

**THE FORM OF DENTAL ARCH IN
ANGLE'S CLASSIFICATION II DIVISION 1 IN THAI**

The image features a large, faint watermark of the Mahidol University logo in the background. The logo is circular with a gold border and contains a central emblem with Thai script. The text '1LT. JURAIORN KMOLVISIT' is centered over the logo.

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Thesis
Entitled

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ANGLE'S CLASSIFICATION II DIVISION 1 IN THAI**

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ABSTRACT

The purpose of the present study is to define a generalized equation describing the Thai's dental arch form specified in Angle's classification II division 1 by applying a computer-curve fitting program. The study is comprised of 40 sets (20 males and 20 females) of dental casts expressing Angle's Classification II division 1 malocclusion. In all models, there are 2 measurement methods 1) Research arch form : twenty-two dental landmarks from the mesial and distal points of each incisal edge of incisor, the cusp tips of canines, premolars, and molars 2) Clinical arch form : fourteen landmarks on the bracket position. Each method was triggered and recorded by using the Coordinate Measuring Machine and reported into their corresponding coordinates (X-, and Y- directions). Those coordinates were processed through a computer curve-fitting program to define the parameter of beta function to describe their dental arches.

The Thai, Class II division 1 dental arches were shown to represent accurately mathematic way by the beta function. The coefficient of the determination between the measured arch-shape data and the mathematical arch shape was expressed by the beta function. The average coefficient was 0.97 in the maxillary arch and 0.96 in the mandibular arch. The clinical arch form is more appropriate to be used as a basis for construction of arch wire than the research arch form but it should include the thickness of bracket's base. Class II maxillary arch form was narrower, deeper and more tapered than that of Class I. Class II mandibular arch form was narrower than that of Class I but their arch depths and taperednesses were similar. Thai dental arch forms have a wider-look than that of Caucasians in Braun's study. Gender differences were found, as they reflected more of a size discrepancy than a shape differences.

The beta function has described accurately the Thai Angle's Classification II division 1 dental arch forms. Due to differences of Class I and II's arch form especially in the maxillary arches, further studies should take place about the post treatment arch form of Class II because it will be the evidence base for a treatment plan.

**KEY WORDS: BETA FUNCTION / DENTAL ARCH FORM / COORDINATE
MEASURING MACHINE / CLASS II DIVISION 1**

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แนวความโค้งการเรียงตัวของฟันในคนไทยที่มีการสบฟันชนิดที่ 2 ดิวิชัน 1 (THE FORM OF DENTAL ARCH IN ANGLE'S CLASSIFICATION II DIVISION 1 IN THAI)

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บทคัดย่อ

วัตถุประสงค์ของการศึกษานี้เพื่อค้นหาสมการทั่วไปทางคณิตศาสตร์ที่จะใช้ในการอธิบายถึงแนวความโค้งการเรียงตัวของฟันในขากรรไกรในคนไทยที่มีการสบฟันแบบที่ 2 ดิวิชัน 1 ข้อมูลมีจำนวน 40 ชุด (เพศชาย 20 ชุด, หญิง 20 ชุด) ในแต่ละชุดจะมีวิธีการวัดแนวความโค้งการเรียงตัวของฟัน 2 แบบ คือ 1) ในทางวิจัย, วัดที่จุดยอดฟันกรามใหญ่, ฟันกรามน้อย, ฟันซี่หัว, จุดใกล้กลางและจุดไกลกลางที่ปลายฟันหน้าจำนวน 22 จุด

2) ในทางคลินิก, วัดที่ตำแหน่งที่ติดเบรคเกต จำนวน 14 จุด ในการวัดจะใช้เครื่องวัดพิกัดสามมิติและรายงานผลการวัดเป็นพิกัดความสัมพันธ์ในแนวระนาบแกนเอกซ์และเวีย ข้อมูลจะถูกนำมาใช้เพื่อหาค่าสัมประสิทธิ์ของสมการเบต้าฟังก์ชันเพื่อใช้ในการอธิบายแนวความโค้งของการเรียงตัวของฟัน

ผลการศึกษาพบว่า เบต้าฟังก์ชันเป็นสมการทางคณิตศาสตร์ที่อธิบายถึงแนวความโค้งการเรียงตัวของฟันในคนไทยที่มีการสบฟันแบบที่ 2 ดิวิชัน 1 ได้ถูกต้องแม่นยำสูง แสดงด้วยค่าสัมประสิทธิ์แห่งการกำหนดเฉลี่ย 0.97 ในขากรรไกรบน และ 0.96 ในขากรรไกรล่าง ในการผลิตลวดจัดฟันการใช้แนวความโค้งการเรียงตัวของฟันในทางคลินิกจะเหมาะสมกว่าแต่ควรเพิ่มความหนาของฐานของเบรคเกตเข้าไปด้วยการเปรียบเทียบการสบฟันแบบที่ 1 และ 2 พบว่า ในขากรรไกรบน การสบฟันแบบที่ 2 จะมีแนวโค้งการเรียงตัวของฟันที่แคบกว่า, ลึกกว่าและเรียวกว่า ในขากรรไกรล่างการสบฟันแบบที่ 2 จะมีแนวโค้งการเรียงตัวของฟันที่แคบกว่าส่วนความลึกและความเรียวใกล้เคียงกับการสบฟันแบบที่ 1

นอกจากนี้ยังพบว่าแนวความโค้งการเรียงตัวของฟันในขากรรไกรในคนไทยจะมีลักษณะที่กว้างกว่าคนผิวขาว โดยเปรียบเทียบกับการศึกษาของ Braun S. ไม่พบความแตกต่างของแนวความโค้งการเรียงตัวของฟันในขากรรไกรระหว่างเพศหญิงและชาย แต่พบว่าเพศชายจะมีขนาดของแนวความโค้งการเรียงตัวของฟันที่ใหญ่กว่าเพศหญิง

เบต้าฟังก์ชันเป็นสมการที่จะใช้อธิบายถึงแนวความโค้งการเรียงตัวของฟันในคนไทยที่มีการสบฟันแบบที่ 2 ดิวิชัน 1 ได้ถูกต้องแม่นยำสูง เนื่องจากมีความแตกต่างของแนวความโค้งการเรียงตัวของฟันในการสบฟันแบบที่ 1 และ 2 ดังนั้นในการศึกษารุ่นต่อไปควรศึกษาถึงแนวความโค้งการเรียงตัวของฟันภายหลังการรักษาของการสบฟันแบบที่ 2 ซึ่งจะเป็นพื้นฐานสำคัญในการวางแผนการรักษาของคนไข้ที่มีการสบฟันผิดปกติแบบที่ 2 ดิวิชัน 1

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CHAPTER 1

INTRODUCTION

Although a number of researchers have attempted to identify an arch form unique to a certain group, most of their studies compared average clinical arch forms derived from normal untreated samples or research arch forms established by measuring arch dimensions using the incisal edges and cusp tips as landmarks.

From the standpoint of clinical orthodontics, however, not only does the determination of the patient's posttreatment arch form help meet esthetic requirements but it also benefits for long-term occlusal stability. Based on previous studies on relapse, it is generally agreed that post-orthodontic original mandibular intercanine widths are preserved from the original arch form.

Little (1) based on more than 35 years of research, recommended as a clinical guideline that the patient's pretreatment arch form be used as a guide to posttreatment arch shape. The application of a single ideal arch form to every member of an ethnic group, despite individual variations, may adversely affect posttreatment occlusal stability.

Meanwhile, with the recent advancements in superelastic wire materials and preadjusted appliance systems, preformed arch wires have been commercially available and frequently used, mainly in the leveling and alignment stages. However, their superelastic property makes customization of arch form and sized difficult. After more than 20 years with the preadjusted appliance, it is apparent to the authors that some customizing of the arch form for individual patients is important. In-out is built into the preadjusted appliance, and this eliminates the need for first order bends. It thus simplifies arch form, but it does not eliminate the need to use different shapes for different individuals. Failure to do some customization will create the probability of relapse in some cases, and can lead to unnatural esthetics.

If a broad arch form is used for an individual with a narrow facial appearance, for example, there will be a risk of relapse and an unnatural look to the smile. A return to the concept of customizing arch form for each patient, but without the need for needless wire bending, is a desirable and sensible approach. Clinically, it seems more reasonable to have several types of preform arch wire available and to select the shape that most closely match the patient's arch form according to their ethnicity and type of malocclusion.

For several years, researchers have been trying to define the ideal arch form, frequently using the concept that the dental arch form is similar to the mathematical formulas as well as the catenary curve. Several studies of the shapes of dental arches have received both praise and criticism, and several have formed the basis for commercially produced arch form, but this is more appropriate to Caucasian than Thais.

In several studies, it also holds that people from different ethnic groups present different modal conditions. Furthermore, the previous study showed that different types of Angle's classification presented different arch forms, and the clinician should anticipate the differences in size and form rather than treating all cases to a single ideal.

In the present day, superelastic preformed arch wire are used widely but there are many problems about a wide variety of shape and size of arch form

Aukvongseree (2) studied arch form of Angle's classification I in Thai and found that the beta function accurately describes it

Generally, arch form of Angle's classification II division 1 exhibited generalized reduced arch width and depth compared with the class I arches

This study aim to quantify the nature of the arch form which specify in Angle's classification II division 1 malocclusion in Thai samples and to define the mathematical equation of the shape of the dental arch which is better suited in the Angle's classification II division 1 group of Thai.

The purposes of the study

1. To define a generalized equation describing the dental arch form of Angle's classification II division 1 in Thai by the application of a computer curve – fitting program.
2. To predict the form of dental arch of individual patient.
3. To determine the dynamic relationships between arch depth and arch width which may yield important clinical applications.
4. To compare the form of dental arch between Thai and Caucasian Angle's classification II division 1 malocclusion (from Braun's study).
5. To compare the form of dental arch between Angle's classification I occlusion and classification II division 1 malocclusion
6. To compare the form of dental arch between measurement at cusp tip and bracket position
7. To compare the dental arch form according to sexual dimorphism.
8. This present work can be used as a database for further studies in arch wire construction.

The expected benefits of this study

1. It is scientific data to establish the mathematical function that describes arch form of Angle's classification II division 1 malocclusion in Thai.
2. It is scientific data to present the difference between arch form of Angle's classification II division 1 malocclusion and normal occlusion in Thai.
3. The measurement at bracket position "clinical arch form" will represent arch form precisely.
4. The deduction of the equation will be useful in determining the dynamic relationships between arch depth and arch width and may yield important clinical applications.
5. It is an appropriated database useful for producing commercial arch forms that are fitting to Thais.

Statement of hypothesis

1. There are correlations between the recorded coordinates and the calculated coordinates from the equation.
2. The beta function fits the Thai dental arch form (Angle's classification II division 1).
3. There are no differences in dental arch forms among Angle's classification I and classification II division 1
4. There are no differences in dental arch forms among measurement at cusp tip and bracket position
5. There are no differences in dental arch forms among sexual dimorphism.

CHAPTER 2

LITERATURE REVIEW

The search for the ideal arch form

In 1885, Bonwill (3) noted the tripod shape of the lower jaw and declared that it formed an equilateral triangle with the base extending from condyle to condyle and the sides extending from each condyle to the midline of the central incisors. He stated that this triangle existed for the proper functioning of the teeth. Importantly, he noted that the bicuspid and molars formed a straight line from the cuspids to the condyles. In 1905, Hawley (4) employed some of Bonwill's principles in proposing a geometric method for constructing the ideal arch form. Hawley suggested that the six anterior teeth be made to lie along a circle whose radius equaled their combined widths. From this circle he created an equilateral triangle, the base of which represented the intercondylar width. It was proposed that the bicuspid and molars should be aligned along these extended straight lines. Hawley did, however, advise against the strict use of this method for determining arch form, and that it be used only as a guide in establishing the arch form.

Numerous authors described other shapes for the dental arches. In 1902, Black (5) stated that the upper teeth are arranged in a semi-ellipse and that the lower teeth were arranged similarly on a smaller curve. Broomell, in the same year, said that "the teeth are arranged in the jaws in the form of two parabolic curves, the superior arch describing the segment of a larger circle than the inferior, as a result of which the upper teeth slightly overhang the lower".

In 1907, Angle (6) discussed in detail the "line of occlusion", which he defined as being "the line with which, in form and position according to type, the teeth must be in harmony if in normal occlusion". The form of this line was said to resemble a parabolic curve but one that varied greatly due to race, type, temperament, etc. of the individual. Because of these variables, Angle did not consider the Bonwill-Hawley arch form to be useful for anything more than a general approximation of the true line

of occlusion. In describing the first order bends needed in the arch form for proper tooth positioning, Angle objected particularly to the straight line proposed from cuspid to third molar. Angle stated that a straight line existed from the cuspid to the mesio-buccal cusp of the first molar, however, there was a natural curvature needed in the molar region.

In 1942, Gray's Anatomy (7) stated the following about human arch form: "The maxillary dental arch forms an elliptical curve...The mandibular dental arch forms a parabolic curve". In 1934, Chuck (8) noted the variation in human arch form and pointed out that arch forms had been referred to as square, round, oval, tapering, etc. He stated that while the Bonwill-Hawley arch form was not suitable for use in each patient, it could serve as a template for the construction of individualized arch forms. Chuck superimposed this arch form on a millimeter grid and used this template for archwire construction according to Angle's method. Chuck suggested that the bicuspid regions should be wider than the cuspids to prevent excessive expansion of the cuspids. In 1963, Boone (9) proposed the similar superimposition of the Bonwill-Hawley arch form on a millimeter template for construction of the individualized edgewise arch form.

Thus, over the years the Bonwill-Hawley arch form has been the most consistently used arch form as a beginning template for edgewise orthodontists. It is the "standard" arch form offered by most orthodontic manufacturers today.

In 1949, MacConaill (10) stated that, in considering the line of occlusion, it would be impossible for an ellipse and a parabola to meet one another at every point. He concluded that the ellipse-parabola description of the two dental arches, although elegant, had no immediate relation to function. He stated that a certain simple and well known curve, the catenary curve, fit so many cases with exactness that it could be taken as the "ideal curve" of common occlusions. The catenary curve was formed simply by suspending a chain of appropriate length from two points of varying width (for example width of the most distal molars in the arch form).

In 1957, Scott (11) also supported the concept of the catenary curve as the shape of the human arch based on the developmental anatomy of the dental arches and surrounding anatomic structures. He pointed out that the basal bone of the maxilla and mandible remains much more constant in form in all mammals and forms a

foundation on which a great deal of variation in form of alveolar processes are constructed. In man, the dentition maintains the primordial catenary form because alveolar process growth does not show a regional differentiation but remains more or in direction in all parts of the arch.

Burdi and Lillie (12) in 1966 further stated that the basic bony arch is established as early as 9.5 weeks in utero and that this form was that of the catenary curve. However, their research actually demonstrated numerous arch forms outside the catenary form. Musich (13) in 1973, supported the concept of the catenary curve as the ideal arch form and suggested the use of the catenometer as a reliable device for construction of arch perimeter. The catenary curve creates a rather tapered arch form and many of the tapered arch forms provide by orthodontic manufacturers today are based on the catenary curve (14).

The last major publication attempting to establish the “ideal” arch form was presented by Brader (15) in 1972. He stated that dental arch form was made up of teeth which assume unique positions along a compound curve representing an equilibrium at all points and delimited by the counterbalancing forces of the tongue and circumoral tissues. The geometry of the curve of the dental arch form was said to be best approximated by a closed curve with the curvilinear properties inherent in the trifocal ellipse, with the teeth occupying only the portion at the constricted end the curve.

Brader recommended an arch guide with five arch forms. The selection of the proper arch form was based on arch width at the second molars as measured at the facial, gingival surface. The maxillary arch form was selected one size larger than the mandibular arch form. While the Brader arch form provided a convenient method of archwire selection, many clinicians found that this arch form created excessive narrowing in the cuspid region of many patients and led to excessive wear of the incisal portion of the cuspids.

Dental arch shapes have been extensively classified both using simple qualitative descriptions and more complex mathematical methods. In the first case the dental arches were described elliptic, parabolic, U-shaped or approximately catenary. More complex procedures involved different curve-fitting mathematical models.

Howes (16) showed that arch width at the first premolar is a practical clinical tool for assessing the taperedness of a dental arch(when the 4 – 4 width is narrow) versus the broad or square arch (where 4 – 4 width is broader).

Lu (17) 1966 showed that a fourth – order orthogonal polynomial produces a close fit with the shape of the human dental arch. A fourth degree orthogonal polynomial may be used to represent the dental arch. The formular was presented as

$$Y = b_1X^1 + b_2X^2 + b_3 X^3 + b_4 X^4$$

Each of the 4 weighting coefficients in the equation was interpretable in terms of an arch's form. The linear (b_1) and cubic terms (b_3) measure the asymmetry. They measure the lopsidedness and the tiltedness of the asymmetry respectively. The quadratic (b_2) and quartic (b_4) terms measure the symmetry. They measure the taperedness and the squaredness of the symmetry respectively. The above procedure may be used in classifying classes of dental arches in terms of dental arch isomorphism.

The sketches of the eight possible combinations of asymmetry were as follows:

1. No asymmetry

If the lines appear to be at right angles with the lines Y, there is no asymmetry, as in figure 1. It corresponds to $b_1=b_3=0$.

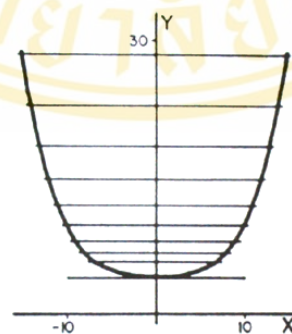


Figure 1 : Dental arch without asymmetry in Lu's study: $b_1 = b_3 = 0$.

2. Linear asymmetry

If the lines appear parallel but tilted upward towards the rights, there is positive linear asymmetry, as illustrated in figure 2. It corresponds to $b_1 > 0$, and $b_3 = 0$. If the lines appear parallel but tilted downward towards the right, there is negative linear asymmetry, as illustrated in figure 3. It corresponds to $b_1 < 0$, and $b_3 = 0$.

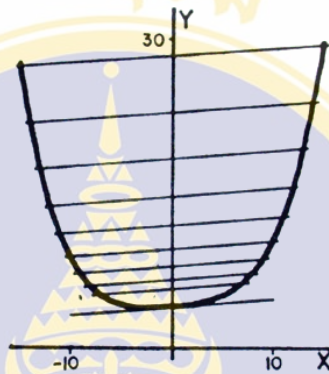


Figure 2: Dental arch with positive linear asymmetry in Lu's study: $b_1 > 0$, $b_3 = 0$.

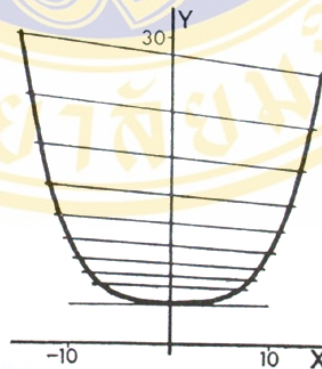


Figure 3: Dental arch with negative linear asymmetry in Lu's study: $b_1 < 0$, $b_3 = 0$.

3. Cubic Asymmetry

If the lines appear downward toward the right, the slope become steeper as x increases and become parallel to x axis as they approach the origin, then there is negative cubic asymmetry, as illustrated in figure 4. It corresponds to $b_1 = 0$ and $b_3 < 0$.

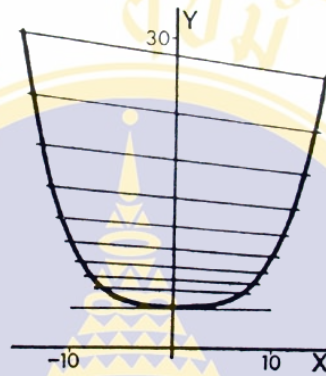


Figure 4: Dental arch with negative cubic asymmetry in Lu's study: $b_1 = 0$, $b_3 < 0$.

If the lines appear upward toward the right, the slopes become steeper as x increases, decrease and become parallel to x axis as they approach the origin, there is positive cubic asymmetry, as showed in figure 5. It corresponds to $b_1 = 0$, and $b_3 > 0$.

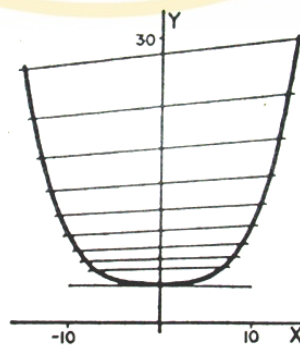


Figure 5: Dental arch with positive cubic asymmetry in Lu's study: $b_1 = 0$, $b_3 > 0$.

4. Linear-cubic asymmetry

4.1. Negative linear-negative cubic asymmetry, the lines tilt downward towards the right, the slopes become steeper as x increase, decrease as they approach the origin, but they do not become parallel to x axis, as illustrated in figure 6. It corresponds to $b_1 < 0, b_3 < 0$.

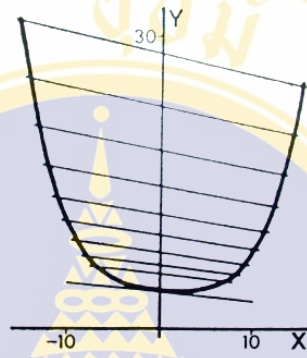


Figure 6: Dental arch with negative linear and negative cubic asymmetries in Lu's study: $b_1 < 0, b_3 < 0$.

4.2. Positive linear-positive cubic asymmetries, the lines tilt upwards towards the right, the slopes become steeper as x increases, decrease as they approach the origin, but they do not become parallel to x axis, as illustrated in figure 7. It corresponds to $b_1 > 0, b_3 > 0$.

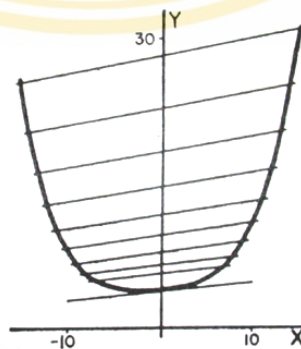


Figure 7: Dental arch with positive linear and positive cubic asymmetries in Lu's study: $b_1 > 0, b_3 > 0$.

4.3. Negative linear-positive cubic asymmetry, the lines do not become parallel to x axis as they approach the origin; the slopes at the lower portion of the arch tilt downward towards the right, become flatter as x increases, and finally become upward toward the right, as illustrated in figure 8. It corresponds to $b_1 < 0$, $b_3 > 0$.

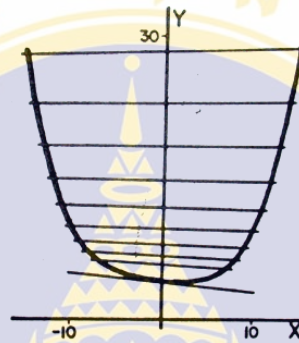


Figure 8: Dental arch with negative linear and positive cubic asymmetries in Lu's study: $b_1 < 0$, $b_3 > 0$.

4.4. Positive linear-negative cubic asymmetry, the lines do not become parallel to x axis as they approach the origin. The slopes at the lower portion of the arch tilt upward toward the right, become flatter as x increases, and finally become downward towards the right, as illustrated in figure 9. It corresponds to $b_1 > 0$, $b_3 < 0$.

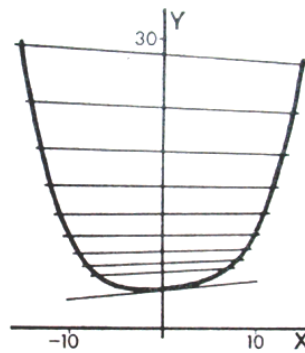


Figure 9: Dental arch with positive linear and negative cubic asymmetries in Lu's study: $b_1 > 0$, $b_3 < 0$.

Biggerstaff 1972 (18) stated of these methods the fourth, fifth, or sixth degree least squares polynomial regression equations give the most accurate and reproducible likeness of arch form. However, the fitted curves were not descriptive and no satisfactory statistical method exist for comparing the coefficients. Furthermore, the statistics of coefficients were difficult to interpret. The known variability of the human dental arch is responsible, at least partially, for the many qualitative descriptions. Generally, one of three curves (ellipsoidal, parabolic, or hyperbolic) will compare favorably with human dental arch. But, some humans may have U-shaped or horseshoe-shaped dental arches. Biggerstaff presented the paper to describe a method for defining the individual arch form as one of three mathematical curves.

Data were derived from a small sample of boys and girls. Photographic negatives registered the location of defined anatomic landmarks on the occlusal and incisal tooth surfaces. The landmarks were converted into rectangular X and Y coordinates and certain of these were selected as collectively representing arch form, e.g., the buccal cusp tips and the tips along the incisal edges.

Computer programs corrected each pair of coordinates to compensate for the enlargement or diminution of the photographic image. Other programs fit the adjusted coordinates with the general quadratic equation:

$$AX^2 + BY^2 + CXY + DX + EY - F = 0$$

The value of F in the equation was arbitrarily set at unity (1), although it could have been set at any value. The F value affects only the values of the coefficients that were computed by a subprogram (a component of the numerical analysis program library supported by the University of Kentucky Computing Center)

The coefficients were then used to solve the equation for Y in terms of X. If coefficients A and B have the same sign (positive or negative) the resulting smoothed curve is an ellipse. If the signs are different, the smoothed curve is hyperbolic. And finally, if either coefficient A or B is zero, the curve is parabolic and degenerate if B is zero. The data can be presented in graphic form using the Calcompplotter.

These methods permit a quantitative definition of dental arch types in a population such that the frequency of hyperbolic, parabolic, and ellipsoidal arch form can be assayed to determine within and between population differences.

Pepe 1975 (19) studied the curve fitting of polynomial and catenary curve to human dental arch. The data base for this study was derived from seven individuals with a permanent dentition and good occlusion were the sample. Inked points were placed on duplicate casts to identify various anatomic landmarks. By means of photogrammetric methods, these points were projected to a two-dimensional plane parallel to the occlusal plane of each cast such that a near 1:1 relationship of cast size and photographic negative image resulted. Semi-automatic methods were then used to convert these points to a Cartesian (that is, rectangular X-Y) coordinate system with the resultant data recorded on computer cards. The computer program was written to fit the polynomial equations (The polynomial equations are $y = \sum a_i x^i$; $2 \leq i \leq 8$) and the catenary equations (that is, $y = A + B \cosh X/B$). The curves were fit by least square error methods, that is, the computer was instructed to approximate the coefficients and the mean square error is minimal. All the curve-generating equations studied in this research, polynomial equation curves were somewhat more accurate than catenary equation curves in describing arch form, they were not remarkably good fits either. Mean square errors for this curve ranged from 1.83 – 17.00 mm². Thus, although these curves may describe arch form, they were not accurate enough to serve as a template for an arch wire.

Application of the Cubic Spline function was described of the dental arch form by Begole (20). This study purposed to use an asymmetric function, to describe the dental arches. Twenty-seven individuals possessing normal occlusions were selected for study. A full complement of permanent teeth was present from at least the first permanent molar to the contralateral first permanent molar. All individuals exhibited a Class I molar, cuspid, and buccal segment relationship. Maxillary models were then photographed individually. The points on the prints were digitized in an X,Y system using the mesiobuccal cusp tips of first permanent molars, cusp tips of the premolars and cuspids, and the midpoint between the incisal edges of the incisors. Five data points were digitized to be utilized as knots of the spline (see in figure 10), they were the mesiobuccal cusp tips of first permanent molars, cusp tips of the cuspids, and the midpoint between the incisal edges of the central incisors. The remaining eight points recorded, which were the buccal cusp tips of bicuspids and midincisal edges of central and lateral incisors, were used as points from which to evaluate the fit of the spline.

Data were subsequently processed using a Fortran computer program which interpolated a cubic spline for each subject.

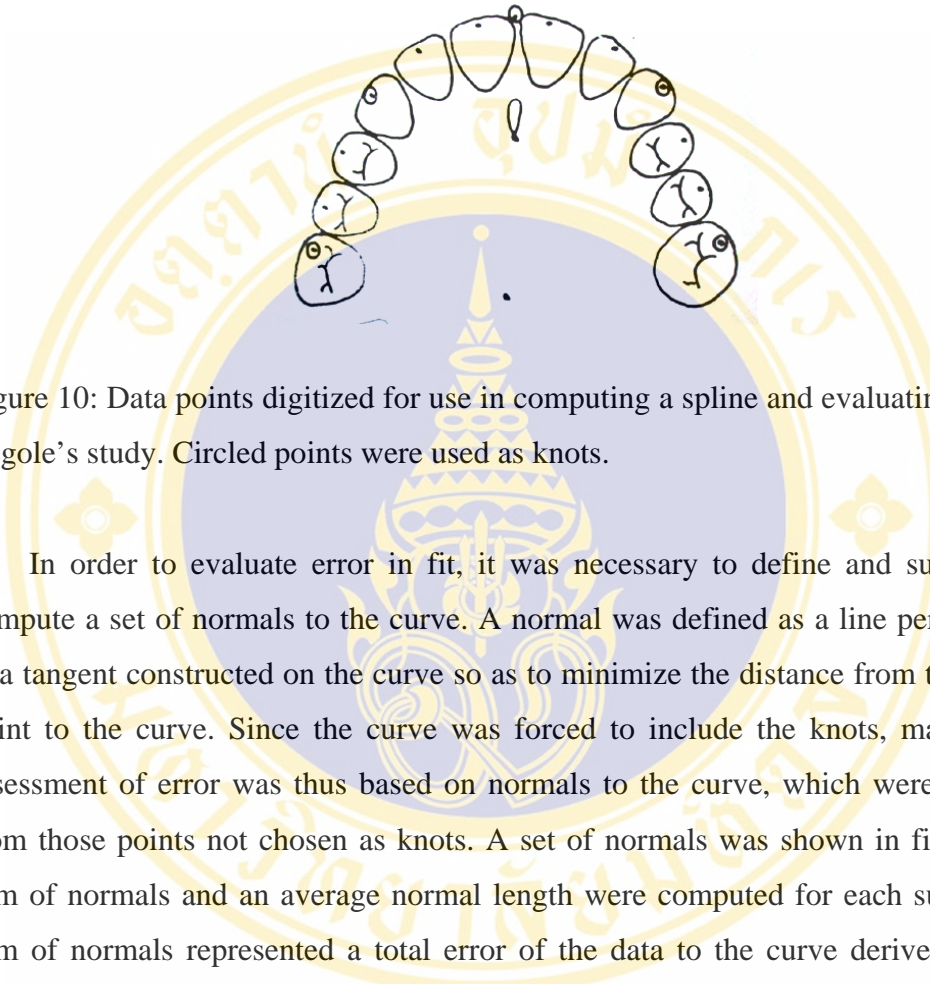


Figure 10: Data points digitized for use in computing a spline and evaluating the fit in Begole's study. Circled points were used as knots.

In order to evaluate error in fit, it was necessary to define and subsequently compute a set of normals to the curve. A normal was defined as a line perpendicular to a tangent constructed on the curve so as to minimize the distance from the exterior point to the curve. Since the curve was forced to include the knots, mathematical assessment of error was thus based on normals to the curve, which were computed from those points not chosen as knots. A set of normals was shown in figure 11. A sum of normals and an average normal length were computed for each subject. The sum of normals represented a total error of the data to the curve derived from the spline function, and the average normal provided a mean value for error for the subject.



Figure 11: A set of normal to the curve in Begole's study. Normal were used to evaluate error in the fit of a spline.

Results of this research suggest that in the maxillary dental arches examined, no particular teeth produced substantially larger error when their distance to the theoretical curve was measured. When the teeth were measured and subsequently grouped into segments for analysis, similar results were obtained.

In regard to symmetry, for almost all the subjects asymmetry was evident upon examination of the computer-generated graphics, which used the midpalatal raphe as the axis of symmetry.

Ferrario et.al. (21) 1994 defined mathematical equation of the shape of dental arches in human permanent healthy dentitions. Fifty male and 45 female subjects aged 20-27 years (mean age 22) with normal occlusion were screened from a group of 160 healthy white Caucasian dental students.

The following criteria were used in the selection

1. Absence of moderate or severe clinical mandibular disorders (no TMJ sounds, no tenderness to palpation of the TMJ or of the masticatory muscles, no painful limitations of mandibular movements);
2. Absence of extensive restorations, cast restorations or cuspal coverage;
3. No previous or current orthodontic treatment;
4. Absence of anterior or lateral cross-bite;
5. Absence of pathologic periodontal condition;
6. Clinical normal arch shapes with minimal dental crowding.

All models of the 95 subjects, the midpoints of the incisal edges and canine cusps, and the buccal and lingual cusps of premolars, first and second molars were

individualized, and traced with a pencil. Standardized photographs of models were obtained (frame 1:1). Co-ordinates of cusp tips and of two points on each axis of symmetry were obtained using a semi-automatic image analyzer. Maxillary and mandibular arches were interpolated by (1) a 'mixed' model and (2) a polynomial models as follows:

1. a mixed model

(a) mid – points of incisal edges and canine cusps by an ellipse:

$$Y^2/f^2 + X^2/g^2 = 1 ;$$

(b) buccal cusps from first premolar to second molar by a parabola:

$$Y = a + bX + cX^2 ;$$

2. 1(a) plus 1(b) points by a fourth – order polynomial

$$Y = aX + bX^2 + cX^3 + dX^4$$

For all three curves, the coefficients were estimated using a dedicated computer program: the algorithm minimized the sum of squared distances from the cusp tip co-ordinates to the fitted curve in the Y direction. The program also calculated the least square correlation coefficients and the residual standard deviations about the fitted curves [$s = \sqrt{\sum (Y - \hat{Y})^2 / (N - 2)}$ where Y is the sample datum, and \hat{Y} is the estimated value] Means and standard deviations were calculated separately for all parabolic, elliptical, and polynomial coefficients in male and female.

In all parabolars the sign of the second degree coefficient, c , indicates the direction of the curve concavity, being positive for upward concavities. High (positive or negative) coefficients describe very concave curves; the more the absolute value of this coefficient approximates 0, the flatter the curve: it became a line for $c = 0$. The first degree coefficient, b , gives information about the symmetry in respect to Y axis: it is positive for left side curves (second and fourth quadrants), negative for right side ones and null when the vertex stays on the Y axis. The a coefficient is the intercept. It is positive when the vertex lays in the first or second quadrants. In the fourth-order polynomial coefficients b (second degree) and d (fourth degree) describe arch shape, while coefficients a (first degree) and c (third degree) describe asymmetry, b and d are negative for downward curves, positive for upward curves.

Statistical comparisons were performed between mean coefficients computed in male and female samples, and in mandibular and maxillary arches using two-tailed Student's t-tests for independent samples. A specially devised computer graphic subroutine allowed the plotting of cusp tips (co-ordinates of digitized point) and of their interpolating curves for each subject, as illustrated in figure 12, thus providing a direct assessment of the goodness of fit suggested by the correlation coefficients.

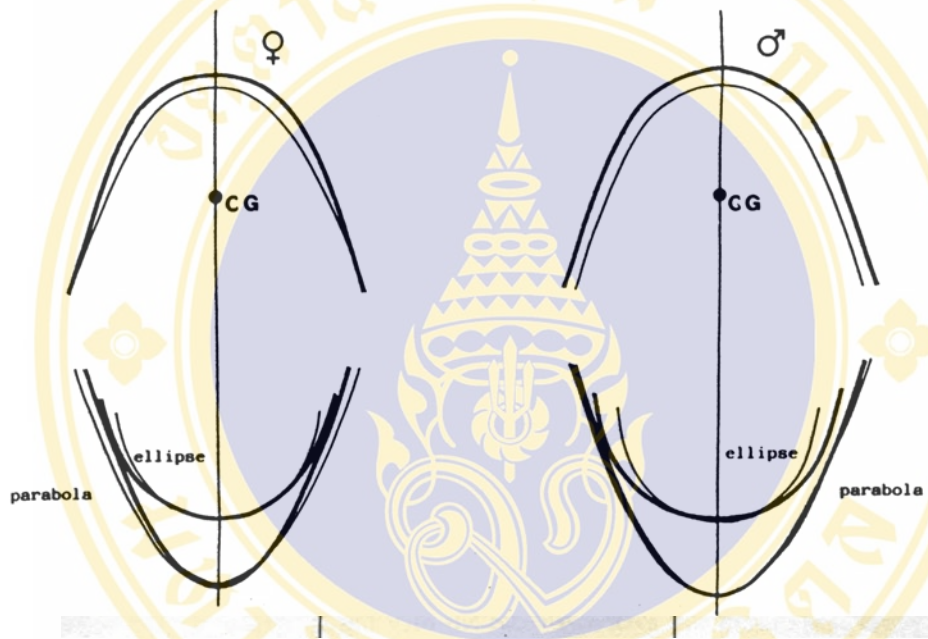


Figure 12: Mean plots of mandibular and maxillary arches in Ferrario's study: Top: maxillary and mandibular curves supposing a right Angle Class I first molar relationship. Bottom: ellipses and parabolas superimposed on there vertices.CG: center of gravity of arches.

The result of this studied, anterior teeth seem to be well fitted by the elliptical model, as test by the mean correlation coefficients ranging from 0.873 to 0.933. Post – canine teeth were well interpolated by their model, mean correlation coefficients of the parabolas were very good in all instances ($r > 0.92$). Fourth – order polynomial well fitted data points in all instances: mean correlation coefficients are all over 0.97, the plots of the mean fourth – order polynomials did not appear to be signification different from the mean plots computed with the mixed model.

Braun S, Hnat WP, Kusnoto B, Hnat FW. (22) proved that the beta function most closely represents the human dental arch. The data were obtained by recording the coordinates of the cusp tips locations of forty sets of casts (15 Class I, 16 Class II, and 9 Class III). Subdivision occlusions were not included in the study. Casts exhibiting incisal or cuspal attrition, fracture of teeth, ectopically erupted teeth, or deciduous teeth were excluded from this study; only casts of fully developed adult dentitions (including second molars) were included. A precision machine tool device (Coordinate Measuring Machine; CMM) was used to record the X-,Y-, and Z-coordinates of selected dental landmarks on all casts to 0.001 mm. accuracy.

Coordinates were recorded at the center of each incisor incisal edge, at the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar. Eighteen points were recorded in each dental arch. The coordinates were processed through a computer curve-fitting program.

The dental arch described by the beta function, was given by

$$Y = 3.031 D \left\{ \frac{x}{w} + \frac{1}{2} \right\}^{0.8} \left\{ \frac{1}{2} - \frac{x}{w} \right\}^{0.8}$$

W represents the cross – arch distance between the second molar contact points

D represents the perpendicular distance from the most anterior point between the two central incisors to the molar cross-arch dimension, as illustrated in figure 13

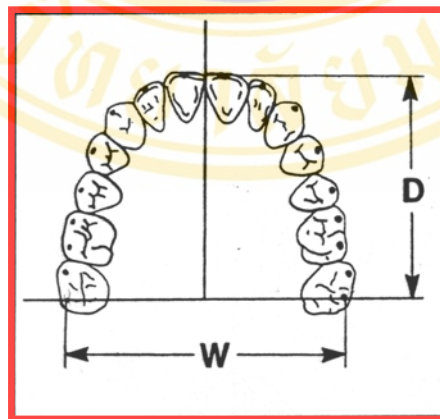


Figure 13: The dental landmarks and parameters in Braun et.al.'s study: W = arch width, D = arch depth.

The beta function has been shown to be an accurate representation of the human dental arches. The mean correlation coefficient of curve fit was found to be 0.98, with a standard deviation of 0.02, for the complete sample of 80 casts. From these study, they concluded that the beta function more accurately described the dental arch form than representations previously reported.

Furthermore comparing mandibular arch shapes (figure 14), it was evident that the dental arches associated with Class III occlusions exhibit a smaller arch depth than the Class I occlusions. Additionally, the mandibular dental arches associated with Class III occlusion were wider than the Class I mandibular arches. When Class II mandibular arches were compared with Class I arches, reduced arch width and arch depth were evident. This could be explained by the fact that some Class II relationships result from a small mandibular body. When comparing maxillary arch shapes (figure 14), it was apparent that arch depths for all Angle's Classifications were essentially the same. However, Class III dental arch widths were greater than Class I widths. This begins in the lateral incisors-canines area and proceeds distally. When Class II maxillary arch widths were compared with Class I widths, they were found to be narrower, beginning in the lateral incisors-canines area.

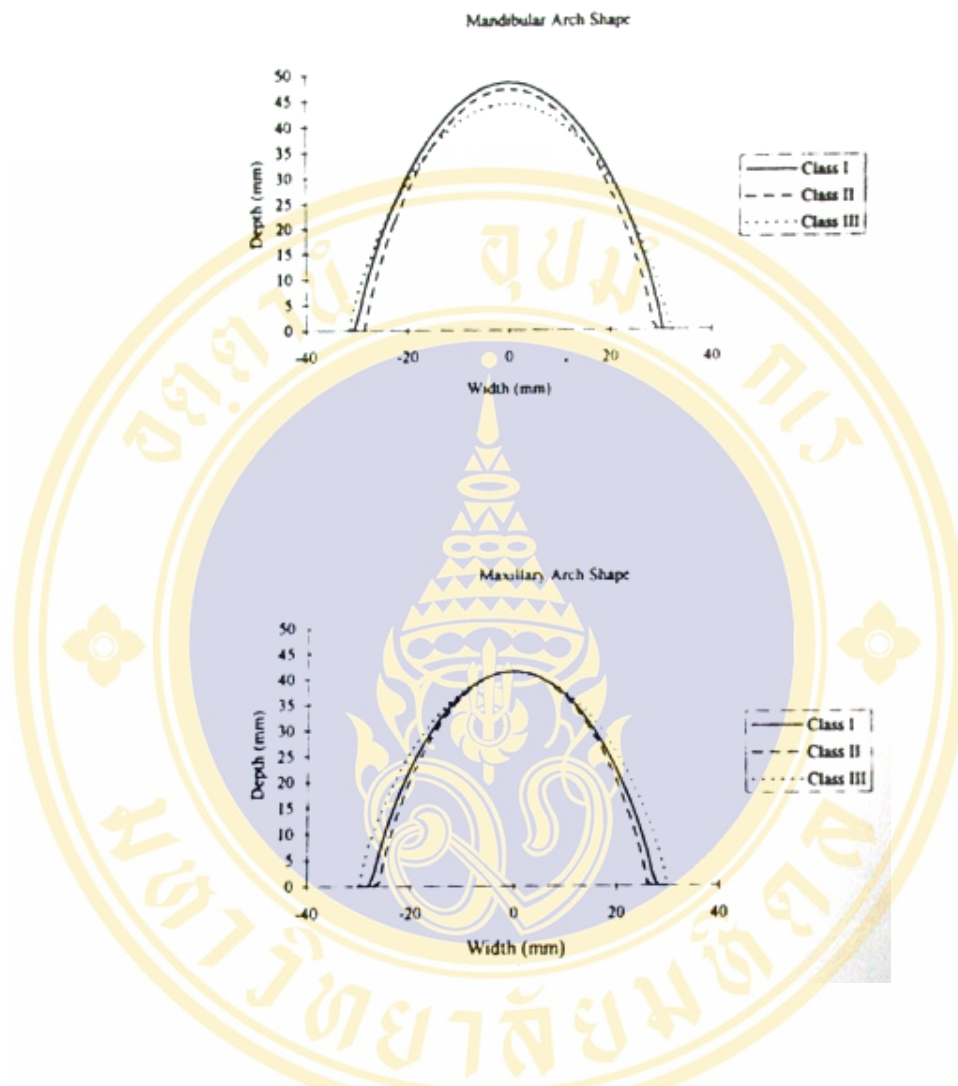


Figure 14: Average (typical) maxillary and mandibular arch shapes for Class I, II, and III occlusion in Braun et.al.’s study.

Braun and Hnat (23, 24) studied the dynamic relationships of the mandibular anterior segment. Twenty-one mandibular casts of untreated patients that showed to minimal anterior segment irregularities were selected. Measurements were made at each canine cusp tip, the center of each incisor incisal edge, and the normal contact between the first bicuspid and canines.

An analytical equation of the anterior segment shape was required to describe the relationships between the intercanine width, anterior segment depth, anterior arch perimeter, and incisor angulation.

Thus the generalized equation of the mandibular anterior segment shape may be expressed as

$$Y = -\cos h \left\{ \frac{x}{b} \cos h^{-1} (h+1) \right\} + 1.0+h$$

b represents $\frac{1}{2}$ the cross arch distance between the normal distal contacts of the right and left canines, as illustrated in figure 15.

h represents the distance measured along a line from the contacts of the central incisors perpendicular to a line connecting the distal contacts of the canine, as illustrated in figure 15.

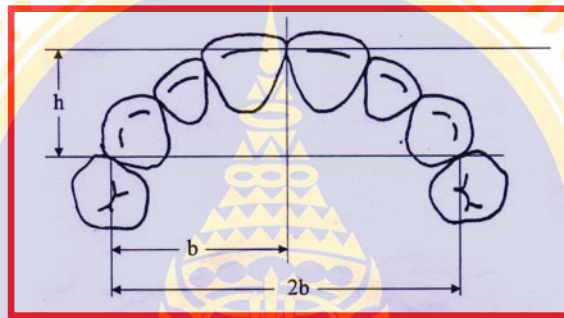


Figure 15: The parameters (h) and (b) used in Braun et.al.'s study.

The arch perimeter may then be calculated using

$$L = \int_{-w}^{+w} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$$

Where W represents half the crossarch distance from the normal canine/first premolar contact on one side to the opposite side in millimeters, and L is the anterior segment arch perimeter from the canine/first premolar normal contact along the arch to the opposite side in millimeters.

The related angular change for a typical mandibular incisor was given by the formula

$$\Delta\theta = \tan^{-1} \frac{\Delta D}{14.2}$$

Where ΔD was the change in depth of the anterior segment in millimeter.

The above relationships were of value in treatment planning. The outcome in anterior arch perimeter, depth, and incisor angular change can be forecast with accuracy ($r = 0.951$) without resorting to trail and error or doing a wax-up.

The variation in human arch form

Over the years the great majority of authors on the subject of arch form have recognized that there is variability in the size and shape of human arch form. Angle (6), for example, stated that the arch form varies within the limits of the normal, according to race, type, temperament etc., of the individual. Because the form of the arches was considered dependent on these variables, Angle did not consider Hawley's method of arch predetermination useful for more than a general approximation as a true line of occlusion. Over the years, however, the majority of edgewise orthodontists used the Bonwill-Hawley arch form as a beginning template for the construction of the edgewise archwire. These construction methods were described by Angle (6), Chuck (8), Boone (9) and others.

Hellman (25) investigated the skulls of apes and human beings, and found no relation between the size of teeth and the form and shape of the dental arches. Therefore, he did not accept the theories of arch predetermination based on measurement of certain teeth. He concluded that mathematical methods for dealing with the question of arch form were unsatisfactory.

Stanton (26) carried out extensive investigations on arch form and pointed out the error in the Bonwill-Hawley method of arch predetermination. He utilized "arthrographic map makers" to study occlusions, and concluded that arch forms are open and closed, that is ellipse, parabolic, and other kindred curves. Izard (27) based his method of arch predetermination on ratios between arch width and facial depth. He concluded that approximately 75% of arch forms were represented by an ellipse, 25% by a parabola and 5% by a U shape. While Wheeler (28) observed that dental arches generally conform to the shapes of parabolic curves, he stated that nothing anatomic could be reduced to the mathematical exactitudes of geometric terms.

Sicher (29) wrote that the shapes of the dental arches vary considerably, but that the upper arch generally took on the appearance of an ellipse, and the lower arch a

parabola. Remsen (30) studied various arch predetermination methods by comparing them with the arches of a sample of “normal” occlusion. He observed that the parabola best represented the anterior curvature of the dental arch, but stated that an arch which fits a precise pattern was the exception rather than the rule.

White (31) reviewed the accuracy of various standardized arch designs to 24 untreated ideal adult occlusions. His findings were as follows:

1. The Bonwill-Hawley arch form had a good fit with 8.33% of the cases, a moderately good fit with 39.58% of the case, and a poor fit with 52.08% of the cases.
2. The Brader arch form had a good fit with 12.50% of the cases, a moderately good fit was 43.74% of the cases, and a poor fit was 43.75% of the cases.
3. The Cantinary curve showed a good fit with 27.08% of the cases, a moderately good fit in 45.82% of the cases, and a poor fit with 27.05% of the cases.
4. The Rocky Mountain data computer derived arch design which is based on measurements of inter-molar width, inter-cuspid width and arch depth, showed a good fit with 8.3% of the cases, and a moderately good fit with 81.57% of the cases. No cases showed a poor fit.

White also pointed out that most theories on arch form consider arch form to be symmetrical. He observed that there was a great deal of asymmetry in the arches and felt that should be considered in arch design. Because of the variability that White found in arch form, he suggested that an occlusion “map-maker” be used to construct the shape of the arch for the individual and used throughout orthodontic treatment.

In order to determine whether a particular ideal orthodontic arch form could be identified, Felton (32) et.al., studied the mandibular casts of 30 untreated normal cases, 30 Class I non-extraction cases, and 30 Class II non-extraction. After computerized digitizing and the use of a mathematical function called a polynomial of the fourth degree, arch forms were generated for each sample and then compared to 17 commercially produced arch forms. Results showed that no particular arch form predominated in any of the three samples. They therefore stated that customizing arch forms appeared to be necessary in many cases to obtain optimum long term stability, because of the great variability in arch form observed in the study.

Nojima and Mclaughlin (33) studied the difference of arch shape in Caucasian and Japanese mandibular in different type of occlusion. The results of this study was

show in table 1. The Caucasian arch tended to be narrower and deeper than the Japanese arch, and they found the difference of arch form in different type of occlusion.

Table 1. The distribution of dental arch shape according to the type of occlusion in Caucasian and Japanese.

Caucasian		Japanese	
Class I	90% ovoid and tapered arch	Class I	90% ovoid and square (a square arch form was observed in more than 50%)
Class II	60% tapered arch form (most of the remainder showing an ovoid arch form)	Class II	Over 50% was ovoid arch form with square and tapered shapes for approximately 25% each
Class III	3 arch forms were almost distributed with square arch form found at the highest frequency of over 40%	Class III	Arch form were of either square or ovoid shape, the former accounting for over 50%

Yamada A. (33) studied the relationship between the capacity of oral cavity proper and skeletal malocclusions, result were showed.

1. Class III cases were statistically larger than class I and II in the capacity of the oral cavity proper but they were no significant difference between those of class I and class II.

2. There were no significant difference among class I, II or III in the maxillary capacity of the oral cavity proper. Class III, however, was significant larger than class I and II in the mandibular capacity of the oral cavity proper.

3. There was no significant difference among the three types of palatal morphology.

4. There were difference for class I, II and III as to the relationship between both the dental arch length and width, and the capacity of the oral cavity proper. The

general indication was that width was more related to the size of the capacity of the oral cavity proper than length.

Ferrario et.al. (21) had already investigated gender differences in dental arch shape in a group of young adults with sound dentitions by Euclidean - distance matrix analysis demonstrated no significant gender difference in the shape of arches. A mathematical interpolation (fourth - order polynomials, semi -ellipses and parabolas) of the same arches produced similar mean curves regardless of gender, showing more a size discrepancy than a shape difference.

Ferrario et.al. (35) studied maxillary versus mandibular arch form differences in the human permanent dentition assessed by Euclidean – distance matrix analysis applied to the selected 57 women and 61 men aged 20-27years. They found that maxillary and mandibular arches had a significantly different shape and size was also different: the upper arch was approximately 12% bigger than the lower arch. The examination of the form – difference matrix allowed separation of teeth into groups that could account for the morphological differences between arch: anterior teeth (incisors and canine) appeared to behave differently from molars and premolar. This finding gives support to the recommendation that anatomical and anthropological description should treat these two sectors separately (21), and that clinical evaluations and treatments should take these differences into account (36,37).

Staley et.al. (38) studied comparison of arch widths and other cast and cephalometric measurements of 36 normal-occlusion subjects (19 males, 17 females) with 39 Class II, Division 1 subjects (20 males, 19 females). None of the subjects had received orthodontic treatment. Analysis of variance demonstrated that 1) subjects with normal occlusion had larger maxillary molar widths, maxillary canine widths, and maxillary alveolar widths than subjects with malocclusion; 2) only male subjects with normal occlusion had larger mandibular molar widths and mandibular alveolar widths than the malocclusion subjects; 3) the normal occlusion and malocclusion groups had similar mandibular canine widths; and 4) when the lower molar and alveolar widths were subtracted from corresponding upper widths, the remainders of the Class II group were negative instead of positive, contrary to the normal group.

Aukvongseree (2) proved that the beta function accurately describes the Thai Angle's Classification I dental arch forms. The study included 40 sets (20 males and

20 females) of dental casts which expressed normal Angle's Classification I occlusion.

Twenty-two dental landmarks from cusp tips of molar, premolar, canine and distal and mesial points of the incisal edges of each dental cast were triggered and recorded using the Coordinate Measuring Machine and reported into their corresponding coordinates (X-, and Y- direction). The coordinates were precessed through a computer curve-fitting program to define the parameter of beta function to describe the dental arch.

The Thai, normal occlusion dental arches were shown to be accurately represented mathematically by the beta function. The coefficient of determination between measured arch-shape data and the mathematical arch shape, expressed by the beta function, was ranging from 0.93-0.99. Thai dental arch forms have a wider-look like than the Caucasian arch form in Braun's study. Gender differences were found, where they reflected more of a size discrepancy than a shape differences. Males arch forms were significantly larger than females at p-value < 0.01 in the maxilla and at p-value < 0.001 in the mandibular arches.

The general formula are generated as following:

The maxillary arch

$$Y = a \left(1 - \frac{4X^2}{c^2} \right)^{0.67}$$

The mandibular arch

$$Y = a \left(1 - \frac{4X^2}{c^2} \right)^{0.77}$$

The equations represent the maxillary and mandibular arches according to the difference in arch width, are following:

The maxillary arch

Small arch size

$$Y = 44.87 \left(1 - \frac{4X^2}{(55.29)^2} \right)^{0.67}$$

Medium arch size

$$Y = 44.87 \left(1 - \frac{4X^2}{(60.43)^2} \right)^{0.67}$$

Large arch size

$$Y = 44.87 \left(1 - \frac{4X^2}{(63.45)^2} \right)^{0.67}$$

The mandibular arch

Small arch size

$$Y = 40.73 \left(1 - \frac{4X^2}{(48.20)^2} \right)^{0.77}$$

Medium arch size

$$Y = 40.73 \left(1 - \frac{4X^2}{(53.77)^2} \right)^{0.77}$$

Large arch size

$$Y = 40.73 \left(1 - \frac{4X^2}{(57.57)^2} \right)^{0.77}$$

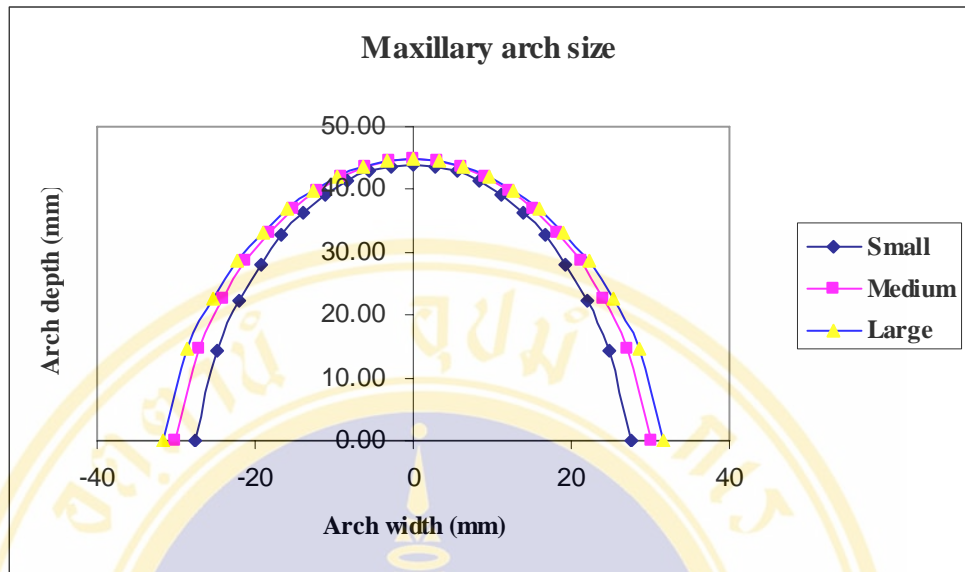


Figure 16: The maxillary dental arch curvature in different sizes (— = Small, — = Medium, — = Large).

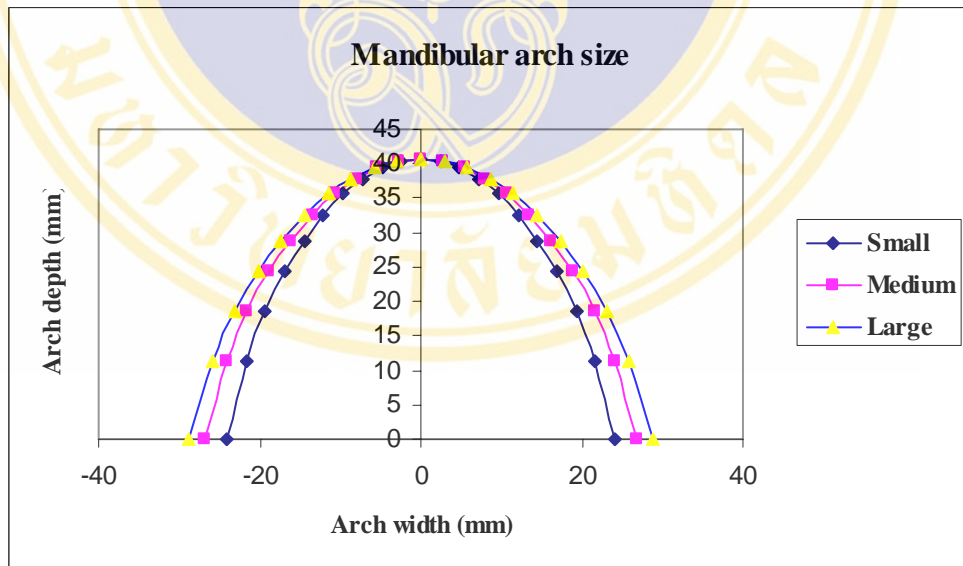


Figure 17: The mandibular dental arch curvature in different sizes (— = Small, — = Medium, — = Large).

One of the limitations of this study was difference between “research arch form” and “clinical arch form”. Research arch form was measured from the mesial and distal point of incisal edge of incisors, the cusp tips of the canines and premolars, and the mesiobuccal and distobuccal cusp tips of the molars. The resulting arch form were not co-ordinate between maxillary and mandibular arch in the same person so it is not appropriate for the direct construction of arch wires.

Summary of review

Contribution of variation in Arch size and shape is defined in the fetus as genetic influence, variability in eruption paths of the teeth, growth of the supporting bone and movement of the teeth after emergence due to habits and unbalanced muscular pressure. The result of genetic and environment factor make the difference of arch shape in the differences in ethnic and the difference type of occlusion.

General method to define arch form in geometric terms:

1. In all models the selective dental landmarks were individualized and marked with a pencil.
2. General study took photography or scanning from dental model: frame 1:1.
3. Coordinated of cusp tips and of two points on each axis of symmetry were obtained using a semi-automatic analyser.
4. For all mathematic function: the coefficients were estimated by using a computer program.

According to the general method, the error of measurement was happen in the process of transferring three-dimensions of dental casts to two-dimensions. In the present day, a highly accurate measuring device (CMM) used in the machine tool industry can be applied to record specific landmarks directly on dental casts (39). The dental landmarks are reported in the corresponding of X, Y, Z. Therefore the processing of collecting and recording the data times were reduced.

The coefficients of the cubic spline and the polynomial equation were difficult to interpret. While the parabola, elliptical equation and hyperbolic cosine function were merely described parts of the whole dental arch curve. The beta function with the feasibility coefficient proved to have a high correlation with the dental arch (22). The

data were obtained by recording the coordinates of the center of each incisal edge, at the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar.

There are the difference of arch form in different type of occlusion. For this reason, study of arch form in other type of occlusion is essential. Using pretreatment arch form as a guide to posttreatment arch shape will create natural esthetic and occlusal stability.

Due to limitation about research arch form in the former study (2). In the present study, we want to solve this problem by add clinical arch form. Clinical arch form was measured from the points where the wire will lie in the bracket slots of correctly positioned brackets. The bracket position need to be considered from the following variable 1) mesiodistal location , 2) vertical height on the crown, 3) angulation of the bracket, 4) labio-lingual positioning (40) .

CHAPTER 3

MATERIALS AND METHODS

Determining sample size

Determining the sample size is a very important issue because samples that are too large may waste time, resources and money, while samples that are too small may lead to inaccurate results. In this study, formula for calculating the appropriate sample size is

$$n = \frac{Z_{\frac{\alpha}{2}}^2}{4E^2}$$

where

$Z_{\frac{\alpha}{2}}$ is known as the critical value, the positive Z value that is at the vertical boundary for the area of $\frac{\alpha}{2}$ in the right tail of the standard normal distribution

E (the margin of error) is the maximum difference between the observed sample mean \bar{x} and the true value of the population mean μ

n is the sample size

A 95% degree confidence corresponds to $\alpha = 0.05$, $Z_{\frac{\alpha}{2}} = 1.96$, $E = 0.2$

$$n = \frac{1.96^2}{4(0.2)^2} = 24.01$$

So we will need to sample at least 24 sets

As a result, based on the above calculation, forty subjects (20 males and 20 females) who have never undergone orthodontic treatment were selected from the Department of Orthodontics, Mahidol University, and private dental clinics

The following criteria were used in the selection:

1. Cast was exhibited Angle class II division 1 malocclusion (without molar shifted)
 - 1.1 bilateral class II molar relationship in centric occlusion with the mesial groove of the mandibular first permanent molar articulates posteriorly (1/4 cusp width – full cusp width) to the mesiobuccal cusp of the maxillary first permanent molar
 - 1.2 protrusive maxillary incisors (overjet > 4 to 9 mm.)
2. Absence of cuspal attrition, fracture of teeth, restoration extending to contact areas, cusp tips or incisal edge
3. Permanent dentition including 2nd molar with normal tooth size and shape
4. a 3-mm or less arch length discrepancy

Dental arches of the 40 subjects were reproduced using an alginate-base hydrocolloid material and transferred to dental models in orthodontic stone (the typical dental casts was show in figure 18). In all models have 2 landmarks were labeled on each dental cast

- 1) the mesial and distal points of each incisal edge of incisors, the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar. Twenty-two points were labeled on each dental cast (figure 19).
- 2) the bracket position: the point on buccal and labial surfaces of teeth that derived from cross section of long axis of teeth and line which was perpendicular with long axis of teeth. The last line was measured from incisal edge for incisors and the cusp tip for other teeth. The distance is 3.5 mm. for upper lateral incisors and lower incisors. Furthermore, 4 mm. for the remaining teeth (40). These points represented position in bonding bracket. Fourteen points were labeled on each dental cast (figure20).

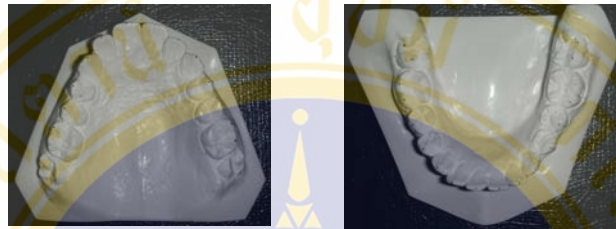


Figure 18: Typical dental models used in this study

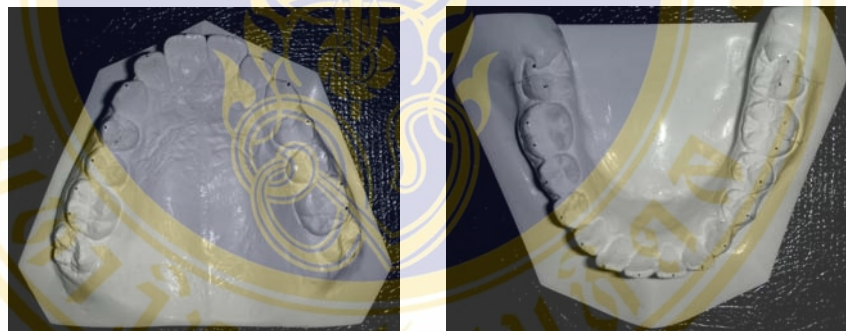


Figure 19: The first dental landmarks used in this study. Twenty-two points were marked on each dental model.



Figure 20: The second dental landmarks used in this study. Fourteen points were marked on each dental model

During the measurement of dental models using Coordinate Measuring Machine (LK G 90C), each cast was oriented in the Coordinate Measuring Machine (CMM). The device was used extensively in the precise machine tool in the Faculty of Engineering, Mahidol university (figure 21). Measurements of dental casts using CMM in this study was done by technician who was trained to use CMM. A frictionless air bearing probe recorded the coordinates of a point in space in each of the three orthogonal axes to 10^{-6} meters. The casts were secured to a fixed plane. The touch trigger probe was used to identify each of the measurement points.

The measurement was done 2 times.

First time: The touch trigger probe was used to identify each of the following measurement points respectively from the

Maxillary dental models:

1. distobuccal cusp tip of right second molar
2. mesiobuccal cusp tip of right second molar
3. distobuccal cusp tip of right first molar
4. mesiobuccal cusp tip of right first molar
5. buccal cusp tip of right second premolar
6. buccal cusp tip of right first premolar
7. cusp tip of right canine
8. mesial point of incisal edge of right lateral incisor
9. distal point of incisal edge of right lateral incisor
10. mesial point of incisal edge of right central incisor
11. distal point of incisal edge of right central incisor
12. mesial point of incisal edge of left central incisor
13. distal point of incisal edge of left central incisor
14. mesial point of incisal edge of left lateral incisor
15. distal point of incisal edge of left lateral incisor
16. cusp tip of left canine
17. buccal cusp tip of left first premolar
18. buccal cusp tip of left second premolar
19. mesiobuccal cusp tip of left first molar
20. distobuccal cusp tip of left first molar

21. mesiobuccal cusp tip of left second molar
22. distobuccal cusp tip of left second molar

Mandibular dental models:

1. distobuccal cusp tip of left second molar
2. mesiobuccal cusp tip of left second molar
3. distobuccal cusp tip of left first molar
4. mesiobuccal cusp tip of left first molar
5. buccal cusp tip of left second premolar
6. buccal cusp tip of left first premolar
7. cusp tip of left canine
8. mesial point of incisal edge of left lateral incisor
9. distal point of incisal edge of left lateral incisor
10. mesial point of incisal edge of left central incisor
11. distal point of incisal edge of left central incisor
12. mesial point of incisal edge of right central incisor
13. distal point of incisal edge of right central incisor
14. mesial point of incisal edge of right lateral incisor
15. distal point of incisal edge of right lateral incisor
16. cusp tip of right canine
17. buccal cusp tip of right first premolar
18. buccal cusp tip of right second premolar
19. mesiobuccal cusp tip of right first molar
20. distobuccal cusp tip of right first molar
21. mesiobuccal cusp tip of right second molar
22. distobuccal cusp tip of right second molar

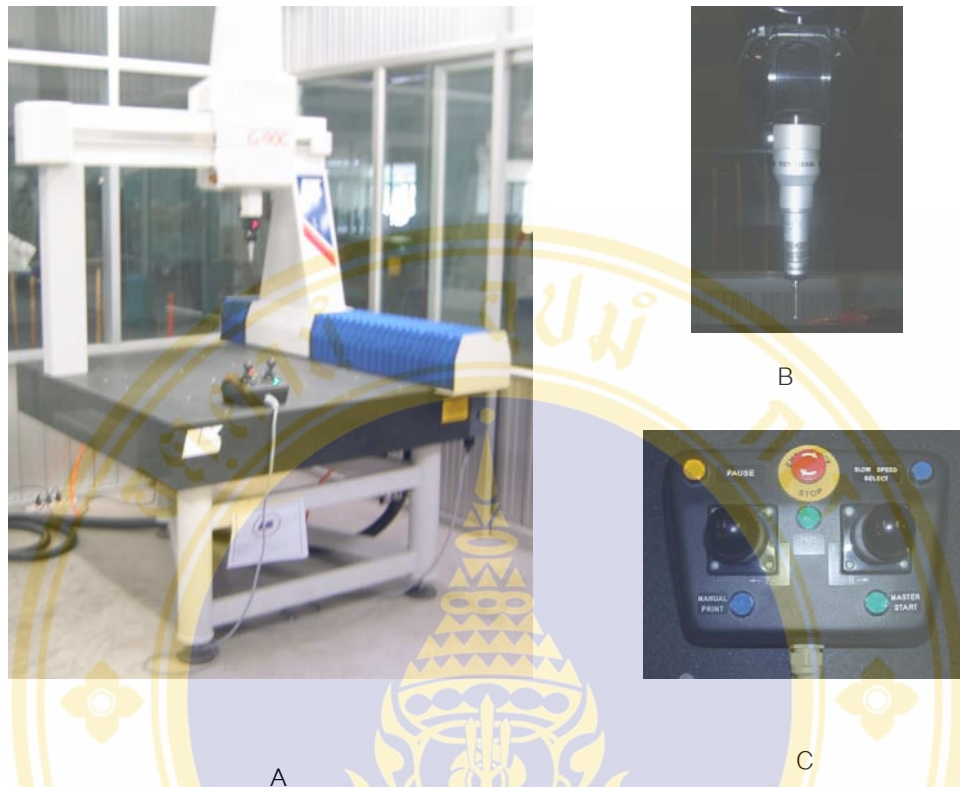


Figure 21: The coordinate measuring machine: A) CMM (LK G-90C), B) Touch trigger probe, C) A hand stick was used to control the direction of trigger probe.

Twenty-two points were recorded in each dental model while the X-and Y-coordinates of each landmark were projected to the Z-plane. The coordinates of the landmarks in three dimensional space for each of the 40 sets of dental casts were recorded in the corresponding X-, Y-, Z-coordinates automatically by using the LK Camio suit 5.5 program (figure 28).

Before recording the dental landmarks, the datum (X, Y, and Z plane) was set at the distobuccal cusp tip of the right second molar on each maxillary dental model and at the distobuccal cusp tip of the left second molar on each mandibular model. The coordinates (X and Y) at these points were defined as zero (figure 22). A planar projection of each cast was subsequently obtained by setting the Z-plane from three-point contact between glass slide and occlusal surface of each dental model. The Z-axis was then perpendicular to this plane (figure 22-23).

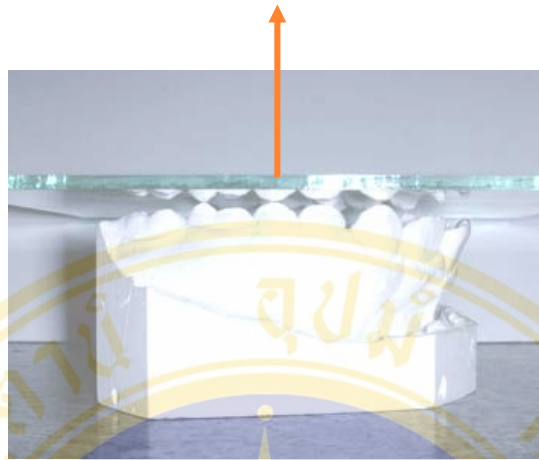


Figure 22: The construction of the Z-axis: the Z- axis was set from the three–point contact between glass slide and occlusal surface of each dental model. The Z axis was perpendicular to this plane.

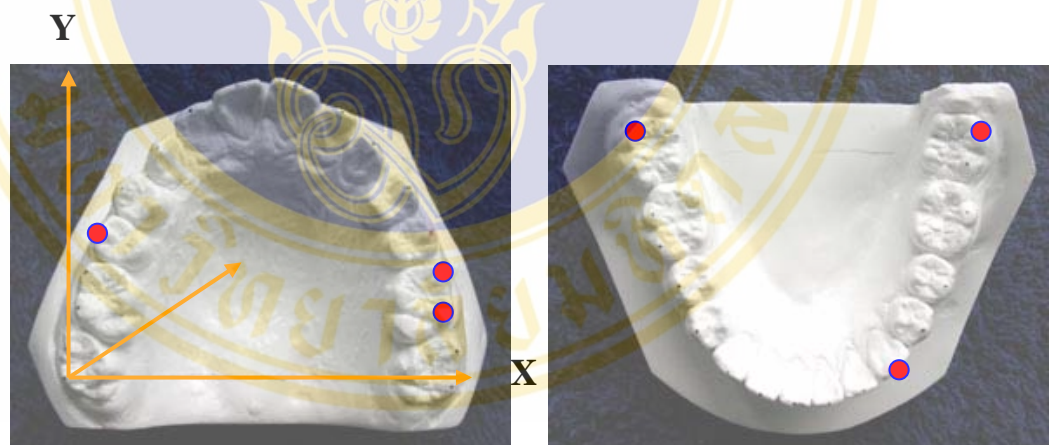


Figure 23: Three points contacted used in constructed of the Z-axis. A planar projection of the Z plane was constructed from the three points (in red) which contacted the glass slide on the occlusal surface of the model.

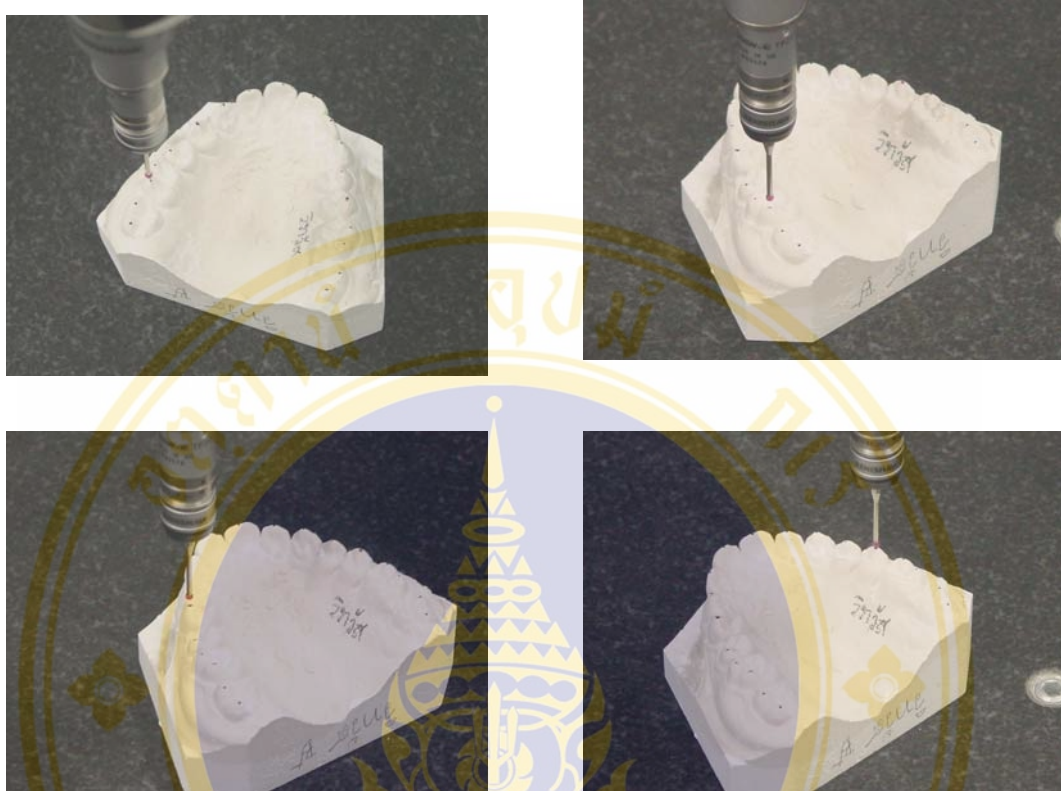


Figure 24: The processing of measuring the dental casts: dental casts were placed on a fixed plane during the recording of the dental landmarks with the Coordinate Measuring Machine.

Second time: The touch trigger probe was used to identify each of the following measurement points respectively from the

Maxillary dental models:

1. point on buccal surface of right second molar
2. point on buccal surface of right first molar
3. point on buccal surface of right second premolar
4. point on buccal surface of right first premolar
5. point on labial surface of right canine
6. point on labial surface of right lateral incisor
7. point on labial surface of right central incisor
8. point on labial surface of left central incisor

9. point on labial surface of left lateral incisor
10. point on labial surface of left canine
11. point on buccal surface of left first premolar
12. point on buccal surface of left second premolar
13. point on buccal surface of left first molar
14. point on buccal surface of left second molar

Mandibular dental models:

1. point on buccal surface of right second molar
2. point on buccal surface of right first molar
3. point on buccal surface of right second premolar
4. point on buccal surface of right first premolar
5. point on labial surface of right canine
6. point on labial surface of right lateral incisor
7. point on labial surface of right central incisor
8. point on labial surface of left central incisor
9. point on labial surface of left lateral incisor
10. point on labial surface of left canine
11. point on buccal surface of left first premolar
12. point on buccal surface of left second premolar
13. point on buccal surface of left first molar
14. point on buccal surface of left second molar

Fourteen points were recorded in each dental model while the X-and Y-coordinates of each landmark were projected to the Z-plane. The coordinates of the landmarks in three dimensional space for each of the 40 sets of dental casts were recorded in the corresponding X-, Y-, Z-coordinates automatically by using the LK Camio suit 5.5 program (figure 28).

Before recording the dental landmarks, the datum (X, Y, and Z plane) was set at the point on buccal surface of the right second molar on each maxillary dental model and at the point on buccal surface of the left second molar on each mandibular model.

The coordinates (X and Y) at these points were defined as zero (figure 25). A planar projection of each cast was subsequently obtained by setting the Z-plane from three-point contact between wire that was bent along arch form and buccal surface of each dental model. The Z-axis was then perpendicular to this plane (figure 25-26).



Figure 25: The construction of the Z-axis: the Z- axis was set from the three–point contact between wire and labial or buccal surface of each dental model. The Z axis was perpendicular to this plane.

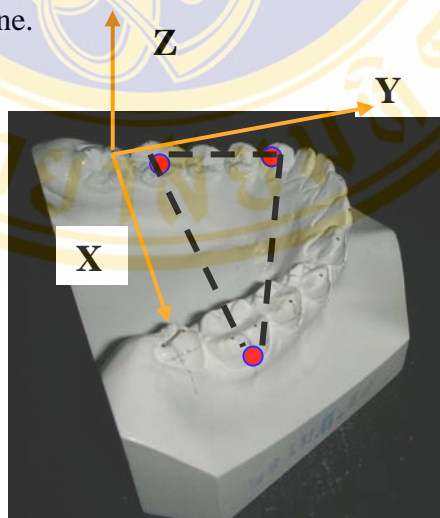


Figure 26: Three points contacted used in constructed of the Z-axis. A planar projection of the Z plane was constructed from the three points (in red) which contacted the wire on the buccal surface of the model.



Figure 27: The processing of measuring the dental casts: dental casts were placed on a fixed plane during the recording of the dental landmarks with the Coordinate Measuring Machine.

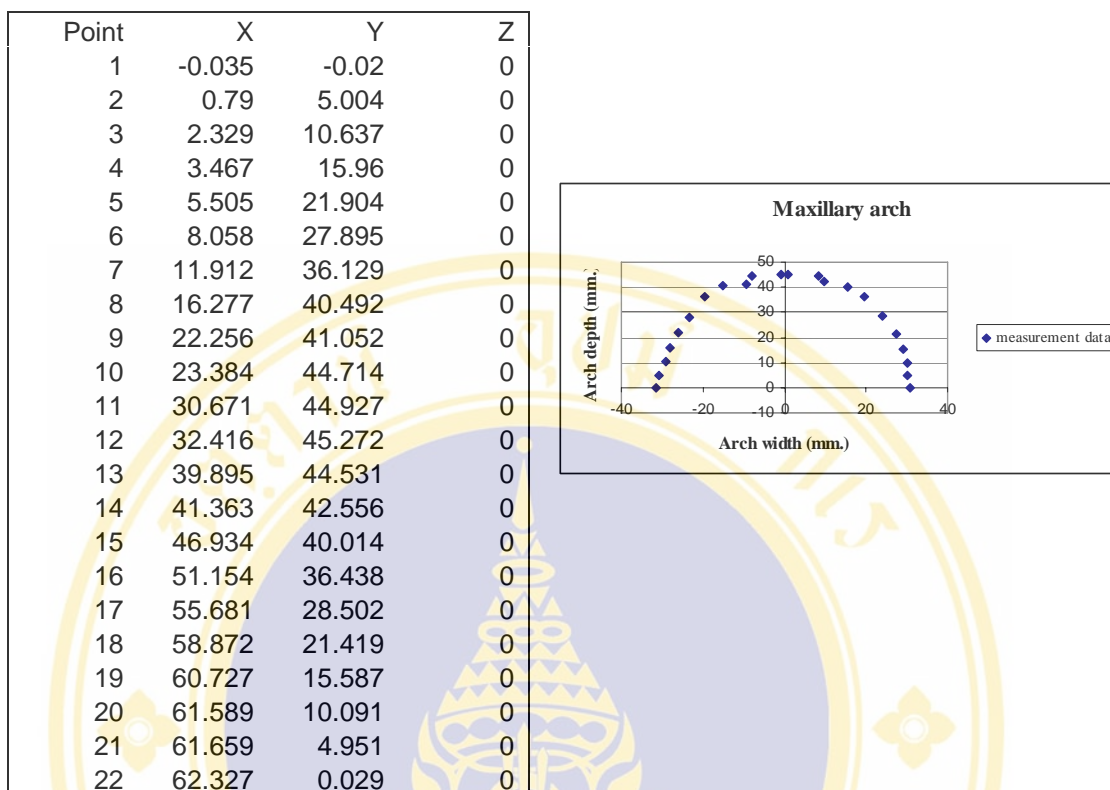


Figure 28: The data of individual landmarks from the CMM measurement: was reported in the X-Y-Z coordinates and a graph was plotted in only two axes from the coordinates X-, and Y-.

Error of measurement

Replicate measurement by the same single examiner were made on 6 sets of randomly selected dental casts after a 1 week interval. The X-, and Y- coordinates of each landmark were compared between two separate of measurements.

The method errors for each of the categories of measurement were calculated according to Dahlberg’s equation.

$$\text{Method error (ME)} = \sqrt{\frac{\sum d^2}{2(n-1)}}$$

d = difference between two successive measurements

n = number of double determinations

Arch interpolation

An analytical equation of the dental arch shape was necessary to describe the relationships between the arch width (X-coordinate) and arch depth (Y-coordinate). Many mathematical functions were investigated as to fit. The beta function most closely represents the dental arch shape (22). Two measurements (molar width and arch depth) were required to generate the dental arch shape. The beta function representing the dental arch shape is given by the general formula (22).

$$Y = \frac{a \left(\frac{X - b + cm}{c} \right)^2 \left(1 - \frac{X - b + cm}{c} \right)}{m^{d-1} n^{e-1}}$$

$$m = \frac{d-1}{d+e-2}; n = \frac{e-1}{d+e-2}$$

Where Y is the arch depth at arch width (X).

Parameter (a) represents the arch depth measured by the perpendicular distance from the most anterior point between two central incisors to the molar cross-arch dimension (figure 23).

The parameter (b) is a offset and is set to zero so the arch centerline is at zero, our reference position.

The parameter (c) is the arch width measured by the cross-arch distance between the second molar distobuccal cusp tips. The parameters (d) and (e) are factors the control the symmetry (figure 23).

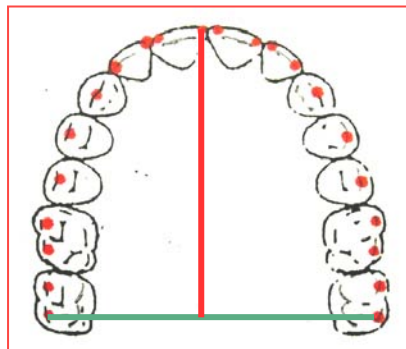


Figure 29: The parameter of beta function: diagram showed the parameter, a; Arch depth (red line), c; Arch width (green line).

If parameter $b = 0$, and $d = e$, the beta function becomes symmetrical about the centerline of the teeth. The beta function then become

$$m = \frac{d-1}{d+e-2} = \frac{d-1}{d+d-2} = \frac{d-1}{2(d-1)} = \frac{1}{2}$$

$$n = \frac{e-1}{d+e-2} = \frac{d-1}{d+d-2} = \frac{d-1}{2(d-1)} = \frac{1}{2}$$

$$Y = \frac{a \left(\frac{X + \frac{c}{2}}{c} \right)^{d-1} \left(1 - \frac{X + \frac{c}{2}}{c} \right)^{d-1}}{\left(\frac{1}{2} \right)^{d-1} \left(\frac{1}{2} \right)^{d-1}}$$

$$= \frac{a \left(\frac{X}{c} + \frac{1}{2} \right)^{d-1} \left(\frac{1}{2} - \frac{X}{c} \right)^{d-1}}{\left(\frac{1}{4} \right)^{d-1}}$$

$$= a \left(4 \left(\frac{1}{4} - \frac{X^2}{c^2} \right) \right)^{d-1}$$

$$Y = a \left(1 - \frac{4X^2}{c^2} \right)^{d-1}$$

For the beta function, the coefficients were estimated using a dedicated computer program, (the computer program was created by Department of Mathematics, Faculty of science, Mahidol university) based on the Differential Algorithm. These also fit standard beta function equations to the data points for each of the eighty dental arches. The curve with a minimal deviation from all data points is

desired. This *best-fitting curve* can be obtained by the least square error methods. That is, the computer was instructed to approximate the coefficients of the beta function of each such that:

$$\sum_{i=0}^n (Y_i - \hat{Y}_i)^2 = 0$$

Where n is the number of data points, Y_i is the Y-coordinates of the data point (X_i, Y_i) on the common line of occlusion and \hat{Y}_i is the computed Y-coordinate of the point on the beta function curve corresponding to the original X_i and the sum of squared distances from the cusp tip coordinates to the fitted curve in the Y direction.

$$\sum_{i=0}^n (Y_i - \hat{Y}_i)^2 = 0 \text{ is minimal}$$

Following this process, each parameter (a, c, d and e) of the beta function were reported for the individual dental arch curvature.

Statistical analysis

1. The mathematics function and coefficient of determination were used to describe the relationship between the recorded and calculated independent variable.
2. Student t-test was used to describe the mean difference of the parameters (a, c, d and e) among the sex and the difference in arch size.
3. Paired t-test was used to describe the mean difference of each coordinate landmark between the first and the second measurements.
4. Paired t-test was used to describe the mean difference of the parameters (a, c, d and e) between landmark on the cusp tips and the bracket positions.

CHAPTER 4

RESULTS

With the least squares method, the curve-fitting program would calculate for every existed. The results were categorized by the type of landmarks as follows:

- 1) landmarks on the cusp tips. The results were shown in Table 2 and 3.
- 2) landmarks on the bracket positions. The results were as shown in Table 4 and 5. The a-parameter represented the arch depth, and the c-parameter represented the arch width at the distobuccal cusp tip of the second molar or at the bracket position of the second molar. While the b-parameter was automatically set to null value by the curve fitting program, the d- and e-parameters were equal. Finally, the beta function became symmetrical at about the centerline of the teeth.

1) landmarks on the cusp tips.

The maxillary arch table (table2) shows the values of each parameter in each individual maxillary dental arch based on the beta function. The mean value of the arch depth (a-parameter) was 48.34 mm. with a standard deviation of ± 2.79 mm. The maximum dimension was 55.66 mm. and the minimum dimension was 42.27 mm. The mean value of the arch width (c-parameter) was 60.29 mm. with a standard deviation of ± 3.12 mm. The maximum width was 69.31 mm. and the minimum width was 52.91 mm. The mean value of the d- and e-parameters were 1.78 with a standard deviation of ± 0.14 . The maximum value was 2.19 and the minimum value was 1.62.

The mandibular arch table (table3) shows the values of each parameter in the individual mandibular dental arch based on the beta function. The mean value of the arch depth (a-parameter) was 41.70 mm. with a standard deviation of ± 2.64 mm. The maximum dimension was 46.94 mm. and the minimum dimension was 35.35 mm. The mean value of the arch width (c-parameter) was 53.93 mm. with a standard deviation of ± 3.48 mm. The maximum width was 59.62 mm. and the minimum width

was 42.19 mm. The mean value of the d- and e-parameters were 1.75 with a standard deviation of ± 0.14 The maximum value was 2.08 and the minimum value was 1.52.

2) landmarks on the bracket positions.

The maxillary arch table (table4) shows the values of each parameter in each individual maxillary dental arch based on the beta function. The mean value of the arch depth (a-parameter) was 41.72 mm. with a standard deviation of ± 2.93 mm. The maximum dimension was 47.68 mm. and the minimum dimension was 36.17 mm. The mean value of the arch width (c-parameter) was 62.56 mm. with a standard deviation of ± 3.19 mm. The maximum width was 71.21 mm. and the minimum width was 55.89 mm. The mean value of the d- and e-parameters were 1.76 with a standard deviation of ± 0.14 . The maximum value was 2.17 and the minimum value was 1.52.

The mandibular arch table (table5) shows the values of each parameter in the individual mandibular dental arch based on the beta function. The mean value of the arch depth (a-parameter) was 37.48 mm. with a standard deviation of ± 3.32 mm. The maximum dimension was 47.43 mm. and the minimum dimension was 31.97 mm. The mean value of the arch width (c-parameter) was 57.32 mm. with a standard deviation of ± 3.39 mm. The maximum width was 64.18 mm. and the minimum width was 46.12 mm. The mean value of the d- and e-parameters were 1.60 with a standard deviation of ± 0.13 The maximum value was 1.88 and the minimum value was 1.38.

As shown in Tables 6 – 9, all values – the measured values, calculated values of maxillary and mandibular dental arches has been well-fit based on the beta function. According to this based sample used, the maxillary and mandibular arches were considered to fit the data well, especially when taking the coefficients of determination average 0.97 in the maxillary arch and 0.96 in the mandibular arch into account .

Table 2. The values of each parameter in individual maxillary dental arch based on the beta function. (landmarks on the cusp tips)

Sample	Maxillary Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
1	52.19	61.35	1.63	1.63
2	48.97	57.77	1.69	1.69
3	47.47	60.70	1.76	1.76
4	48.17	61.18	1.79	1.79
5	47.78	61.89	1.67	1.67
6	45.39	59.79	1.60	1.60
7	51.97	59.14	1.69	1.69
8	50.89	52.91	1.90	1.90
9	52.46	59.49	1.56	1.56
10	42.27	61.92	2.19	2.19
11	50.25	61.24	1.72	1.72
12	50.79	65.22	1.70	1.70
13	48.44	63.40	1.89	1.89
14	46.88	62.01	1.99	1.99
15	47.46	63.26	1.69	1.69
16	55.66	60.19	1.62	1.62
17	47.53	57.77	1.79	1.79
18	51.46	61.54	1.79	1.79
19	52.02	69.31	2.10	2.10
20	46.49	64.06	2.02	2.02
21	46.82	60.70	1.82	1.82
22	46.58	62.32	1.66	1.66
23	47.39	59.01	1.99	1.99
24	47.78	56.51	1.68	1.68
25	48.57	58.67	1.62	1.62
26	46.73	65.89	1.97	1.97
27	46.974	56.44	1.71	1.71
28	43.12	57.20	1.78	1.78
29	49.28	56.59	1.68	1.68
30	48.42	60.38	1.74	1.74
31	47.43	57.66	1.65	1.65
32	49.89	63.62	1.64	1.64
33	45.09	59.80	1.79	1.79
34	49.21	55.20	1.74	1.74
35	45.58	56.38	1.71	1.71
36	45.84	59.41	1.95	1.95
37	43.95	60.20	1.76	1.76
38	47.44	62.48	1.81	1.81
39	53.16	59.73	1.72	1.72
40	49.70	59.06	1.88	1.88
Mean	48.34	60.29	1.78	1.78
SD	2.79	3.12	0.14	0.14

Table 3. The values of each parameter in individual mandibular dental arch based on the beta function. (landmarks on the cusp tips)

Sample	Mandibular Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
1	45.18	55.71	1.67	1.67
2	41.86	52.97	1.79	1.79
3	38.75	54.32	1.74	1.74
4	41.123	55.03	1.79	1.79
5	41.80	59.62	1.53	1.53
6	42.02	56.07	1.69	1.69
7	41.85	52.22	1.80	1.80
8	44.92	42.19	1.85	1.85
9	43.39	55.52	1.67	1.67
10	38.78	52.12	1.74	1.74
11	46.94	54.18	1.76	1.76
12	42.35	56.99	1.61	1.61
13	44.21	59.31	2.03	2.03
14	38.12	54.19	1.73	1.73
15	40.37	56.62	1.71	1.71
16	44.89	54.86	1.65	1.65
17	41.51	53.53	1.78	1.78
18	42.15	52.62	1.72	1.72
19	44.45	59.36	1.90	1.90
20	35.35	53.80	1.65	1.65
21	39.87	53.79	1.76	1.76
22	44.64	56.09	1.89	1.89
23	39.56	54.02	1.98	1.98
24	41.21	50.49	1.76	1.76
25	42.84	51.71	1.53	1.53
26	42.15	58.86	1.90	1.90
27	40.67	51.16	1.73	1.73
28	37.37	50.35	1.69	1.69
29	39.33	49.97	1.59	1.59
30	42.85	54.21	1.61	1.61
31	45.00	52.99	1.76	1.76
32	45.53	59.34	1.73	1.73
33	40.69	48.29	1.52	1.52
34	39.28	49.01	1.60	1.60
35	40.52	54.75	2.04	2.04
36	39.69	50.86	1.79	1.79
37	38.28	54.89	1.71	1.71
38	39.75	58.56	2.08	2.08
39	46.55	52.05	1.61	1.61
40	42.22	54.43	1.90	1.90
Mean	41.70	53.93	1.750	1.754
SD	2.64	3.48	0.14	0.14

Table 4. The values of each parameter in individual maxillary dental arch based on the beta function. (landmarks on the bracket position)

Sample	Maxillary Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
1	42.85	63.44	1.58	1.58
2	44.11	60.82	1.77	1.77
3	38.69	61.93	1.66	1.66
4	42.96	64.11	1.88	1.88
5	41.25	64.22	1.72	1.72
6	38.61	63.06	1.68	1.68
7	47.64	61.22	1.66	1.66
8	38.46	55.89	1.81	1.81
9	47.68	62.75	1.64	1.64
10	36.17	62.38	1.92	1.92
11	45.55	62.37	1.61	1.61
12	44.44	67.61	1.73	1.73
13	41.84	65.06	1.64	1.64
14	40.52	63.17	1.92	1.92
15	44.34	66.34	1.65	1.65
16	45.34	63.52	1.71	1.71
17	43.69	61.20	1.76	1.76
18	44.72	63.42	1.86	1.86
19	45.49	71.21	2.17	2.17
20	38.60	66.89	2.06	2.06
21	38.82	63.35	1.89	1.89
22	38.78	61.93	1.52	1.52
23	41.29	61.56	2.02	2.02
24	42.97	58.07	1.69	1.69
25	40.32	61.48	1.67	1.67
26	41.78	67.64	1.97	1.97
27	37.97	59.29	1.73	1.73
28	37.87	58.59	1.59	1.59
29	41.91	59.38	1.68	1.68
30	42.78	63.05	1.68	1.68
31	42.16	60.05	1.58	1.58
32	39.87	68.69	1.87	1.87
33	38.36	59.75	1.68	1.68
34	41.00	56.37	1.62	1.62
35	42.02	60.49	1.72	1.72
36	38.07	59.97	1.89	1.89
37	37.79	62.31	1.73	1.73
38	45.37	65.23	1.79	1.79
39	43.35	63.59	1.82	1.82
40	43.56	61.22	1.78	1.78
Mean	41.72	62.56	1.76	1.76
SD	2.93	3.19	0.14	0.14

Table 5. The values of each parameter in individual mandibular dental arch based on the beta function. (landmarks on the bracket position)

Sample	Mandibular Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
1	41.33	59.66	1.47	1.47
2	36.48	54.28	1.46	1.46
3	34.21	57.51	1.63	1.63
4	37.89	59.54	1.72	1.72
5	38.08	64.18	1.56	1.56
6	38.32	57.76	1.49	1.49
7	47.43	61.32	1.65	1.65
8	40.70	46.12	1.66	1.66
9	39.67	55.95	1.43	1.43
10	36.38	56.76	1.68	1.68
11	42.29	57.79	1.61	1.61
12	37.18	62.06	1.61	1.61
13	41.86	61.22	1.88	1.88
14	33.99	57.53	1.72	1.72
15	35.39	60.93	1.59	1.59
16	38.98	56.34	1.38	1.38
17	35.25	60.43	1.79	1.79
18	37.03	57.12	1.49	1.49
19	41.69	62.85	1.78	1.78
20	31.97	57.14	1.52	1.52
21	34.65	57.04	1.72	1.72
22	40.41	58.58	1.84	1.84
23	33.07	57.32	1.71	1.71
24	35.35	53.06	1.55	1.55
25	35.69	57.14	1.49	1.49
26	35.12	61.48	1.79	1.79
27	36.95	53.56	1.53	1.53
28	33.96	55.06	1.46	1.46
29	38.21	53.43	1.42	1.42
30	38.52	56.20	1.46	1.46
31	40.76	55.94	1.52	1.52
32	43.49	61.64	1.58	1.58
33	36.14	53.49	1.55	1.55
34	34.44	52.46	1.42	1.42
35	34.95	55.09	1.63	1.63
36	36.29	54.77	1.76	1.76
37	34.89	58.46	1.63	1.63
38	34.28	58.06	1.74	1.74
39	41.54	56.09	1.55	1.55
40	34.43	57.48	1.67	1.67
Mean	37.48	57.32	1.60	1.60
SD	3.32	3.39	0.13	0.13

Table 6. The measured values, calculated value, and the coefficient of determinant of maxillary arches; in all instances of maxillary arch are judged to fit the data well. (landmarks on the cusp tips)

Sample	Measured values (mm)		Calculated values (mm)		R ²
	Depth	Width	Depth	Width	
Maxillary arch					
1	51.2	60.09	54.18	62.94	0.96
2	48.46	57.31	49.59	58.43	0.98
3	46.29	60.42	47.54	60.77	0.96
4	47.6	59.6	48.16	60.95	0.99
5	47.37	61.9	48.80	62.34	0.98
6	44.99	58.89	46.11	62.17	0.96
7	51.77	59.14	53.02	60.06	0.98
8	48.36	52.91	49.74	52.91	0.91
9	52.06	58.17	54.22	61.91	0.95
10	39.95	60.75	39.87	60.75	0.95
11	50.09	57.80	51.36	61.79	0.96
12	50.51	62.85	52.29	65.81	0.94
13	46.36	63.35	47.49	63.37	0.96
14	46.24	62.01	45.62	62.01	0.98
15	45.97	63.09	47.72	63.68	0.99
16	55.09	59.25	58.45	61.98	0.97
17	46.22	56.22	47.48	57.76	0.97
18	51.08	61.54	51.31	61.54	0.99
19	51.84	67.41	50.65	67.44	0.97
20	44.93	59.6	44.96	62.36	0.94
21	45.7	60.7	46.11	60.70	0.99
22	45.22	62.32	46.99	62.32	0.94
23	45.89	56.89	45.74	57.64	0.97
24	47.07	55.001	49.48	56.72	0.97
25	47.68	58.19	49.66	59.30	0.98
26	45.22	64.33	45.91	64.48	0.97
27	44.56	56.43	47.11	56.44	0.99
28	42.09	54.52	43.13	57.20	0.95
29	49.28	54.71	49.93	56.98	0.97
30	47.34	60.02	48.89	60.51	0.97
31	46.57	56.77	49.55	57.94	0.98
32	50.03	63.33	50.41	65.44	0.97
33	44.84	56.95	44.96	59.57	0.92
34	48.47	55.19	49.22	55.20	0.95
35	44.81	56.38	46.30	56.42	0.95
36	43.89	58.77	43.91	58.77	0.98
37	42.73	60.18	44.63	60.20	0.99
38	46.78	61.23	47.27	61.60	0.98
39	50.75	58.19	53.18	60.54	0.95
40	46.84	59.04	47.14	59.05	0.96
Average	47.29	59.26	48.45	60.45	0.97
SD	3.06	3.06	3.47	3.02	0.02

Table 7. The measured values, calculated value, and the coefficient of determinant of mandibular arches; in all instances of mandibular arch are judged to fit the data well. (landmarks on the cusp tips)

Sample	Measured values (mm)		Calculated values (mm)		R ²
	Depth	Width	Depth	Width	
Mandibular arch					
1	44.86	55.71	45.83	56.16	0.97
2	40.73	52.03	41.65	52.62	0.96
3	38.23	54.32	38.81	54.32	0.97
4	40.52	54.72	40.69	54.72	0.99
5	41.27	57.73	42.14	63.57	0.88
6	40.99	54.29	42.07	56.34	0.95
7	40.51	52.22	41.75	52.22	0.95
8	42.21	42.19	43.73	42.19	0.92
9	42.88	55.51	44.01	55.52	0.96
10	38.25	52.12	38.65	52.12	0.99
11	45.95	53.92	46.90	54.08	0.99
12	42.76	56.99	43.10	58.47	0.96
13	42.53	58.55	43.25	58.56	0.94
14	38.07	54.18	38.20	54.19	0.98
15	40.28	56.62	40.63	56.78	0.97
16	42.79	54.86	46.03	54.86	0.96
17	40.71	50.89	41.32	53.20	0.93
18	41.87	52.62	42.28	52.62	0.98
19	45.16	57.89	43.71	57.91	0.98
20	33.37	53.79	36.27	53.81	0.96
21	39.11	53.77	39.87	53.79	0.97
22	43.86	54.59	44.36	55.50	0.95
23	37.88	54.02	38.52	54.02	0.95
24	40.53	49.11	41.17	50.47	0.96
25	41.69	51.71	44.77	53.95	0.93
26	41.37	57.14	41.59	57.18	0.97
27	38.82	51.16	40.96	51.16	0.99
28	36.39	50.35	37.35	51.06	0.94
29	38.25	48.09	41.25	50.82	0.92
30	42.62	52.74	43.38	54.88	0.94
31	43.81	50.75	45.00	52.99	0.95
32	44.62	58.14	45.75	59.74	0.96
33	40.41	46.86	41.38	51.63	0.89
34	39.17	49.01	40.15	49.87	0.97
35	40.71	51.45	40.45	52.77	0.90
36	39.54	50.86	39.44	50.86	0.99
37	38.13	54.89	38.70	55.05	0.98
38	39.41	55.84	37.85	55.85	0.94
39	45.62	52.05	47.30	52.05	0.97
40	41.59	54.43	41.77	54.43	0.97
Average	40.99	53.21	41.80	54.06	0.96
SD	2.51	3.34	2.71	3.39	0.03

Table 8. The measured values, calculated value, and the coefficient of determinant of maxillary arches; in all instances of maxillary arch are judged to fit the data well. (landmarks on the bracket position)

Sample	Measured values (mm)		Calculated values (mm)		R ²
	Depth	Width	Depth	Width	
Maxillary arch					
1	43.06	63.44	43.66	64.54	0.97
2	43.56	60.82	44.10	60.82	0.98
3	38.39	61.93	38.86	61.93	0.97
4	42.12	63.48	42.39	63.42	0.99
5	40.19	64.22	42.16	64.22	0.99
6	38.17	62.07	39.08	63.93	0.97
7	46.02	61.21	48.81	61.94	0.99
8	39.91	55.89	38.43	55.89	0.92
9	47.33	62.75	48.31	64.74	0.97
10	35.77	61.29	35.66	61.29	0.97
11	44.68	62.37	47.81	63.72	0.98
12	43.93	67.58	44.75	67.78	0.93
13	38.17	64.46	42.94	65.53	0.95
14	39.06	62.42	39.74	62.52	0.99
15	43.11	66.33	45.29	67.14	0.99
16	44.05	62.83	45.39	63.82	0.98
17	42.12	60.53	43.71	61.26	0.98
18	43.42	62.18	44.72	62.94	0.98
19	43.54	68.75	43.00	68.75	0.96
20	38.15	62.01	38.02	64.31	0.93
21	37.76	62.59	37.87	62.59	0.96
22	38.35	61.93	40.84	64.40	0.91
23	40.57	60.23	40.64	60.23	0.95
24	42.22	58.07	43.55	58.39	0.98
25	38.98	61.48	41.62	61.48	0.99
26	39.66	67.56	40.29	67.58	0.97
27	37.08	59.29	38.08	59.29	0.99
28	37.56	58.59	38.94	58.61	0.97
29	41.43	58.84	42.43	60.38	0.98
30	41.40	63.05	43.55	63.61	0.98
31	41.79	60.05	45.30	60.81	0.97
32	39.65	65.87	39.56	67.84	0.96
33	38.05	57.80	39.72	60.06	0.96
34	40.53	56.37	41.58	57.03	0.97
35	41.05	60.47	42.03	60.63	0.98
36	36.59	59.97	37.00	59.97	0.98
37	37.18	61.50	37.97	62.42	0.98
38	44.05	64.22	44.99	66.17	0.96
39	40.29	60.57	42.06	61.01	0.95
40	39.87	61.22	42.75	61.22	0.98
Average	40.72	61.90	41.94	62.61	0.97
SD	2.76	2.85	3.15	2.98	0.02

Table 9. The measured values, calculated value, and the coefficient of determinant of mandibular arches; in all instances of mandibular arch are judged to fit the data well.(landmarks on the bracket position)

Sample	Measured values (mm)		Calculated values (mm)		R ²
	Depth	Width	Depth	Width	
Mandibular arch					
1	41.17	59.66	41.67	60.51	0.96
2	36.19	54.28	37.11	55.51	0.95
3	34.14	57.51	34.20	57.51	0.98
4	37.23	59.54	36.98	59.54	0.98
5	38.21	61.45	38.25	64.23	0.92
6	38.17	57.76	39.33	57.76	0.99
7	46.89	61.32	46.67	61.32	0.99
8	39.35	46.12	40.39	46.12	0.94
9	39.54	55.95	40.31	57.67	0.94
10	34.91	56.74	35.97	56.74	0.99
11	41.69	57.79	42.30	57.80	0.99
12	37.22	62.06	37.18	62.06	0.98
13	40.79	61.16	40.84	61.16	0.94
14	34.19	57.53	33.36	57.53	0.99
15	34.98	60.38	35.42	61.01	0.98
16	38.05	56.34	40.77	58.47	0.91
17	35.12	56.36	34.97	57.75	0.93
18	36.89	57.11	37.53	57.12	0.99
19	41.22	62.84	40.04	62.84	0.97
20	31.32	57.14	32.55	57.15	0.97
21	34.42	57.04	34.51	57.04	0.96
22	37.94	58.49	38.22	58.51	0.94
23	32.92	57.32	33.02	57.32	0.96
24	35.08	53.06	35.58	53.06	0.98
25	35.22	55.82	36.50	57.71	0.93
26	34.98	61.40	32.59	61.42	0.96
27	36.43	53.56	37.88	53.56	0.996
28	33.45	55.06	33.99	55.08	0.96
29	37.83	53.43	39.22	54.24	0.96
30	38.59	56.20	39.46	57.06	0.96
31	40.35	55.94	41.71	55.94	0.98
32	43.18	61.64	43.60	61.64	0.99
33	35.74	52.14	36.78	53.77	0.94
34	35.38	52.46	35.68	53.37	0.96
35	34.72	55.08	34.79	55.09	0.95
36	36.02	54.77	35.40	54.77	0.98
37	34.91	57.32	34.69	58.31	0.96
38	34.42	58.06	33.63	58.06	0.98
39	41.36	56.09	42.46	56.09	0.98
40	34.46	57.45	34.31	57.47	0.97
Average	37.12	57.11	37.50	57.48	0.96
SD	3.18	3.28	3.38	3.28	0.02

A sample of the beta function, which has landmark on the cusp tips, was calculated from the measured data points of maxillary and mandibular arches as shown in figures 30 and 31 respectively. Arch width was measured by the gap between the coordinate distance of the second molar distobuccal cusp tips. Then arch depth was measured by height from the peak of the arch to the arch width's line. As seen from the curve fit analysis, the true values underestimated the calculated values, by approximately 1.2 mm. in the maxillary and 0.8 mm. in the mandibular arches (table 6 and 7).

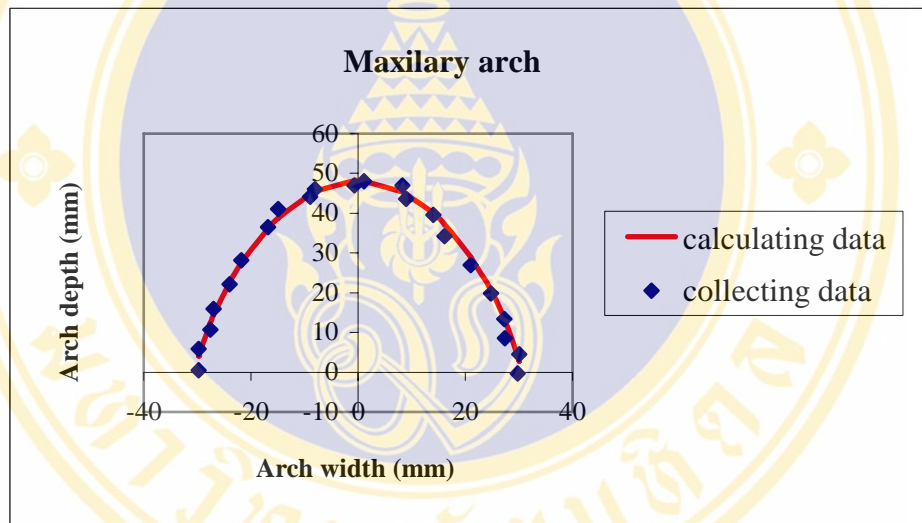


Figure 30: Coordinates that were plotted at the cusp tips of maxillary dental arch as data- collected in this study. The superimposition of the collected data (blue dot) and the calculated data from the beta function (red line) are as shown in this figure.

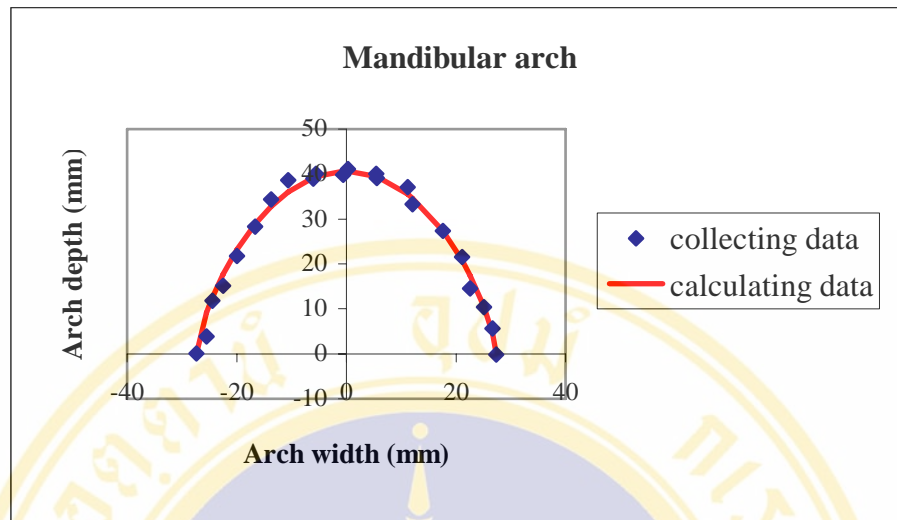


Figure 31: Coordinates were plotted at the cusp tips of mandibular dental arch as data- collected in this study. The superimposition of the collected data (blue dot) and the calculated depth data from the beta function (red line) are as shown in this figure.

A sample of the beta function, which has landmark on the bracket positions, was calculated from the measured data points of maxillary and mandibular arches as shown in figures 32 and 33 respectively. Arch width was measured by the gap between the coordinate distance of the second molar bracket position. Then arch depth was measured by height from the peak of the arch to the arch width's line. As seen from the curve fit analysis, the true values underestimated the calculated values of arch width, by approximately 0.7 mm. in the maxillary arches and 0.4 mm. in the mandibular arches. The measured values underestimated the calculated values of arch depth, by approximately 1.2 mm. in the maxillary arches and 0.4 mm. in the mandibular arches. (table 8 and 9).

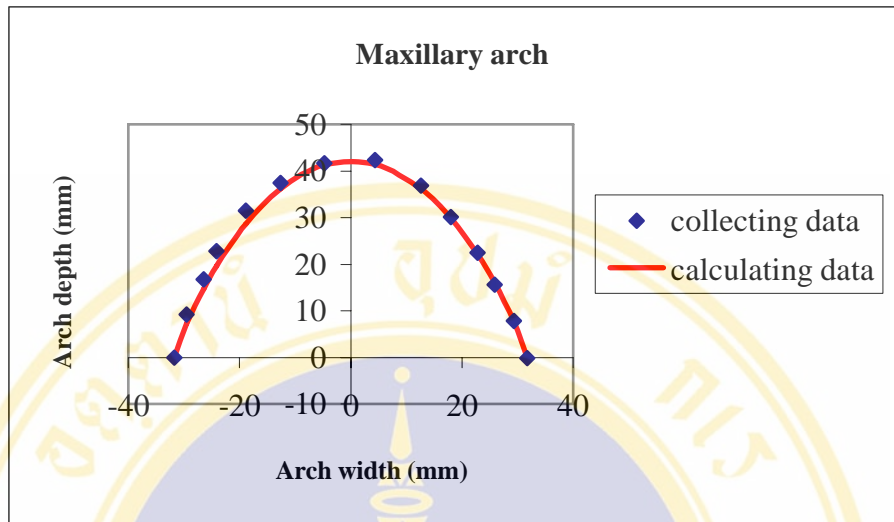


Figure 32: Coordinates were plotted at the bracket positions of maxillary dental arch as data-collected in this study. The superimposition of the collected data (blue dot) and the calculated data from the beta function (red line) as shown in this figure.

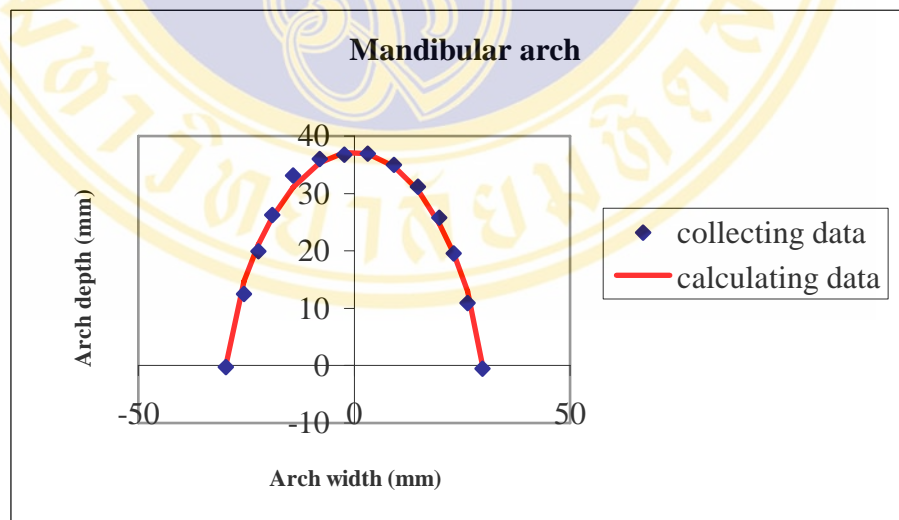


Figure 33: Coordinates were plotted at the bracket positions of mandibular dental arch as data-collected in this study. The superimposition of the collecting data (blue dot) and the calculated data from the beta function (red line) as shown in this figure.

Table 10. Descriptive statistics (Means and SD) of each parameter both in the maxillary and mandibular arches between males and females. (landmarks on the cusp tips)

Parameter	Female (n=20)		Male (n=20)	
	Mean (mm)	SD	Mean (mm)	SD
<i>Maxilla</i>				
a-parameter	46.29	2.27	48.32	3.44
c-parameter	58.46	2.90	60.12	3.05
d-parameter	1.77	0.11	1.79	0.17
<i>Mandibular</i>				
a-parameter	40.68	2.47	41.20	2.84
c-parameter	52.35	3.00	54.06	3.51
d-parameter	1.76	0.16	1.74	0.11

Table 10 was the descriptive statistics (Means and SD) of each parameter, which came from recording at the cusp tips, both in the maxillary and mandibular arches between males and females. All of the parameters indicated the small difference in values between the sex dimorphism. The comparison of each instance between males and females are as shown in table 11. No statistically significant difference of any parameter was found between males and females at p-value < 0.05 both in the maxillary and mandibular arches except at the a-parameter in the maxillary arch. It was found that the arch depth in maxillary arch from males were significantly greater than females average 2.03 mm. at p-value < 0.05.

Table 11. Statistical difference of each instance between males and females. (landmarks on the cusp tips)

Parameter	Mean diff (mm)	t	p-value
<i>Maxillary</i>			
A	2.03	2.205*	0.035
C	1.66	1.760	0.086
D	0.027	0.591	0.559
<i>Mandibular arch</i>			
A	0.52	0.618	0.54
C	1.71	1.657	0.106
D	.019	-0.430	0.67

* Significant at p-value < 0.05

Table 12 was the descriptive statistics (Means and SD) of each parameter, which came from recording at the bracket positions, both in the maxillary and mandibular arches between males and females. All of the parameters indicated the small difference in values between the sex dimorphism. The comparison of each instance between males and females are as shown in table 13. No statistically significant difference of any parameter was found between males and females at p-value < 0.05 in the mandibular arches. There was statistically significant difference of the a- and c-parameter between males and females at p-value <0.05 in the maxillary arch. It was found that the arch depth and arch width in maxillary arch from males were significantly greater than females at p-value < 0.05. The differences were 2.03 mm. in the arch depth and 1.84 mm. in the arch width.

Table 12. Descriptive statistics (Means and SD) of each parameter both in the maxillary and mandibular arches between males and females. (landmarks on the bracket position)

Parameter	Female (n=20)		Male (n=20)	
	Mean (mm)	SD	Mean (mm)	SD
<i>Maxilla</i>				
a-parameter	39.70	1.99	41.74	3.08
c-parameter	60.98	2.73	62.83	2.72
d-parameter	1.75	0.13	1.77	0.16
<i>Mandibular</i>				
a-parameter	36.37	2.71	37.86	3.49
c-parameter	56.12	2.61	57.95	3.64
d-parameter	1.60	0.13	1.61	0.13

Table 13. Statistical difference of each instance between males and females. (landmarks on the bracket position)

Parameter	Mean diff (mm)	t	p-value
Maxillary			
A	2.03	2.482*	0.018
C	1.84	2.139*	0.039
D	0.025	0.537	0.594
Mandibular arch			
A	1.49	1.511	0.139
C	1.84	1.832	0.076
D	0.0074	0.182	0.856

* Significant at p-value < 0.05

Table 14 and 15 were the comparisons of the parameters from data-recording on the cusp tips and bracket positions. All of the parameters indicated the difference in values between two recorded landmarks. There was statistically significant difference of every parameters between two recorded landmarks at p-value <0.05 except at the d-parameter in the maxillary arch. The results was shown that the a-parameter in both arches recorded on the cups tips were significantly greater than those on the bracket positions at p-value <0.05. The c-parameter in both arches recorded on the bracket positions were significantly greater than those on the cusp tips at p-value <0.05. The d-parameter recorded on the cusp tips were significantly greater than that on the bracket positions only in the mandibular arch at p-value <0.05.

Table 14. Descriptive statistics (Means and SD) of each parameter both in the maxillary and mandibular arches between recorded on the cusp tips and on the bracket positions

Parameter	Cusp tips (n=40)		Bracket position (n=40)	
	Mean (mm)	SD	Mean (mm)	SD
<i>Maxilla</i>				
a-parameter	47.30	3.05	40.72	2.76
c-parameter	59.26	3.06	61.90	2.85
d-parameter	1.78	0.144	1.76	0.143
<i>Mandible</i>				
a-parameter	40.99	2.51	37.11	3.18
c-parameter	53.21	3.34	57.11	3.28
d-parameter	1.75	0.14	1.60	0.13

Table 15. Statistical difference of each instance between recorded on the cusp tips and on the bracket positions

Parameter	Mean diff (mm)	t	p-value
<i>Maxillary arch</i>			
A	6.583	21.443**	0.000
C	-2.642	12.582**	0.000
D	0.019	1.17	0.249
<i>Mandibular arch</i>			
A	3.871	11.264**	0.000
C	-3.896	16.077**	0.000
D	0.146	8.832**	0.000

** Significant at p-value < 0.01

Table 16 and 17 was the comparisons of the parameters in the Angle's classification I from Aukvongseree's study (41) and the Angle's classification II division 1 from this study. The difference of every parameters in the maxillary arch between two Angle's classification was statistically significant at p-value <0.05. The results has shown that the a- and d-parameter of class II were significantly greater than those of class I at p-value <0.05. The c-parameter of class II were significantly lesser than that of class I at p-value <0.05. In the mandibular arch , only the c-parameter of class II was significantly lesser than that of class I at p-value <0.05.

Table 16. Descriptive statistics (Means and SD) of each parameter both in the maxillary and mandibular arches between class I and II

Parameter	Class I (n=40)		Class II (n=40)	
	Mean (mm)	SD	Mean (mm)	SD
<i>Maxilla</i>				
a-parameter	44.87	2.07	47.296	3.06
c-parameter	60.62	2.79	59.26	3.06
d-parameter	1.67	0.095	1.78	0.14
<i>Mandible</i>				
a-parameter	40.74	2.12	40.987	2.51
c-parameter	55.21	3.17	53.21	3.34
d-parameter	1.77	0.11	1.75	0.14

Table17 . Statistical difference of each instance between classification I and II malocclusion.

Parameter	Mean diff (mm)	t	p-value
<i>Maxillary</i>			
A	-2.43	-4.157**	0.000
C	1.358	2.075*	0.041
D	-0.108	-3.944**	0.000
<i>Mandibular arch</i>			
A	0.249	-0.480	0.632
C	1.998	2.743**	0.008
D	0.025	0.883	0.380

** Significant at p-value < 0.01, * Significant at p-value < 0.05

The curves were generated by putting the values of the mean arch widths, the mean arch depth (table 6-9) and the mean of d-parameter (table 2-5) received from the samples into the equation as follows:

$$Y = a \left(1 - \frac{4X^2}{c^2} \right)^{d-1}$$

The maxillary and mandibular dental arch equations for each arch are as shown below:

The maxillary arch

Landmarks on the cusp tips

$$Y = 47.29 \left(1 - \frac{4x^2}{59.26^2} \right)^{0.78}$$

Landmarks on the bracket positions

$$Y = 40.72 \left(1 - \frac{4x^2}{61.9^2} \right)^{0.76}$$

The mandibular arch

Landmarks on the cusp tips

$$Y = 40.99 \left(1 - \frac{4x^2}{53.21^2} \right)^{0.75}$$

Landmarks on the bracket positions

$$Y = 37.12 \left(1 - \frac{4x^2}{57.11^2} \right)^{0.6}$$

Error of measurement

The method error was calculated by repeating the registration of dental landmark randomly with 6 sets of study models by keeping the period of timing differs approximately one week. Double determination reveal errors in the measurements, including mean and standard deviations. Tables 18 to 21 were the summaries of these measurements. No statistically significant difference (at p -value < 0.05) was found between the two series of recording. Table 22 has shown the errors of measurement which were smaller and acceptable.

Table 18. The statistical differences of the X-coordinate from maxillary dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Maxillary arch						
X-coordinate						
X1	-0.02	0.07	-0.01	0.05	-0.01	0.703
X2	1.69	1.37	1.70	1.33	-0.01	0.821
X3	4.72	1.67	4.69	1.64	0.03	0.236
X4	7.10	1.77	7.10	1.77	0.00	0.954
X5	12.00	2.21	12.03	2.09	-0.03	0.583
X6	18.93	2.00	18.88	2.07	0.05	0.148
X7	27.01	2.73	27.03	2.70	-0.02	0.743
X8	36.70	1.94	36.87	2.03	-0.17	0.097
X9	44.97	2.40	44.90	2.48	0.07	0.351
X10	51.20	2.47	51.24	2.51	-0.04	0.237
X11	56.08	2.40	56.13	2.37	-0.05	0.463
X12	58.73	2.20	58.72	2.22	0.01	0.885
X13	61.70	1.04	61.70	1.07	0.00	0.874
X14	63.74	1.96	63.69	1.92	0.05	0.099

Table 19. The statistical differences of the Y-coordinate from maxillary dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Maxillary arch						
Y-coordinate						
Y1	-0.01	0.34	0.01	0.35	-0.02	0.434
Y2	9.14	0.68	9.17	0.64	-0.03	0.155
Y3	17.46	1.71	17.47	1.67	-0.01	0.842
Y4	24.25	1.89	24.28	1.87	-0.03	0.420
Y5	32.29	2.70	32.28	2.70	0.01	0.686
Y6	37.58	2.41	37.55	2.44	0.03	0.264
Y7	42.18	3.59	42.16	3.61	0.02	0.135
Y8	41.81	3.60	41.81	3.56	0.00	0.945
Y9	36.43	3.43	36.48	3.49	-0.05	0.422
Y10	31.45	2.33	31.44	2.37	0.01	0.773
Y11	23.01	2.30	22.98	2.32	0.02	0.453
Y12	16.05	2.24	15.99	2.23	0.06	0.169
Y13	8.39	1.93	8.37	1.96	0.02	0.677
Y14	-0.00	0.20	-0.05	0.25	0.05	0.142

Table 20. The statistical differences of the X-coordinate from mandibular dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Mandibular arch						
X-coordinate						
X1	0.22	0.20	0.27	0.09	-0.05	0.219
X2	2.00	1.78	2.02	0.74	-0.02	0.597
X3	6.02	2.35	5.96	2.41	0.06	0.334
X4	8.59	1.99	8.55	2.03	0.04	0.403
X5	13.52	1.54	13.46	1.50	0.06	0.134
X6	19.95	1.42	19.91	1.37	0.04	0.190
X7	26.39	1.70	26.37	1.68	0.02	0.619
X8	32.19	1.50	32.19	1.55	0.00	0.985
X9	38.09	1.88	38.09	1.88	0.00	0.909
X10	44.91	1.59	44.83	1.54	0.08	0.185
X11	49.52	1.90	49.50	1.90	0.02	0.339
X12	51.32	2.83	51.35	2.84	-0.03	0.098
X13	56.31	2.66	56.30	2.63	0.01	0.958
X14	58.21	3.14	58.19	3.12	0.02	0.413

Table 21. The statistical differences of the Y-coordinate from mandibular dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Mandibular arch						
Y-coordinate						
Y1	0.16	0.20	0.19	0.17	-0.03	0.464
Y2	10.30	1.35	10.31	1.33	-0.01	0.746
Y3	19.86	1.53	19.83	1.44	0.03	0.630
Y4	26.60	2.09	26.64	2.11	-0.04	0.102
Y5	32.99	2.08	32.98	2.19	0.01	0.848
Y6	36.74	1.84	36.73	1.82	0.01	0.778
Y7	38.52	1.59	38.56	1.53	-0.04	0.259
Y8	38.56	1.64	38.57	1.61	-0.01	0.739
Y9	36.57	1.55	36.53	1.61	0.04	0.499
Y10	32.99	1.63	33.06	1.60	-0.06	0.173
Y11	26.96	2.48	26.98	2.43	-0.02	0.469
Y12	21.29	1.64	21.22	1.62	0.07	0.162
Y13	11.71	1.26	11.76	1.22	-0.04	0.073
Y14	0.03	0.23	0.09	0.15	-0.06	0.257

Table 22. The measurement errors calculated using Dahlberge's equation.

Dental landmark	Maxillary arch		Mandibular arch	
	X-coordinate (mm)	Y-coordinate (mm)	X-coordinate (mm)	Y-coordinate (mm)
1	0.076	0.095	0.072	0.065
2	0.052	0.040	0.053	0.044
3	0.046	0.036	0.099	0.095
4	0.043	0.054	0.087	0.055
5	0.090	0.025	0.076	0.093
6	0.070	0.056	0.057	0.050
7	0.111	0.035	0.069	0.055
8	0.196	0.039	0.058	0.042
9	0.119	0.101	0.048	0.092
10	0.067	0.059	0.099	0.083
11	0.127	0.056	0.049	0.051
12	0.084	0.082	0.041	0.084
13	0.047	0.081	0.047	0.046
14	0.054	0.057	0.050	0.080

CHAPTER 5

DISCUSSION

It is proven that, the stability of the mandibular dental arch's dimension and form is as same as a factor in the stability of the therapeutic results itself. Technological advance in material, with the use of new arch wires and appliances, is capable to generate rapid results during phases of alignment and during leveling between arches. However, the choice of the dental arches form has become essential when using these elastic arch wires.

Nowadays in Thailand, the clinician always use preform arch wires, which are based on Caucasian arch forms, and choose the one that closely match their patient's arch form. Based on the previous studies on stability and relapse (1, 41), it's agreed that preservation of the mandibular intercanine width and original arch form during orthodontic treatment will be resulted in the good post-treatment occlusal stability. In the study of Nojima and McLaughlin (33), it appears that the frequency of a particular arch form varies among ethnic groups. Benjamin et.al. (42) has found that the differences of ethnic (American Blacks and Whites) also seem to correlate with those of arch form. Clinically, it seems more reasonable to have several types of preformed arch wire available and then select the shape that closely matches the most with the patient's pretreatment arch form based on his or her ethnicity and type of malocclusion.

There are several studies associating a geometric curve form with the dental arch, some have predetermined a mathematical and geometric arch form on the basis of landmarks, which are recorded on systems of coordinates (21-24); While the other studies has also tend to individualize the arch form to respect the original mandibular intercanine width during treatment, by using arch guides (see Raberin(43) and references therein) or by using a computer-assisted determination of an ideal dental arch form.

The present work aims at defining the mathematical equation of the dental arch's shape in the Angle's classification II division 1 that best fit to Thai and clearly clarify differences between normal occlusion and class II division 1 malocclusion.

A sample of 40 subjects (20 males and 20 females) with class II division 1 malocclusion was selected from patient records at the Department of Orthodontics, Faculty of Dentistry, Mahidol University and from private dental clinics. The models included the full permanent teeth (excluded the third molar).

In this study, the measurement was done by using the CMM (Coordinate Measuring Machine) to record the landmarks on dental model and report these landmarks in the corresponding X-, Y-, and Z- coordinates. CMM is used extensively in the precision machinetool industry. As previously described, Braun et al (22) and Qiong et al (44) have used CMM to record the X-, Y- coordinates of dental landmarks for the study of human dental arch forms and the comparison of intermaxillary tooth size discrepancies among different malocclusion groups. The study of the reliability of CMM in measuring dental models (39) had concluded that the Coordinate Measuring Machine can be applied to measuring dental casts in three dimensions with highly accurate, reliable and reproducible results.

Replicated measurements by the same single examiner were made on six sets of study casts with a time lapse of one week each. The paired t-test, which was used to assess any systematic error of the machine, was reliable and reproducible. The method error of the coordinates's measurements of dental landmarks were calculated based on Dahlberg's equation. As a result, very small error range has been shown (0.025-0.196 mm.). Intraexamination has also reconfirmed that the results were accurate and well-calibrated.

Lu (17) has used the multiple regression, correlation, and the coefficient of determination to define how well-fit of the fourth order polynomial equation in the analysis of the form, symmetry and asymmetry of the dental arch. In his research, however, the observed values were (except for small errors of measurement) the truth, while the calculated values were only the approximations to the truth. Resulting from his efforts to obtain an effective representation, neither can properly be regarded as a random variable, which loosed their more time. While Bookstein's algorithm for fitting conic sections to data, or Sampson's modification of that algorithm (see

Sampson (45) and the references therein) was well-suited to modeling the shape of the dental arch. Previous approaches to modeling the outline of the dental arch have used fitting algorithms which depend on the exact specification of a coordinate system.

Many of the algorithms are classic least-squares methods, minimizing the sum of squared distances from data points to a fitted curve in the direction parallel to an arbitrarily chosen Y-axis.

In the present, we used the evolution algorithm, a good optimization method (46), to calculate the parameters of the beta function from the collected data of each dental arch. It was based on the minimized sum of squared errors of the calculated values to the measurement values (details about the estimation of its parameters from a sample of data were given by Department of Mathematics, Faculty of Science, Mahidol university). The goal was to obtain the measure of the closeness of agreement between the true arch form and the approximation. The total sums of squares accounted for by the regression was a measure of closeness of an approximation. The ratio

$$R^2 = 1 - \frac{\sum_{i=1}^n e_i^2}{\sum_{i=1}^n (y_i - \bar{y})^2} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

(where y_i is the Y-coordinate of the data point and \hat{y}_i is the computed Y-coordinate of the point on the beta function curve corresponding to the original X_i)

was equivalent to the coefficient of determination in the multiple-correlation sense. As a rule, the beta function will fit the arch form quit nicely, the investigation may require that $R^2 \geq a$; for $0 < a < 1$ to suit this purpose (16).

In this study, we show the goodness of fit of the beta function to represent each of the 40 maxillary and mandibular arches, as also verified by the high coefficient of determination (R^2), average 0.97 with a standard deviation of 0.03 (tables 6-9). This is in agreement with Braun et.al. (21), who has been the first to offer the beta function to describe the dental arch, with an average correlation coefficient of 0.98 with a standard deviation of 0.02.

According to the geographic mathematic, the beta function describing the human arch form, the general formula (the relationship between the width and depth of the dental arch) can be expressed as:

$$Y = a \left(1 - \frac{4X^2}{c^2} \right)^{d-1}$$

As described earlier, If the a-parameter (arch depth) and c-parameter (arch width) were set to 40 mm. and 55 mm. respectively while the d-parameter was put into the different value, the resulting curve would express into different arch forms, as illustrated in figure 34.

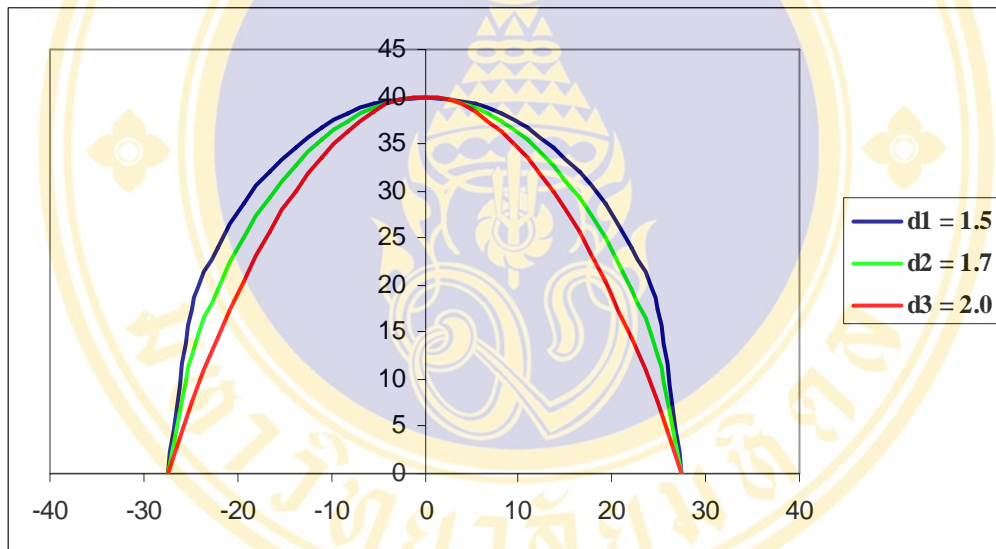


Figure 34: The curves expressed different arch forms according to the difference in d-values.

Since the d-parameter measures the quadratic portion, the larger the magnitude of d-parameter, the more the arch would assume the form of a taper; roughly, we may say it is the measuring of “taperedness”. The consequence of small d-parameter in physical terms would tend to give the arch a more square-like look. In this description, we may say that d-parameter is a description of the form of dental arch. The more the values of the d-parameter, the narrower the arch form is to be. The less the values of the d-parameter, the wider the arch form is to be.

According to the d-parameter, we found that the arch form of the Thai subjects have a more wider-like look than that of the Caucasian subjects in the study of Braun both the Angle's classification I and II division 1. Because the maxilla d-parameter of class I and II are 1.67 and 1.78 respectively, the mandibular d-parameter of class I and II are 1.77 and 1.75 respectively, meanwhile the d-parameter of the Caucasian subjects in the study of Braun equal to 1.8 in both maxillary and mandibular arches (22). The superimposition has shown in figure 35-38. It is agreed by Nojima and MaLaughlin (33) who compared the Caucasian and Japanese mandibular clinical arch forms. Their study has concluded that the Caucasian have a statistically significant decreased arch width and increased arch depth compared with the Japanese. While the subjects were regrouped by arch form, no statistically significant difference in arch dimension was observed between the two ethnic groups in any of the arch form samples. As a result, it is suggested that there was no single arch form specific to any of the Angle's classifications or ethnic groups.

Kook et. al. (47) has compared Korean and North American white clinical arch forms. The result has shown that, among the Korean, the most frequent arch form is square, whereas among the white, the tapered arch form has been predominated. Therefore the current preformed superelastic wires are too narrow for many patients in the Asian and should be modified when they are being treated.

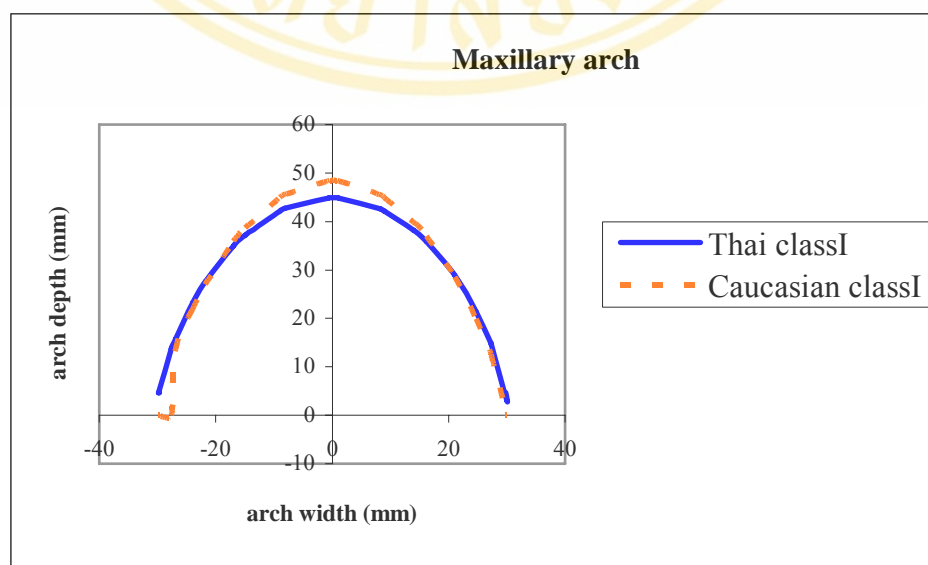


Figure 35: The superimposition of class I maxillary arch form that were calculated from formula of Braun's and this study

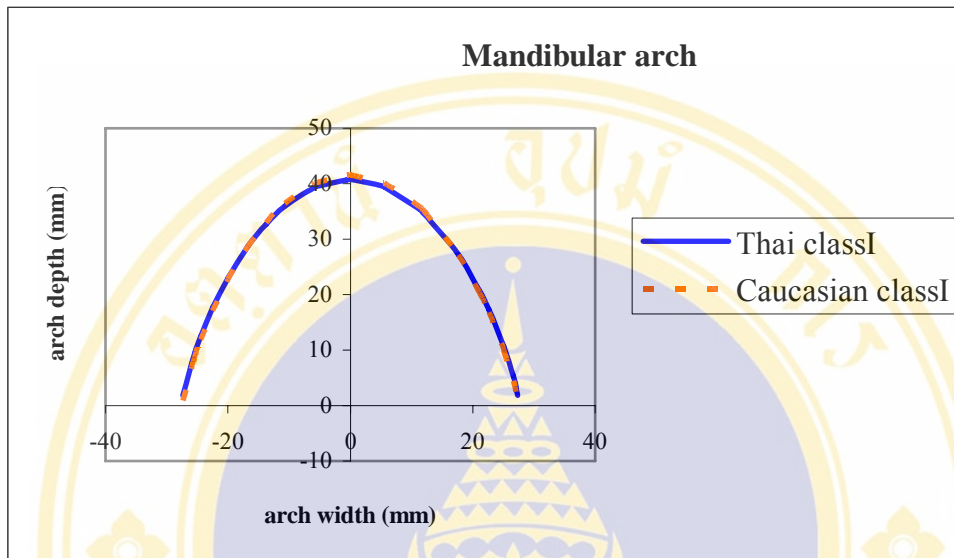


Figure 36: The superimposition of class I mandibular arch form that were calculated from formula of Braun's and this study

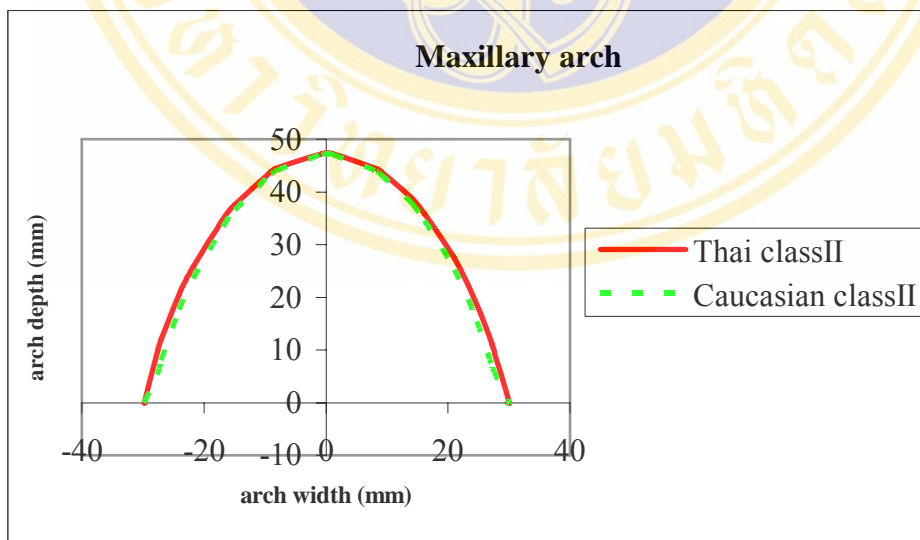


Figure 37: The superimposition of class II maxillary arch form that were calculated from formula of Braun's and this study

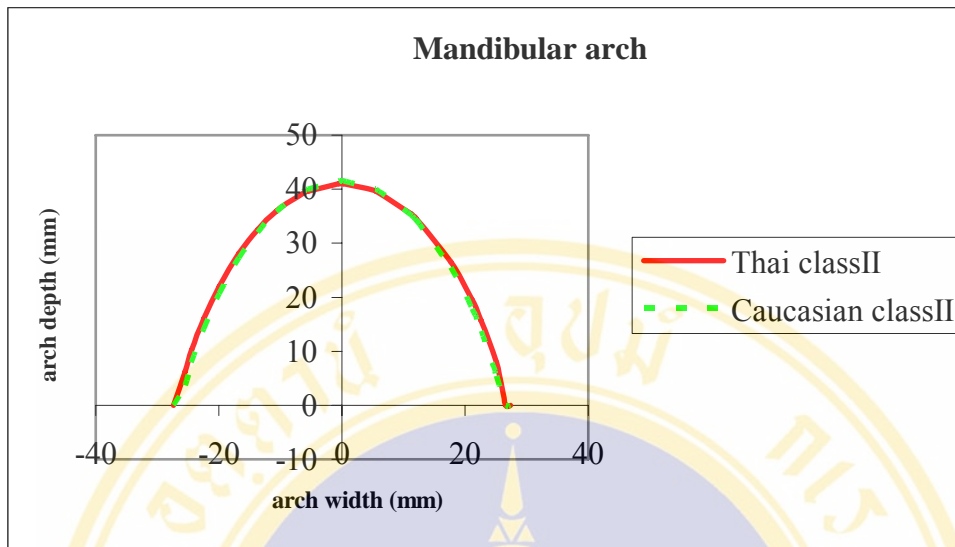


Figure 38: The superimposition of class II mandibular arch form that were calculated from formula of Braun's and this study

When arch form was derived from the measurement at the cusp tips, the difference between male and female was only the arch depth (a-parameters) in maxillary arch. Also, when arch form was derived from measurement at the bracket positions, the differences between genders are both in the arch depth (a-parameters) and in the arch width (c-parameters) in maxillary arch (Table 10-13). This observation was as same as that from Ferrario et.al. (21, 35), who defined the mathematical formulas of the shape of dental arches in human permanent healthy dentitions. The maxillary arch were wider than the mandibular arch regardless of gender. Gender differences were found majorly in the maxillary arch, where they reflected more of a size discrepancy than a shape difference. Gender seemed to have some influence on arch sized differences (men has shown larger differences than women). The differences of the arch depth are because of differences of samples' overjet.

There are important differences between "Research arch form" and "Clinical arch form". For example, the work of Braun et al. (22) has reported that the human arch form could be represented by a complex mathematical formula, known as the beta function. This was calculated by entering measurements of dental landmarks on orthodontic models into a computer curve-fitting program. Braun et al. (22) has measured "the center of each incisor incisal edge, the cusp tips of the canines and

premolars, and the mesiobuccal and distobuccal cusp tips of the molars” and this is called the “research arch form”.

In contrast, the clinician’s arch wire shape must be based on the points where the wire will lie in the bracket slots of correctly positioned brackets. This “clinical arch form” relates to the mid-points on the labial surface of the clinical crowns of the teeth, and should include an estimation for the in-out which is built into the bracket system. The “Clinical arch form” is not related to the incisal edge or the cusp tips which are used to establish the “Research arch form”.

Therefore, in this study, there were two measurement methods. The comparison of the two measurement methods has shown that the a-parameter (arch depth) in both arches as recorded on the cusp tips were significantly greater than that on the bracket positions at p-value <0.05. The c-parameter (arch width) in both arches as recorded on the bracket positions were significantly greater than that on the cusp tips at p-value <0.05. The d-parameter (taperedness) as recorded on the cusp tips were significantly greater than that on the bracket positions only in the mandibular arch at p-value <0.05. This agreed with McLaughlin (14) who has shown that the research arch form is more tapered than the clinical arch form. Due to the superimposition between maxillary and mandibular arches, there were no co-ordination between two arches in the research arch form but there were co-ordination between two arches in the clinical arch form. Therefore, it is appropriated to use clinical arch form as a basis for the construction of arch wire but it should include the thickness of bracket’s base.

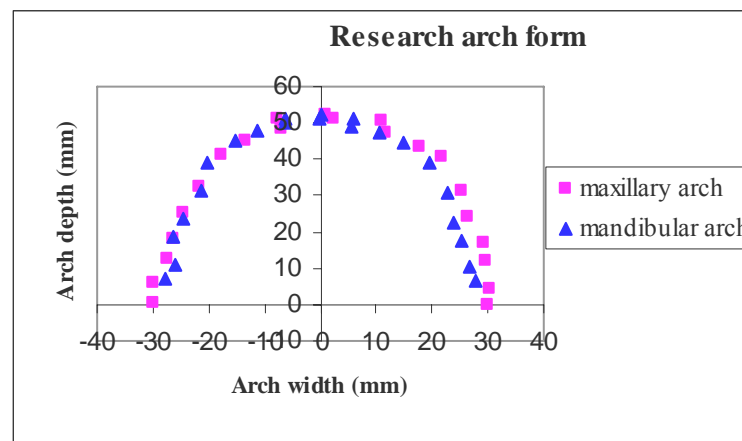


Figure39: superimposition of maxillary and mandibular research arch form

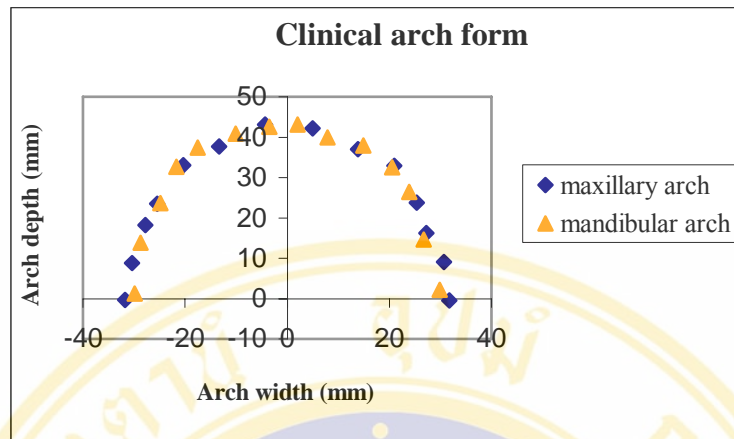


Figure40: superimposition of maxillary and mandibular clinical arch form

In this study, the arch size cannot be categorized into small, medium, and large like in the Augvongseree's study (2) because the samples of this study are classified in Angle's Classification II division 1 which express protrusive maxillary incisors. Therefore, a-parameter (arch depth) should be more concentrated than c-parameter (arch width).

As a result from the comparison of Class I and Class II, when the Class II mandibular arches are compared to the Class I arches, an average generalized reduced arch width of 2 mm. is observed and it is apparent that arch depth and taperedness are essentially the same.

Comparing Class II maxillary arches to Class I arches, an average generalized reduced arch width of 1.4 mm. is evident, and an average increased arch depth of 2.4 mm. is observed. Perhaps this can be explained by the fact that some Class II relationships are resulted from a large maxillary body. The d-parameter of Class II is more than that of Class I, which means that arch form of Class II is more tapered than that of Class I. Clinicians have speculated on the reasons for this difference—nasal obstruction, finger habits, tongue thrusting, low tongue position, abnormal swallowing and sucking behaviors. Interestingly, the Class I and Class II groups had similar mandibular canine widths.

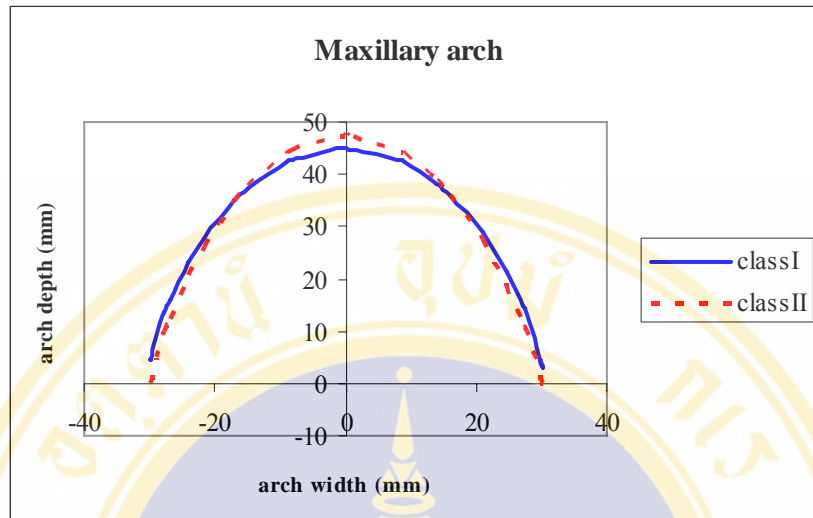


Figure 41: The superimposition of the calculating data from the beta function of class I from Augvongseree’s study (blue line) and class II from this study (red point) are showing in this figure.

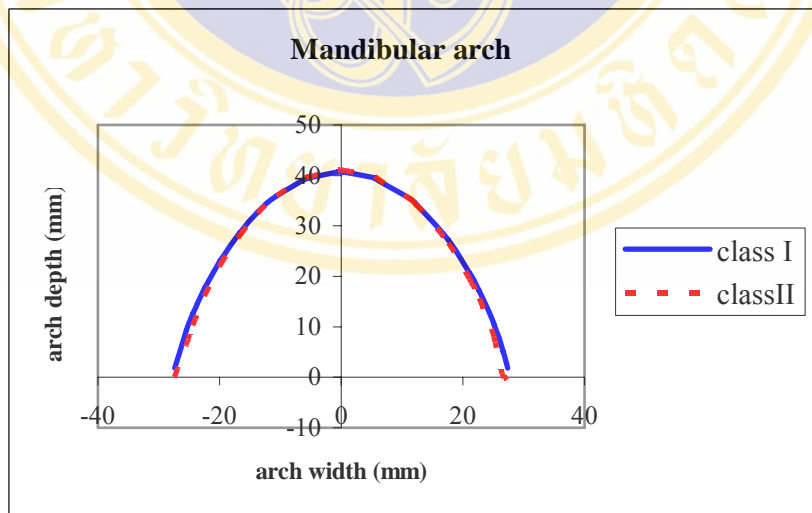


Figure 42: The superimposition of the calculating data from the beta function of class I from Augvongseree’s study (blue line) and class II from this study (red point) are showing in this figure.

Clinical application

According to the mean of d-parameter, the general formula which represent the maxillary arch are as follows:

Research arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.78}$$

Clinical arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.76}$$

The general formula which represent the mandibular arch are as follows:

Research arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.75}$$

Clinical arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.6}$$

The a-(arch depth) and c-(arch width) parameter are varies depending on the arch dimension of the individual. If that patient has maxillary arch depth of 47 mm. and arch width of 59 mm., the arch form of these patient can be predicted by putting the a-parameter with 47 mm. and the c-parameter with 59 mm. in the equation. In addition, the arch form can be predicted after arch expansion, comparing to the pre-treatment arch form.

Due to the differences between Class I and II arch form especially in the maxillary arch, this knowledge for selection of arch wires can be applied as follows:

- In the leveling and movement phases, Class II arch wire should be selected. Because if choosing the Class I arch wire, the arch will be expanded.

- After the contraction phase, Class I arch wire should be selected. Because occlusion was changed from Class II (large overjet) to Class I (normal overjet)

Limitations and Suggestions

The beta function is an accurate representative of Thai dental arches in the Angle's classification II division 1. The precision of these equations, to represent the dental arch of the population, depends on the form of those dental arches. The more the arch shape resembles that of the study samples, the higher the precision of the beta function.

The present study has been focused an attention on the dental malocclusion regardless of the skeletal pattern that coincided with it. Further studies, therefore, should select the samples by the criteria of Angle's classification and skeletal discrepancies

Due to the small sample sizes in this study, the further studies is needed to fit the curves of the beta function of dental arches and to examine the variability from average shapes in a population.

The clinical arch form is appropriate to be used as a basis for the construction of arch wire than the research arch form. Also, it should include an estimation for the in-out which is built into the bracket system.

Comparing Class I to II groups. The mandibular Class I arch wire can be used in mandibular Class II arch wire but the arch width that is suitable with the original arch width of patients should be selected. In maxillary arches, the use of predicted arch form after finishing of treatment is more appropriate. Therefore, further studies should be about the post treatment arch form of Angle's classification II division 1.

Finally, previous studies (21, 22, 38) have been focused on the variation of the dental arches form among the different types of Angle's classification. Hence additional studies to evaluate arch form in Angle's classification III in Thais would be desirable to provide better understanding of the Thai dental arch forms.

CHAPTER 6

CONCLUSIONS

The samples is comprised of forty sets of dental casts (20 males and 20 females) , which expressed Angle's classification II division 1. The results has been revealed that:

1. From the current study, the beta function accurately described Thai dental arch forms with the high coefficient of determination, average 0.97 in the maxillary arch and average 0.96 in the mandibular arch. The general formula are generated as follows:

The maxillary arch

Research arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.78}$$

Clinical arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.76}$$

The mandibular arch

Research arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.75}$$

Clinical arch form

$$Y = a \left(1 - \frac{4x^2}{c^2} \right)^{0.6}$$

The equations represent the maxillary and mandibular arches based on the difference in arch width, as follows:

The maxillary arch

Research arch form

$$Y = 47.29 \left(1 - \frac{4x^2}{59.26^2} \right)^{0.78}$$

Clinical arch form

$$Y = 40.72 \left(1 - \frac{4x^2}{61.9^2} \right)^{0.76}$$

The mandibular arch

Research arch form

$$Y = 40.99 \left(1 - \frac{4x^2}{53.21^2} \right)^{0.75}$$

Clinical arch form

$$Y = 37.12 \left(1 - \frac{4x^2}{57.11^2} \right)^{0.6}$$

2. The d-parameter of beta function is a description of the form of dental arch. The higher the values of d-parameter, the more tapered the arch form, and the less the values of d-parameter, the more squared the arch form.

3. According to the d-parameter, it is found that the Thai dental arch forms have a wider-look like (maxillary d-parameter = 1.78, mandibular d-parameter = 1.75) than the Caucasian arch form in Braun's study when d-parameter is equal to 1.8 in both maxillary and mandibular arches.

4. Gender seemed to have some influences on arch size differences, with the men showing larger difference than the women.

5. In the construction of archwires, the clinical arch form is more appropriate to use than the research arch form.

6. Class II maxillary arch form was narrower, deeper and more tapered than that of Class I. Class II mandibular arch form was narrower than that of Class I but their arch depths and taperednesses were similar.



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