)=RR1(1)
3390 RI1(1)=(-SQR(-DIS))/(2*QUF1(3))
3400 RI1(2)=(SQR(-DIS))/(2*QUF1(3))
3410 RETURN
3420 DIS=QUF2(2)*QUF2(2)-4*QUF2(3)*QUF2(1)
3430 IF DIS < 0 THEN 3490
3440 RR2(1)=(-QUF2(2)+SQR(DIS))/(2*QUF2(3))
3450 RR2(2)=(-QUF2(2)-SQR(DIS))/(2*QUF2(3))
3460 RI2(1)=0
3470 RI2(2)=0
3480 GOTO 3530
3490 RR2(1)=-QUF2(2)/(2*QUF2(3))
3500 RR2(2)=RR2(1)
3510 RI2(1)=(-SQR(-DIS))/(2*QUF2(3))
3520 RI2(2)=(SQR(-DIS))/(2*QUF2(3))
3530 RETURN
3540 PRINT
3550 PRINT"     ROOTS OF POLYNOMIAL       "
3560 PRINT
3570 FOR I=1 TO 4
3580 PRINT "ROOT(";I;") = ";ROOTR(I);" + J ";ROOTI(I)
3590 NEXT I
3600 RETURN
3610 W1=SQR(QUF1(1)) : W2=SQR(QUF2(1))
3620 IF W1 > W2 THEN 3660
3630 WNSP=W2 : WNP=W1
3640 ZNSP=QUF2(2)/(2*WNSP) : ZNP=QUF1(2)/(2*WNP)
3650 GOTO 3680
3660 WNSP=W1 : WNP=W2
3670 ZNSP=QUF1(2)/(2*WNSP) : ZNP=QUF2(2)/(2*WNP)
3680 T12SP=.693147/(ZNSP*WNSP)
3690 T12P=.693147/(ZNP*WNP)
3700 PRINT : PRINT "                         "," SHORT PERIOD";"          ";"PHUGOID"
3710 PRINT
3720 PRINT "NATURAL FREQUENCY (RAD/SEC) = ";WNSP;"            ";WNP
3730 PRINT "DAMPING RATIO               = ";ZNSP;"            ";ZNP
3740 PRINT "TIME FOR 1/2 DAMPING (SEC)  = ";T12SP;"            ";T12P
3750 RETURN
3760 PRINT : PRINT "******* THIS A/C MODEL IS NOT FOUND ******* "
3770 CLOSE #1
3780 GOTO 120
Ok
```

Appendix B

CORFIL computer program

CORFIL is the associated program of LOPROG which is developed for the change in the value of any input data stored in data file, which created by LOPROG, and restoring the new value of specifically changed variable into the same data file. The source listing of CORFIL is shown as follow.

```
LIST
10 REM PROGRAM FOR CHANGING INPUT DATA OF LOPROG PROGRAM
20 REM ***********************************************
30 CLS
40 PRINT : PRINT : PRINT
50 PRINT "*************************"
60 PRINT "ENTER NAME OF DATA FILE "
70 PRINT "*************************"
80 A$=INKEY$
90 INPUT A$
100 A$=A$+".LOP"
110 ON ERROR GOTO 640
120 OPEN A$ FOR INPUT AS#1
130 INPUT #1,RHO,U,MS,IYY,THRUST,ZJ,GCOSGM,GSINGM,COSXZ,SINXZ,S,ST,B,BT,EFF,CHE,
TR,TRT,CL2DW,ITAIL,CLA2DW,CLA2DT,CDPIE,CDA2DW,ALPHA,IWING,LT,LT1,XA,ZA,LB,WFUS,X
FUS,XCG,ZT,SELEV
140 CLOSE #1
150 PRINT"ENTER NAME OF VARIABLE YOU WISH TO CHANGE ITS VALUE "
160 B$=INKEY$
170 INPUT B$
180 PRINT : PRINT"ENTER NEW VALUE FOR ABOVE VARIABLE"
190 IF B$="RHO" THEN INPUT RHO
200 IF B$="U" THEN INPUT U
210 IF B$="MS" THEN INPUT MS
220 IF B$="IYY" THEN INPUT IYY
230 IF B$="THRUST" THEN INPUT THRUST
240 IF B$="ZJ" THEN INPUT ZJ
250 IF B$="GCOSGM" THEN INPUT GCOSGM
260 IF B$="GSINGM" THEN INPUT GSINGM
270 IF B$="COSXZ" THEN INPUT COSXZ
280 IF B$="SINXZ" THEN INPUT SINXZ
290 IF B$="S" THEN INPUT S
300 IF B$="ST" THEN INPUT ST
310 IF B$="B" THEN INPUT B
320 IF B$="BT" THEN INPUT BT
330 IF B$="EFF" THEN INPUT EFF
340 IF B$="CHE" THEN INPUT CHE
350 IF B$="TR" THEN INPUT TR
360 IF B$="TRT" THEN INPUT TRT
370 IF B$="CL2DW" THEN INPUT CL2DW
380 IF B$="ITAIL" THEN INPUT ITAIL
390 IF B$="CLA2DW" THEN INPUT CLA2DW
400 IF B$="CLA2DT" THEN INPUT CLA2DT
```

```
410 IF B$="CDPIE" THEN INPUT CDPIE
420 IF B$="CDA2DW" THEN INPUT CDA2DW
430 IF B$="ALPHA" THEN INPUT ALPHA
440 IF B$="IWING" THEN INPUT IWING
450 IF B$="LT" THEN INPUT LT
460 IF B$="LT1" THEN INPUT LT1
470 IF B$="XA" THEN INPUT XA
480 IF B$="ZA" THEN INPUT ZA
490 IF B$="LB" THEN INPUT LB
500 IF B$="WFUS" THEN INPUT WFUS
510 IF B$="XFUS" THEN INPUT XFUS
520 IF B$="XCG" THEN INPUT XCG
530 IF B$="ZT" THEN INPUT ZT
540 IF B$="SELEV" THEN INPUT SELEV
550 PRINT : PRINT"DONE ! , ANOTHER VARIABLE ? (Y/N)"
560 C$=INKEY$
570 INPUT C$
580 IF C$ < > "Y" THEN 600
590 GOTO 150
600 OPEN A$ FOR OUTPUT AS #1
610 PRINT #1,RHO;U;MS;IYY;THRUST;ZJ;GCOSGM;GSINGM;COSXZ;SINXZ;S;ST;B;BT;EFF;CHE;
TR;TRT;CL2DW;ITAIL;CLA2DW;CLA2DT;CDPIE;CDA2DW;ALPHA;IWING;LT;LT1;XA;ZA;LB;WFUS;X
FUS;XCG;ZT;SELEV
620 CLOSE #1
630 END
640 PRINT : PRINT "******  NO FILE  ****** " : PRINT
650 GOTO 10
Ok
```

Appendix C

Output listing produced by LOPROG

```
               PERTINENT AIRPLANE CHARACTERISTICS
               ----------------------------------

               AIRCRAFT MODEL : RTAF-5

DENSITY (SLUGS/FT**3)                  =   .00205
VELOCITY (FT/SEC)                      =   243.7
MASS (SLUGS)                           =   108.7
IYY (SLUG-FT**2)                       =   3000
THRUST (POUNDS)                        =   0
ZJ (FT)                                =   .85
G*COS(GAMMA) (FT/SEC/SEC)              =   32.2
G*SIN(GAMMA) (FT/SEC/SEC)              =   0
COS(XZ)                                =   .99863
SIN(XZ)                                =   .05234

WING AREA (FT**2)                      =   108
HORZ. TAIL AREA (FT**2)                =   30
WING SPAN (FT)                         =   32.3
HORZ. TAIL SPAN (FT)                   =   10.58
WING CHORD (FT)                        =   5.20124
HORZ. TAIL CHORD                       =   2.83554
WING ASPECT RATIO                      =   6.21006
HORZ. TAIL ASPECT RATIO                =   3.73121
WING TAPER RATIO                       =   .87
HORZ. TAIL TAPER RATIO                 =   1
WING ALPHA (DEGREES)                   =   3
TAIL ALPHA (DEGREES)                   =   .750574
IWING (DEGREES)                        =   3
ITAIL (DEGREES)                        =   2
DOWNWASH ANGLE (DEGREES)               =   1.24943
DOWNWASH/ALPHA                         =   .444846
ELEVATOR ANGLE (DEGREES)               = -.565025
ELEVATOR AREA (FT**2)                  =   13.45
TAIL EFFICIENCY                        =   .9
ELEVATOR CHORD (FT)                    =   3.46
2-D WING CLA                           =   .112
2-D TAIL CLA                           =   .108
CDPIE                                  =   .035
2-D WING CDA                           =   0
2-D WING CL                            =   .296

                    DISTANCES

LENGTH OF FUSELAGE                          =   18.42
WIDTH OF FUSELAGE                           =   3.08
C.G. TO TAIL QUARTER-CHORD (FT)             =   14.99
```

```
C.G. TO WING A.C.(CHORDWISE) (FT) =  .26
C.G. TO WING A.C.(VERTICAL) (FT)  =  .85
NOSE TO WING QUARTER-CHORD (FT)   =  12.56
C.G. TO WING QUARTER-CHORD (FT)   = -.26
C.G. TO THRUST AXIS (FT)          =  .85

     LONGITUDINAL STABILITY DERIVATIVES

CL   =   0.3000      CLA  =   4.5677      CLDA  =   1.4938
CLQ  =   2.8442      CLDE =   0.6820      CLU   =   0.0000
CT   =   0.0000      CD   =   0.0398      CDA   =   0.1457
CDDA =   0.0000      CDQ  =   0.0000      CDDE  =   0.0000
CDU  =   0.0000      CTU  =   0.0000      CM    =   0.0000
CMA  =  -0.5000      CMDA =  -4.3052      CMQ   =  -9.5130
CMDE =  -1.9655      CMU  =   0.0000      CTRPM =   0.0000

          DIMENSIONAL STABILITY DERIVATIVES

XU  =  -0.0307       ZU   =  -0.2316      MU  =   0.0000
TU  =   0.0000       XW   =   0.0596      ZW  =  -1.7788
MW  =  -0.0364       XDW  =   0.0000      ZDW =  -0.0062
MDW =  -0.0033       XQ   =   0.0000      ZQ  =  -2.8556
MQ  =  -1.8000       TRPM =   0.0000      XIN =   0.0000
ZIN = -64.1658       MIN  = -34.8509

   POLYNOMIAL COEFFICIENTS FOR THE DENOMINATOR
   -------------------------------------------
DS( 5 ) =  1.00615
DS( 4 ) =  4.42579
DS( 3 ) =  12.1121
DS( 2 ) =  .417262
DS( 1 ) =  .271338

     ROOTS OF POLYNOMIAL

ROOT( 1 ) = -.0132619  + J -.149961
ROOT( 2 ) = -.0132619  + J  .149961
ROOT( 3 ) = -2.1861  + J -2.6684
ROOT( 4 ) = -2.1861  + J  2.6684

                               SHORT PERIOD          PHUGOID

NATURAL FREQUENCY (RAD/SEC) =   3.44955              .150546
DAMPING RATIO               =   .633735              .0880922
TIME FOR 1/2 DAMPING (SEC)  =   .31707              52.266
Ok
```

Appendix D

D.1 CLASSIFICATION OF AIRPLANES

REQUIREMENT

Classification of airplanes. For the purpose of this specification, an airplane shall be placed in one of the following Classes :

Class I Small, light airplanes such as

- Light utility
- Primary trainer
- Light observation

Class II Medium weight, low-to-medium maneuverability airplanes such as

- Heavy utility/search and rescue
- Light or medium transport/cargo/tanker
- Early warning/electronic countermeasures/airborne command, control, or communications relay
- Antisubmarine
- Assualt transport
- Reconnaissance
- Tactical bomber
- Heavy attack
- Trainer for Class II

Class III Large, heavy, low-to-medium maneuverability airplanes such as

- Heavy transport/cargo/tanker
- Heavy bomber

Patrol/early warning/electronic countermeasures/airborne command, control, or communications relay

Trainer for Class III

Class IV High-maneuverability airplanes such as

Fighter/interceptor

Attack

Tactical reconnaissance

Observation

Trainer for Class IV

The procuring activity will assign an airplane to one of these Classes, and the requirements for that Class shall apply. When no Class is specified in a requirement, the requirement shall apply to all Classes. When operational missions so dictate, an airplane of one Class may be required by the procuring activity to meet selected requirements ordinarily specified for airplanes of another Class.

D.2 FLIGHT PHASE CATEGORIES

REQUIREMENT

Flight Phase Categories. The Flight Phases have been combined into three Categories which are referred to in the requirement statements. These Flight Phases shall be considered in the context of total missions so that there will be no gap between successive Phases of any flight and so that transition will be smooth. When no Flight Phase or Category is stated in a requirement, that requirement shall apply to all three Categories. In certain cases, requirements are directed at specific Flight Phases identified in the requirement. Flight Phases descriptive of most military airplane missions are :

Nonterminal Flight Phases :

Category A - Those nonterminal Flight Phases that require rapid maneuvering, precision tracking, or precise flight-path control. Included in this Category are :

a. Air-to-air combat (CO)

b. Ground attack (GA)

c. Weapon delivery/launch (WD)

d. Aerial recovery (AR)

e. Reconnaissance (RC)

f. In-flight refueling (receiver) (RR)

g. Terrain following (TF)

h. Antisubmarine search (AS)

i. Close formation flying (FF)

Category B - Those nonterminal Flight Phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in this Category are :

a. Climb (CL)

b. Cruise (CR)

c. Loiter (LO)

d. In-flight refueling (tanker) (RT)

e. Descent (D)

f. Emergency descent (ED)

g. Emergency deceleration (DE)

h. Aerial delivery (AD)

Terminal Flight Phases :

Category C - Terminal Flight Phases are normally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this Category are :

a. Takeoff (TO)

b. Catapult takeoff (CT)

c. Approach (PA)

d. Wave-off/go-around (WO)

e. Landing (L)

When necessary, recategorization or addition of Flight Phases or delineation of requirements for special situations, e.g., zoom climbs, will be accomplished by the procuring activity.

D.3 LEVELS OF FLYING QUALITIES

REQUIREMENT

Levels of flying qualities. Where possible, the requirements of Section 3 have been stated in terms of three values of the stability or control parameter being specified. Each value is a minimum condition to meet one of three Levels of acceptability related to the ability to complete the operational missions for which the airplane is designed. The levels are :

Level 1 Flying qualities clearly adequate for the mission Flight Phase

Level 2 Flying qualities adequate to accomplish the mission Flight Phase, but some increase in pilot workload or degradation in mission effectiveness, or both exists

Level 3 Flying qualities such that the airplane can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate, or both. Category A Flight Phases can be terminated safely, and Category B and C Flight Phases can be completed.

D.4 PHUGOID STABILITY

REQUIREMENT

Phugoid stability. The long-period airspeed oscillations which occur when the airplane seeks a stabilized airspeed following a disturbance shall meet the following requirements :

Table Phugoid damping ratio requirements

a.	Level 1	ζ_ρ at least 0.04
b.	Level 2	ζ_ρ at least 0
c.	Level 3	T_2 at least 55 seconds.

D.5 SHORT-PERIOD FREQUENCY AND ACCELERATION SENSITIBITY REQUIREMENT

Short-period frequency and acceleration sensitivity. The short-period undamped natural frequency, ω_{nsp}, shall be within the limits shown in figures 1, 2 and 3. If suitable means of directly controlling normal force are provided, the lower bounds on ω_{nsp} and n/α of Figure 3 may be relaxed if approved by the procuring activity.

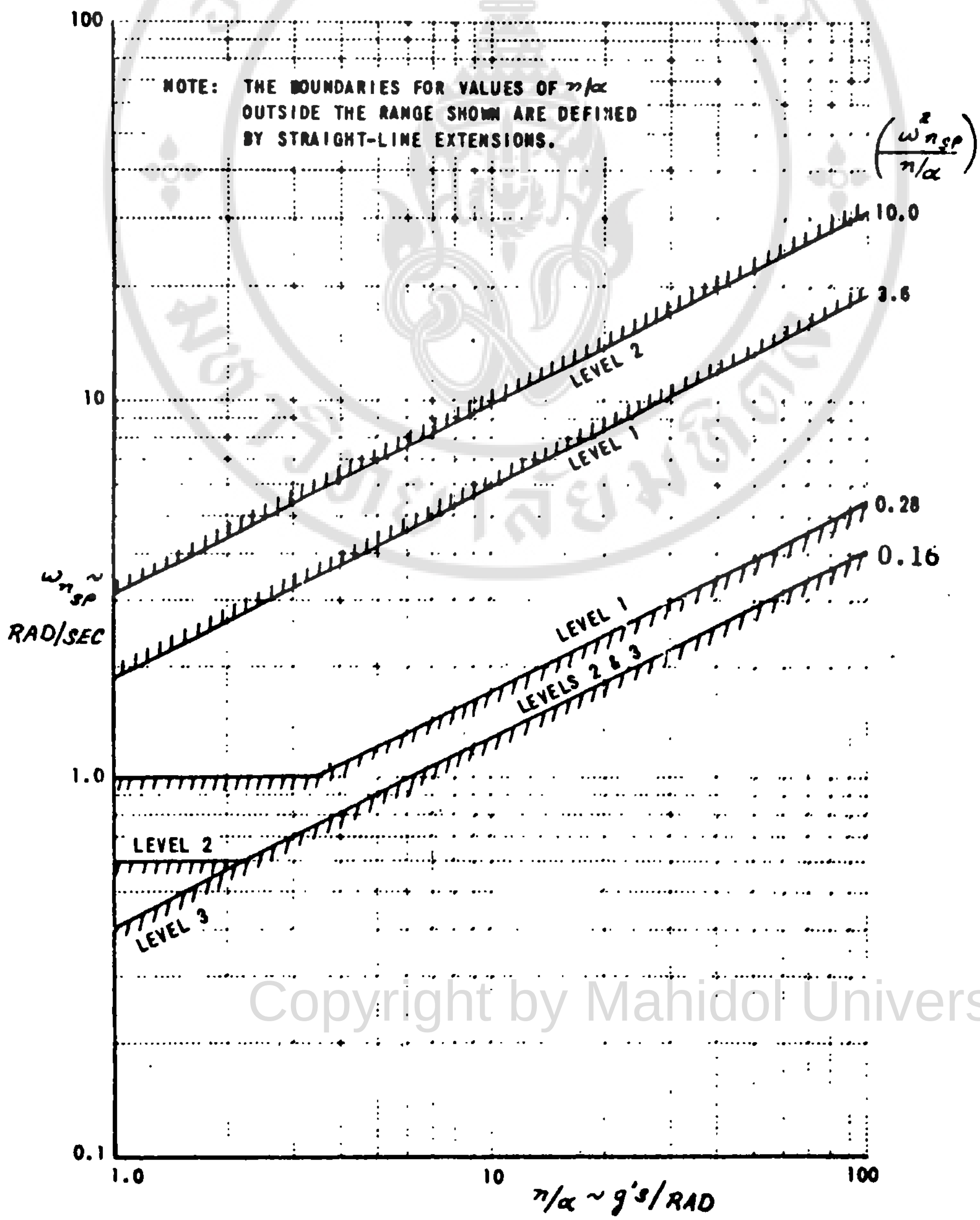

FIGURE 11

SHORT-PERIOD FREQUENCY REQUIREMENTS - CATEGORY A FLIGHT PHASES

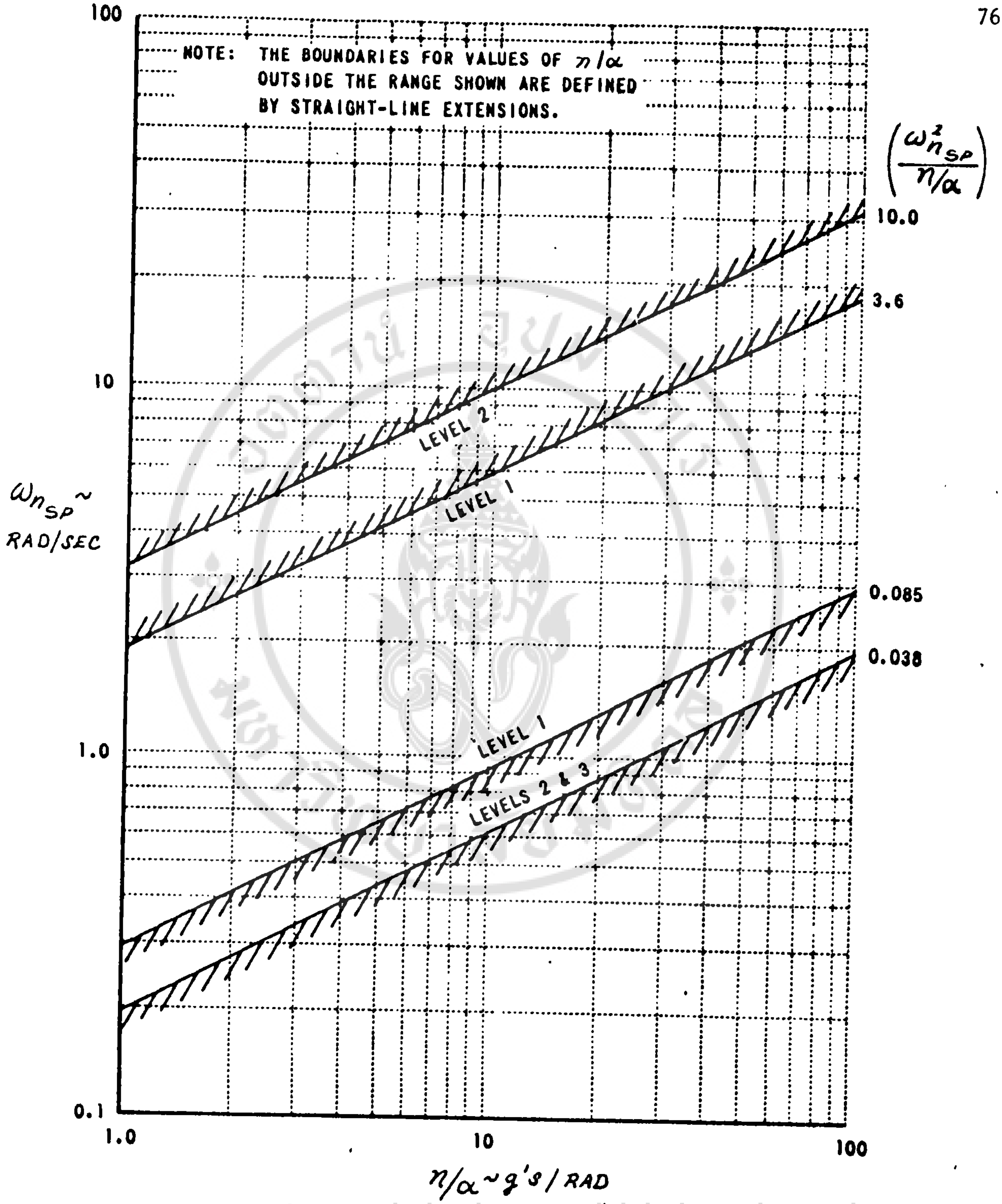

FIGURE 12

SHORT-PERIOD FREQUENCY REQUIREMENTS-CATEGORY B FLIGHT PHASES

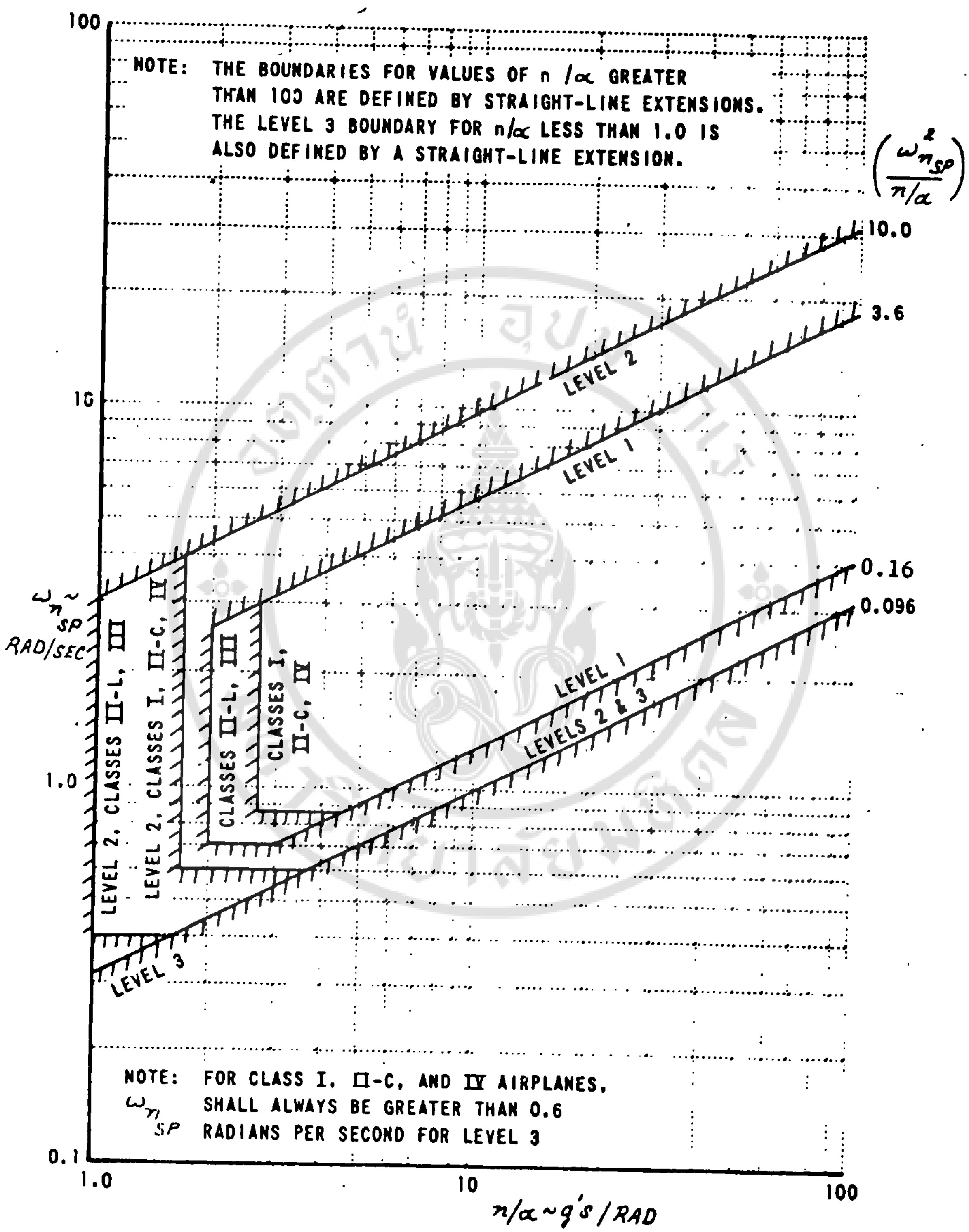

FIGURE 13

SHORT - PERIOD FREQUENCY REQUIREMENTS- CATEGORY C FLIGHT PHASES

D.6 SHORT-PERIOD DAMPING

REQUIREMENT

Short-period damping. The short-period damping ratio, ζ_{SP}, shall be within the limits of table IV.

TABLE IV Short-period Damping Ratio Limits

Level	Category A and C Flight Phases		Category B Flight Phases	
	Minimum	Maximum	Minimum	Maximum
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	0.15*	-	0.15*	-

* May be reduced at altitudes above 20,000 feet if approved by the procuring activity.

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10. Lang, J.D.; Aircraft Performance, stability and control., USAF Academy

11. USAF test pilot school, Flying Qualities theory and Flight test techniques, "Dynamics"., January 1979

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