THE FORM OF DENTAL ARCH IN ANGLE'S CLASSIFICATION I IN THAI

LT. CHANTIRA AUKVONGSEREE

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ABSTRACT

The purpose of the present study is to define a generalized equation describing the dental arch form of Thais by applying a computer-curve fitting program. The study included 40 sets (20 males and 20 females) of dental casts which expressed normal Angle's Classification I occlusion. Twenty-two dental landmarks from cusp tips of molar, premolar, canine and distal and mesial points of the incisal edges of each dental cast were triggered and recorded using the Coordinate Measuring Machine and reported into their corresponding coordinates (X-, and Y- direction). The coordinates were processed through a computer curve-fitting program to define the parameter of beta function to describe the dental arch.

The Thai, normal occlusion dental arches were shown to be accurately represented mathematically by the beta function. The coefficient of determination between measured arch-shape data and the mathematical arch shape, expressed by the beta function, was ranging from 0.93-0.99. Thai dental arch forms have a wider-look like than the Caucasian arch form in Braun's study. Gender differences were found, where they reflected more of a size discrepancy than a shape differences. Males arch forms were significantly larger than females at p-value < 0.01 in the maxilla and at p-value < 0.001 in the mandibular arches.

The beta function accurately describes the Thai Angle's Classification I dental arch forms. These curves were geometrically simple and their coefficient in the beta function can be more easily understood than previous studies.

KEY WORDS: BETA FUNCTION / DENTAL ARCH FORM / COORDINATE MEASURING MACHINE

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บทคัดย่อ

วัตถุประสงค์ของการศึกษาครั้งนี้เพื่อค้นหาสมการทั่วไปทางคณิตศาสตร์ที่จะใช้ในการอธิบายถึงแนวความโค้งการเรียงตัวของฟันในขากรรไกรในคนไทยที่มีการสบฟันแบบที่ 1 ข้อมูลได้จากการวัดแบบจำลองฟันที่มีการสบฟันที่ปกติแบบที่ 1 จำนวน 40 ชุด (เพศชาย 20 ชุด, หญิง 20 ชุด) โดยเครื่องวัดพิกัดสามมิติวัดที่จุดยอดฟันกรามใหญ่, ฟันกรามน้อย, ฟันเขี้ยว, จุดใกล้กลาง และจุดไกลกลางที่ปลายฟันหน้าจำนวน 22 จุด บนแบบจำลองฟันแต่ละชิ้นและรายงานผลการวัดจากเครื่องวัดพิกัดสามมิติเป็นพิกัดความสัมพันธ์ในแนวระนาบแกนเอกซ์และวาย ข้อมูลจะถูกนำมาใช้เพื่อหาค่าสัมประสิทธิ์ของสมการ เบต้า ฟังก์ชัน เพื่อใช้ในการอธิบายแนวความโค้งของการเรียงตัวของฟัน

ผลการศึกษาพบว่า เบต้า ฟังก์ชันเป็นสมการทางคณิตศาสตร์ที่จะใช้ในการอธิบายถึงแนวความโค้งการเรียงตัวของฟันในคนไทยที่มีการสบฟันแบบที่ 1 โดยมีความถูกต้องแม่นยำสูง แสดงด้วยค่าสัมประสิทธิ์แห่งการกำหนดช่วงกว้างจาก 0.93-0.99 นอกจากนี้ยังพบว่าแนวความโค้งการเรียงตัวของฟันในขากรรไกรในคนไทยจะมีลักษณะที่กว้างกว่าคนผิวขาวโดยเปรียบเทียบกับการศึกษาของ Braun S. ไม่พบความแตกต่างของแนวความโค้งการเรียงตัวของฟันในขากรรไกรระหว่างเพศหญิงและชาย แต่พบว่าเพศชายจะมีขนาดของแนวความโค้งการเรียงตัวของฟันที่ใหญ่กว่าหญิงอย่างมีนัยสำคัญทางสถิติที่ $p\text{-value} < 0.01$ ในฟันบน และ $p\text{-value} < 0.001$ ในฟันล่าง

เบต้า ฟังก์ชันเป็นสมการที่จะใช้ในการอธิบายถึงแนวความโค้งการเรียงตัวของฟันในคนไทยที่มีการสบฟันแบบที่ 1 โดยมีความถูกต้องแม่นยำสูง สัมประสิทธิ์ที่อยู่ในสมการสามารถอธิบายและเข้าใจได้ง่ายกว่าการศึกษาที่ผ่านมา

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CHAPTER 1

INTRODUCTION

Although a number of researchers have attempted to identify an arch form unique to a certain group, most of their studies compared average clinical arch forms derived from normal untreated samples or research arch forms established by measuring arch dimensions using the incisal edges and cusp tips as landmarks.

From the standpoint of clinical orthodontics, however, not only does the determination of the patient's posttreatment arch form help meet esthetic requirements but it also benefits for long–term occlusal stability. Based on previous studies on relapse, it is generally agreed that post-orthodontic original mandibular intercanine widths are preserved from the original arch form.

Little (1) based on more than 35 years of research, recommended as a clinical guideline that the patient's pretreatment arch form be used as a guide to posttreatment arch shape. The application of a single ideal arch form to every member of an ethnic group, despite individual variations, may adversely affect posttreatment occlusal stability.

Meanwhile, with the recent advancements in superelastic wire materials and preadjusted appliance systems, preformed arch wires have been commercially available and frequently used, mainly in the leveling and alignment stages. However, their superelastic property makes customization of arch form and sized difficult. After more than 20 years with the preadjusted appliance, it is apparent to the authors that some customizing of the arch form for individual patients is important. In-out is built into the preadjusted appliance, and this eliminates the need for first order bends. It thus simplifies arch form, but it does not eliminate the need to use different shapes for different individuals. Failure to do some customization will create the probability of relapse in some cases, and can lead to unnatural esthetics.

If a broad arch form is used for an individual with a narrow facial appearance, for example, there will be a risk of relapse and an unnatural look to the smile. A return to the concept of customizing arch form for each patient, but without the need for needless wire bending, is a desirable and sensible approach. Clinically, it seems more reasonable to have several types of preform arch wire available and to select the shape that most closely match the patient's arch form according to their ethnicity and type of malocclusion.

For several years, researchers have been trying to define the ideal arch form, frequently using the concept that the dental arch form is similar to the mathematical formulas as well as the catenary curve. Several studies of the shapes of dental arches have received both praise and criticism, and several have formed the basis for commercially produced arch form, but this is more appropriate to Caucasian than Thais.

In several studies, it also holds that people form different ethnic groups present different modal conditions. Furthermore, the previous study showed that different types of Angle's classification presented different arch forms, and the clinician should anticipate the differences in size and form rather than treating all cases to a single ideal.

This study aim to quantify the nature of the arch form in Thai samples and to define the mathematical equation of the shape of the dental arch which is better suited to Thais.

The purposes of the study

1. To define a generalized equation describing the dental arch form of Thais by the application of a computer curve – fitting program.
2. To predict the form of dental arch of individual patient.
3. To determine the dynamic relationships between arch depth and arch width which may yield important clinical applications.
4. To compare the form of dental arch between Thai and Caucasian Angle's classification I occlusion (from Braun's study).
5. To compare the dental arch form according to sexual dimorphism.

6. This present work can be used as a database for further studies in arch wire construction.

The expected benefits of this study

1. This is a good opportunity to be the first to establish the stratregies of the definition the mathematical function that describes the Thai dental arch.
2. It is scientific data to present what exactly the Thai dental arch form is.
3. The deduction of the equation will be useful in determining the dynamic relationships between arch depth and arch width and may yield important clinical applications.
4. It is an appropriate database useful for producing commercial arch forms that are fitting to Thais.

Statement of hypothesis

1. There are correlations between the recorded coordinates and the calculated coordinates from the equation.
2. The beta function fits the Thai dental arch form.
3. There are no differences in dental arch forms among sexual dimorphism.

CHAPTER 2
LITERATURE REVIEW

The search for the ideal arch form

In 1885, Bonwill (2) noted the tripod shape of the lower jaw and declared that it formed an equilateral triangle with the base extending from condyle to condyle and the sides extending from each condyle to the midline of the central incisors. He stated that this triangle existed for the proper functioning of the teeth. Importantly, he noted that the bicuspids and molars formed a straight line from the cuspids to the condyles. In 1905, Hawley (3) employed some of Bonwill's principles in proposing a geometric method for constructing the ideal arch form. Hawley suggested that the six anterior teeth be made to lie along a circle whose radius equaled their combined widths. From this circle he created an equilateral triangle, the base of which represented the inter-condylar width. It was proposed that the bicuspids and molars should be aligned along these extended straight lines. Hawley did, however, advised against the strict use of this method for determining arch form, and that it be used only as a guide in establishing the arch form.

Numerous authors described other shapes for the dental arches. In 1902, Black (4) stated that the upper teeth are arranged in a semi-ellipse and that the lower teeth were arranged similarly on a smaller curve. Broomell, in the same year, said that " the teeth are arranged in the jaws in the form of two parabolic curves, the superior arch describing the segment of a larger circle than the inferior, as a result of which the upper teeth slightly overhang the lower".

In 1907, Angle (5) discussed in detail the "line of occlusion", which he defined as being "the line with which, in form and position according to type, the teeth must be in harmony if in normal occlusion". The form of this line was said to resemble a parabolic curve but one that varied greatly due to race, type, temperament, etc.of the individual. Because of these variables, Angle did not consider the Bonwill-Hawley arch form to be useful for anything more than a general approximation of the true line of occlusion. In describing the first order bends needed in the arch form for proper tooth positioning, Angle objected particularly to the straight line proposed from cuspid to third molar. Angle stated that a straight line existed from the cuspid to the mesiobuccal cusp of the first molar, however, there was a natural curvature needed in the molar region.

In 1942, Gray's Anatomy (6) stated the following about human arch form: "The maxillary dental arch forms an elliptical curve...The mandibular dental arch forms a parabolic curve". In 1934, Chuck (7) noted the variation in human arch form and pointed out that arch forms had been referred to as square, round, oval, tapering, etc. He stated that while the Bonwill-Hawley arch form was not suitable for use in each patient, it could serve as a template for the construction of individualized arch forms. Chuck superimposed this arch form on a millimeter grid and used this template for archwire construction according to Angle's method. Chuck suggested that the bicuspid regions should be wider than the cuspids to prevent excessive expansion of the cuspids. In 1963, Boone (8) proposed the similar superimposition of the Bonwill-Hawley arch form on a millimeter template for construction of the individualized edgewise arch form.

Thus, over the years the Bonwill-Hawley arch form has been the most consistently used arch form as a beginning template for edgewise orthodontists. It is the "standard" arch form offered by most orthodontic manufacturers today.

In 1949, MacConaill (9) stated that, in considering the line of occlusion, it would be impossible for an ellipse and a parabola to meet one another at every point. He concluded that the ellipse-parabola description of the two dental arches, although elegant, had no immediate relation to function. He stated that a certain simple and well known curve, the catenary curve, fit so many cases with exactness that it could be taken as the "ideal curve"of common occlusions. The catenary curve was formed simply by suspending a chain of appropriate length from two points of varying width (for example width of the most distal molars in the arch form).

In 1957, Scott (10) also supported the concept of the catenary curve as the shape of the human arch based on the developmental anatomy of the dental arches and surrounding anatomic structures. He pointed out that the basal bone of the maxilla and mandible remains much more constant in form in all mammals and forms a foundation on which a great deal of variation in form of alveolar processes are constructed. In man, the dentition maintains the primordial catenary form because alveolar process growth does not show a regional differentiation but remains more or in direction in all parts of the arch.

Burdi and Lillie (11) in 1966 further stated that the basic bony arch is established as early as 9.5 weeks in utero and that this form was that of the catenary curve. However, their research actually demonstrated numerous arch forms outside the catenary form. Musich (12) in 1973, supported the concept of the catenary curve as the ideal arch form and suggested the use of the catenometer as a reliable device for construction of arch perimeter. The caternary curve creates a rather tapered arch form and many of the tapered arch forms provide by orthodontic manufacturers today are based on the catenary curve (13).

The last major publication attempting to establish the “ideal” arch form was presented by Brader (14) in 1972. He stated that dental arch form was made up of teeth which assume unique positions along a compound curve representing an equilibrium at all points and delimited by the counterbalancing forces of the tongue and circumoral tissues. The geometry of the curve of the dental arch form was said to be best approximated by a closed curve with the curvilinear properties inherent in the trifocal ellipse, with the teeth occupying only the portion at the constricted end the curve.

Brader recommended an arch guide with five arch forms. The selection of the proper arch form was based on arch width at the second molars as measured at the facial, gingival surface. The maxillary arch form was selected one size larger than the mandibular arch form. While the Brader arch form provided a convenient method of archwire selection, many clinicians found that this arch form created excessive narrowing in the cuspid region of many patients and led to excessive wear of the incisal portion of the cuspids.

Dental arch shapes have been extensively classified both using simple qualitative descriptions and more complex mathematical methods. In the first case the dental arches were described elliptic, parabolic, U-shaped or approximately catenary. More complex procedures involved different curve-fitting mathematical models.

Howes (15) showed that arch width at the first premolar is a practical clinical tool for assessing the taperedness of a dental arch(when the 4 – 4 width is narrow) versus the broad or square arch (where 4 – 4 width is broader).

Lu (16) 1966 showed that a fourth – order orthogonal polynomial produces a close fit with the shape of the human dental arch. A fourth degree orthogonal polynomial may be used to represent the dental arch. The formular was presented as

$$Y = b_1X^1 + b_2X^2 + b_3 X^3 + b_4 X^4$$

Each of the 4 weighting coefficients in the equation was interpretable in terms of an arch's form. The linear (b_1) and cubic terms (b_3) measure the asymmetry. They measure the lopsidedness and the tiltedness of the asymmetry respectively.The quadratic (b_2) and quartic (b_4) terms measure the symmetry. They measure the tapperedness and the squaredness of the symmetry respectively. The above procedure may be used in classifying classes of dental arches in terms of dental arch isomorphism.

The sketches of the eight possible combinations of asymmetry were as follows:

1. No asymmetry

If the lines appear to be at right angles with the lines Y, there is no asymmetry, as in figure 1. It corresponds to $b_1=b_3=0$.

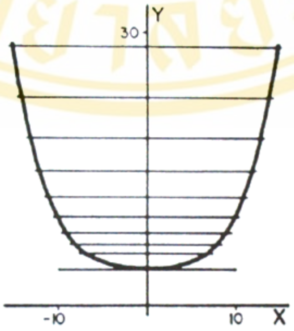

Figure 1 : Dental arch without asymmetry in Lu's study: $b_1 = b_3 = 0$.

2. Linear asymmetry

If the lines appear parallel but tilted upward towards the rights, there is positive linear asymmetry, as illustrated in figure 2. It corresponds to $b_1 > 0$, and $b_3 = 0$. If the lines appear parallel but tilted downward towards the right, there is negative linear asymmetry, as illustrated in figure 3. It corresponds to $b_1 < 0$, and $b_3 = 0$.

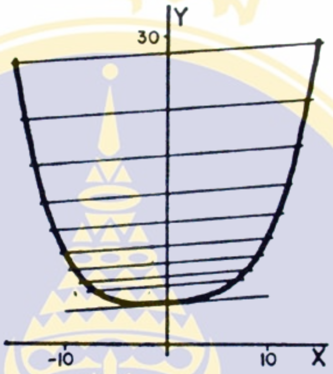

Figure 2: Dental arch with positive linear asymmetry in Lu's study: $b_1 > 0$, $b3 = 0$.

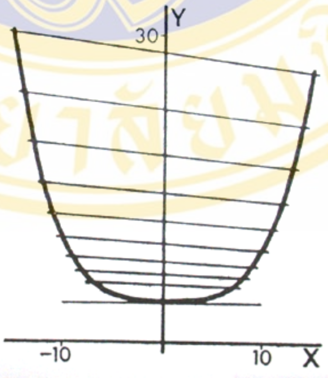

Figure 3: Dental arch with negative linear asymmetry in Lu's study: $b_1 < 0$, $b_3 = 0$.

3. Cubic Asymmetry

If the lines appear downward toward the right, the slope become steeper as x increases and become parallel to x axis as they approach the origin, then there is negative cubic asymmetry, as illustrated in figure 4. It corresponds to $b_1 = 0$ and $b_3 < 0$.

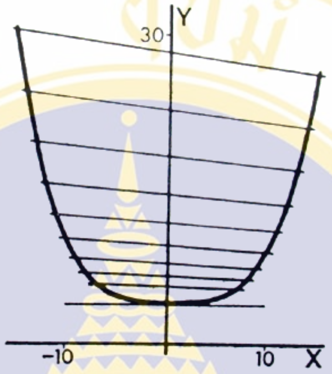

Figure 4: Dental arch with negative cubic asymmetry in Lu's study: $b_1 = 0$, $b_3 < 0$.

If the lines appear upward toward the right, the slopes become steeper as x increases, decrease and become parallel to x axis as they approach the origin, there is positive cubic asymmetry, as showed in figure 5. It corresponds to $b_1 = 0$, and $b_3 > 0$.

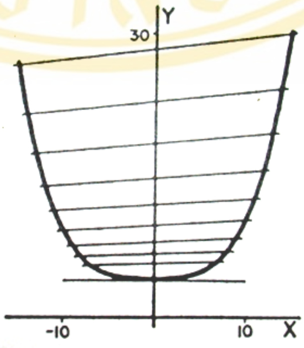

Figure 5: Dental arch with positive cubic asymmetry in Lu's study: $b_1 = 0$, $b_3 > 0$.

4. Linear-cubic asymmetry

4.1. Negative linear-negative cubic asymmetry, the lines tilt downward towards the right, the slopes become steeper as x increase, decrease as they approach the origin, but they do not become parallel to x axis, as illustrated in figure 6. It corresponds to $b_1 < 0$, $b_3 < 0$.

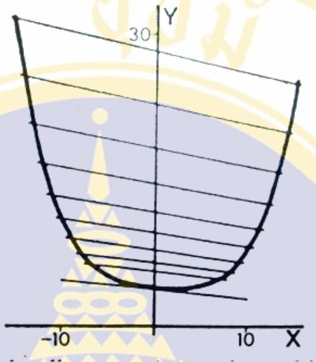

Figure 6: Dental arch with negative linear and negative cubic asymmetries in Lu's study: $b_1<0$, $b_3 < 0$.

4.2. Positive linear-positive cubic asymmetries, the lines tilt upwards towards the right, the slopes become steeper as x increases, decrease as they approach the origin, but they do not become parallel to x axis, as illustrated in figure 7. It corresponds to $b_1 > 0$, $b_3 > 0$.

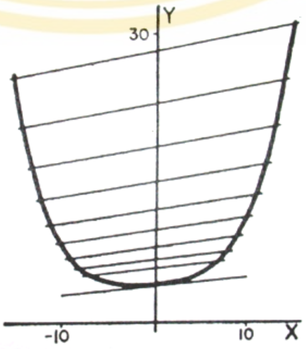

Figure 7: Dental arch with positive linear and positive cubic asymmetries in Lu's study: $b_1 > 0$, $b_3 > 0$.

4.3. Negative linear-positive cubic asymmetry, the lines do not become parallel to x axis as they approach the origin; the slopes at the lower portion of the arch tilt downward towards the right, become flatter as x increases, and finally become upward toward the right, as illustrated in figure 8. It corresponds to $b_1 < 0$, $b_3 > 0$.

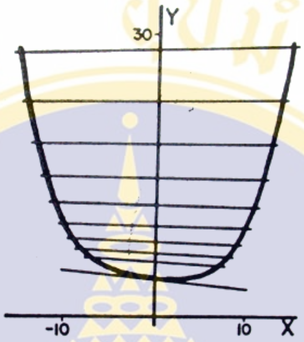

Figure 8: Dental arch with negative linear and positive cubic asymmetries in Lu's study: $b_1 < 0$, $b_3 > 0$.

4.4. Positive linear-negative cubic asymmetry, the lines do not become parallel to x axis as they approach the origin. The slopes at the lower portion of the arch tilt upward toward the right, become flatter as x increases, and finally become downward towards the right, as illustrated in figure 9. It corresponds to $b_1 > 0$, $b_3 < 0$.

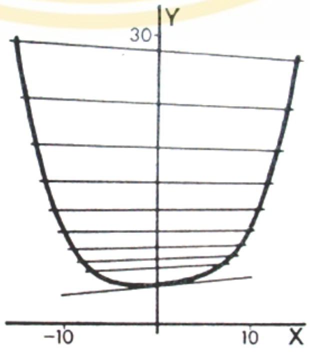

Figure 9: Dental arch with positive inear and negative cubic asymmetries in Lu's study: $b_1 > 0$, $b_3 < 0$.

Biggerstaff 1972 (17) stated of these methods the fourth, fith, or sixth degree least squares polynomial regression equations give the most accurate and reproducible likeness of arch form. However, the fitted curves were not descriptive and no satisfactory statistical method exist for comparing the coefficients. Furthermore, the statistics of coefficients were difficult to interpret. The known variability of the human dental arch is responsible, at least partially, for the many qualitative descriptions. Generally, one of three curves (ellipsoidal, parabolic, or hyperbolic) will compare favorably with human dental arch. But, some humans may have U-shaped or horseshoe-shaped dental arches. Biggerstaff presented the paper to describe a method for defining the individual arch form as one of three mathematical curves.

Data were derived from a small sample of boys and girls. Photographic negatives registered the location of defined anatomic landmarks on the occlusal and incisal tooth surfaces. The landmarks were converted into rectangular X and Y coordinates and certein of these were selected as collectively representing arch form, e.g., the buccal cusp tips and the tips along the incisal edges.

Computer programs corrected each pair of coordinates to compensate for the enlargement of diminution of the photographic image. Other programs fit the adjusted coordinates with the general quadratic equation:

$$AX^2 + BY^2 + CXY + DY + EY - F = 0$$

The value of F in the equation was arbitrarily set at unity (1), although it could have been set at any value. The F value affects only the values of the coefficients that were computed by a subprogram (a component of the numerical analysis program library supported by the University of Kentucky Computing Center)

The coefficients were then used to solve the equation for Y in terms of X. If coefficients A and B have the same sign (positive or negative) the resulting smoothed curve is an ellipse. If the signs are different, the smoothed curve is hyperbolic. And finally, if either coefficient A or B is zero, the curve is parabolic and degenerate if B is zero. The data can be presented in graphic form using the Calcompplotter.

These methods permit a quantitative definition of dental arch types in a population such that the frequency of hyperbolic, parabolic, and ellipsodal arch form can be assayed to determine within and between population differences.

Pepe 1975 (18) studied the curve fitting of polynomial and catenary curve to human dental arch. The data base for this study was derived from seven individuals with a permanent dentition and good occlusion were the sample. Inked points were placed on duplicate casts to identify various anatomic landmarks. By means of photogrammetric methods, these points were projected to a two-dimensional plane parallel to the occlusal plane of each cast such that a near 1:1 relationship of cast size and photographic negative image resulted. Semi-automatic methods were then used to convert these points to a Cartesian (that is, rectangular X-Y) coordinate system with the resultant data recorded on computer cards. The computer program was written to fit the polynomial equations (The polynomial equations are $y = \Sigma a_i x^i$; $2 \leq i \leq 8$) and the catenary equations (that is, $y = A + B \cosh X/B$). The curves were fit by least square error methods, that is, the computer was instructed to approximate the coefficients and the mean square error is minimal. All the curve-generating equations studied in this research, polynomial equation curves were somewhat more accurate than catenary equation curves in describing arch form, they were not remarkably good fits either. Mean square errors for this curve ranged from 1.83 –17.00 mm^2. Thus, although these curves may describe arch form, they were not accurate enough to serve as a template for an arch wire.

Application of the Cubic Spline function was described of the dental arch form by Begole (19). This study purposed to use an asymmetric function, to describe the dental arches. Twenty-seven individuals possessing normal occlusions were selected for study. A full complement of permanent teeth was present from at least the first permanent molar to the contralateral first permanent molar. All individuals exhibited a Class I molar, cuspid, and buccal segment relationship. Maxillary models were then photographed individually. The points on the prints were digitized in an X,Y system using the mesiobuccal cusp tips of first permanent molars, cusp tips of the premolars and cuspids, and the midpoint between the incisal edges of the incisors. Five data points were digitized to be utilized as knots of the spline (see in figure 10), they were the mesiobuccal cusp tips of first permanent molars, cusp tips of the cuspids, and the midpoint between the incisal edges of the central incisors. The remaining eight points recorded, which were the buccal cusp tips of bicuspids and midincisal edges of central and lateral incisors, were used as points from which to evaluate the fit of the spline.

Data were subsequently processed using a Fortran computer program which interpolated a cubic spline for each subject.

Figure 10: Data points digitized for use in computing a spline and evaluating the fit in Begole's study. Circled points were used as knots.

In order to evaluate error in fit, it was necessary to define and subsequently compute a set of normals to the curve. A normal was defined as a line perpendicular to a tangent constructed on the curve so as to minimize the distance from the exterior point to the curve. Since the curve was forced to include the knots, mathematical assessment of error was thus based on normals to the curve, which were computed from those points not chosen as knots. A set of normals was shown in figure 11. A sum of normals and an average normal length were computed for each subject. The sum of normals represented a total error of the data to the curve derived from the spline function, and the average normal provided a mean value for error for the subject.

Figure 11: A set of normal to the curve in Begole's study. Normal were used to evaluate error in the fit of a spline.

Results of this research suggest that in the maxillary dental arches examined, no particular teeth produced substantially larger error when their distance to the theoretical curve was measured. When the teeth were measured and subsequently grouped into segments for analysis, similar results were obtained.

In regard to symmetry, for almost all the subjects asymmetry was evident upon examination of the computer-generated graphics, which used the midpalatal raphe as the axis of symmetry.

Ferrario et.al. (20) 1994 defined mathematical equation of the shape of dental arches in human permanent healthy dentitions. Fifty male and 45 female subjects aged 20-27 years (mean age 22) with normal occlusion were screened from a group of 160 healthy white Caucasian dental students.

The following criteria were used in the selection

1. Absence of moderate or severe clinical mandibular disorders (no TMJ sounds, no tenderness to palpation of the TMJ or of the masticatory muscles, no painful limitations of mandibular movements);
2. Absence of extensive restorations, cast restorations or cuspal coverage;
3. No previous or current orthodontic treatment;
4. Absence of anterior or lateral cross-bite;
5. Absence of pathologic periodontal condition;
6. Clinical normal arch shapes with minimal dental crowding.

All models of the 95 subjects, the midpoints of the incisal edges and canine cusps, and the buccal and lingual cusps of premolars, first and second molars were individualized, and traced with a pencil. Standardized photographs of models were obtained (frame 1:1). Co-ordinates of cusp tips and of two points on each axis of symmetry were obtained using a semi-automatic image analyzer. Maxillary and mandibular arches were interpolated by (1) a 'mixed' model and (2) a polynomial models as follows:

1. a mixed model
    (a) mid – points of incisal edges and canine cusps by an ellipse:
    $$Y^2/f^2 + X^2/g^2 = 1 \;;$$
    (b) buccal cusps from first premolar to second molar by a parabola:
    $$Y = a + b \cdot X + c \cdot X^2 \;;$$
2. 1(a) plus 1(b) points by a fourth – order polynomial
    $$Y = a \cdot X + b \cdot X^2 + c \cdot X^3 + d \cdot X^4$$

For all three curves, the coefficients were estimated using a dedicated computer program: the algorithm minimized the sum of squared distances from the cusp tip co-ordinates to the fitted curve in the Y direction. The program also calculated the least square correlation coefficients and the residual standard deviations about the fitted curves [$s = \sqrt{\sum (Y - \hat{Y})^2 / (N-2)}$ where Y is the sample datum, and $\hat{Y}$ is the estimated value] Means and standard deviations were calculated separately for all parabolic, elliptical, and polynomial coefficients in male and female.

In all parabolars the sign of the second degree coefficient, c, indicates the direction of the curve concavity, being positive for upward concavities. High (positive or negative) coefficients describe very concave curves; the more the absolute value of this coefficient approximates 0, the flatter the curve: it became a line for c = 0. The first degree coefficient, b, gives information about the symmetry in respect to Y axis: it is positive for left side curves (second and fourth quadrants), negative for right side ones and null when the vertex stays on the Y axis. The a coefficient is the intercept. It is positive when the vertex lays in the first or second quadrants. In the fourth-order polynomial coefficients b (second degree) and d (fourth degree) describe arch shape, while coefficients a (first degree) and c (third degree) describe asymmetry, b and d are negative for downward curves, positive for upward curves.

Statistical comparisons were performed between mean coefficients computed in male and female samples, and in mandibular and maxillary arches using two-tailed Student's t-tests for independent samples. A specially devised computer graphic subroutine allowed the plotting of cusp tips (co-ordinates of digitized point) and of their interpolating curves for each subject, as illustrated in figure 12, thus providing a direct assessment of the goodness of fit suggested by the correlation coefficients.

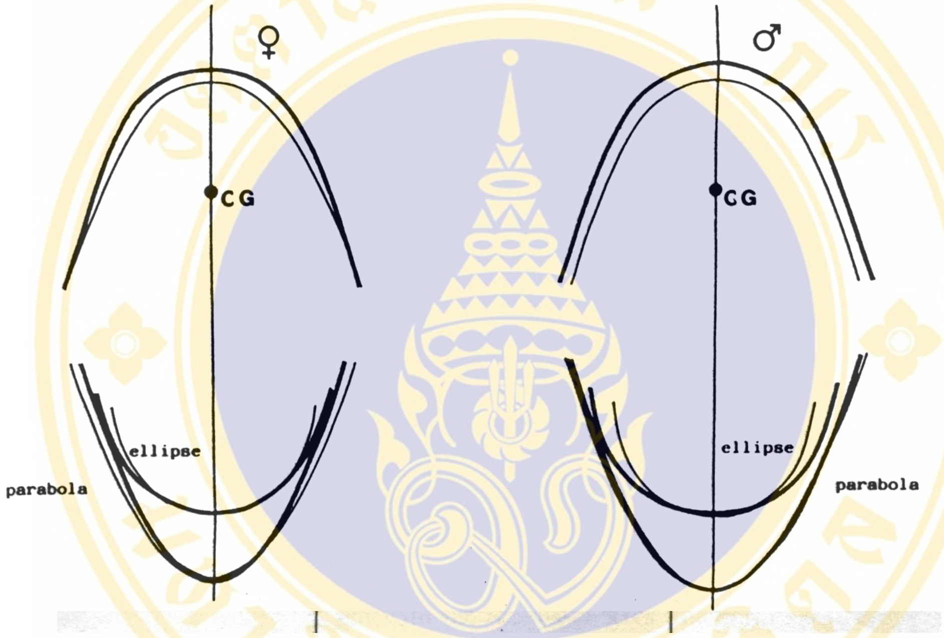

Figure 12: Mean plots of mandibular and maxillary arches in Ferrario's study: Top: maxillary and mandibular curves supposing a right Angle Class I first molar relationship. Bottom: ellipses and parabolas superimposed on there vertices.CG: center of gravity of arches.

The result of this studied, anterior teeth seem to be well fitted by the elliptical model, as test by the mean correlation coefficients ranging from 0.873 to 0.933. Post – canine teeth were well interpolated by their model, mean correlation coefficients of the parabolas were very good in all instances (r >0.92). Fourth – order polynomial well fitted data points in all instances: mean correlation coefficients are all over 0.97, the plots of the mean fourth – order polynomials did not appear to be signification different from the mean plots computed with the mixed model.

Braun S, Hnat WP, Kusnoto B, Hnat FW. (21) proved that the beta function most closely represents the human dental arch. The data were obtained by recording the coordinates of the cusp tips locations of forty sets of casts (15 Class I, 16 Class II, and 9 Class III). Subdivision occlusions were not included in the study. Casts exhibiting incisal or cuspal atrrition, fracture of teeth, ectopically erupted teeth, or deciduous teeth were excluded from this study; only casts of fully developed adult dentitions (including second molars) were included. A precision machine tool device (Coordinate Measuring Machine; CMM) was used to record the X-,Y-, and Z-coordinates of selected dental landmarks on all casts to 0.001 mm. accuracy.

Coordinates were recorded at the center of each incisor incisal edge, at the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar. Eeighteen points were recorded in each dental arch. The coordinates were processed through a computer curve-fitting program.

The dental arch described by the beta function, was given by

$$Y = 3.031\ D\ \{x/\ w + ½\ \}^{0.8}\{1/2 - x/\ w\}^{0.8}$$

W represents the cross – arch distance between the second molar contact points

D represents the perpendicular distance form the most anterior point between the two central incisors to the molar cross-arch dimension, as illustrated in figure 13

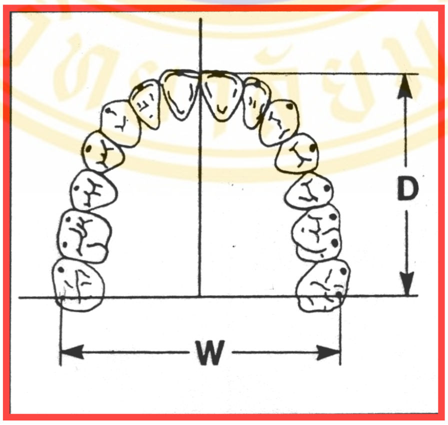

Figure 13: The dental landmarks and parameters in Braun et.al.'s study: W = arch width, D = arch depth.

The beta function has been shown to be an accurate representation of the human dental arches. The mean correlation coefficient of curve fit was found to be 0.98, with a standard deviation of 0.02, for the complete sample of 80 casts. From these study, they concluded that the beta function more accurately described the dental arch form than representations previously reported.

Furthermore comparing mandibular arch shapes (figure 14), it was evident that the dental arches associated with Class III occlusions exhibit a smaller arch depth than the Class I occlusions. Additionally, the mandibular dental arches associated with Class III occlusion were wider than the Class I mandibular arches. When Class II mandibular arches were compared with Class I arches, reduced arch width and arch depth were evident. This could be explained by the fact that some Class II relationships result from a small mandibular body. When comparing maxillary arch shapes (figure 14), it was apparent that arch depths for all Angle's Classifications were essentially the same. However, Class III dental arch widths were greater than Class I widths. This begins in the lateral incisors-canines area and proceeds distally. When Class II maxillary arch widths were compared with Class I widths, they were found to be narrower, beginning in the lateral incisors-canines area.

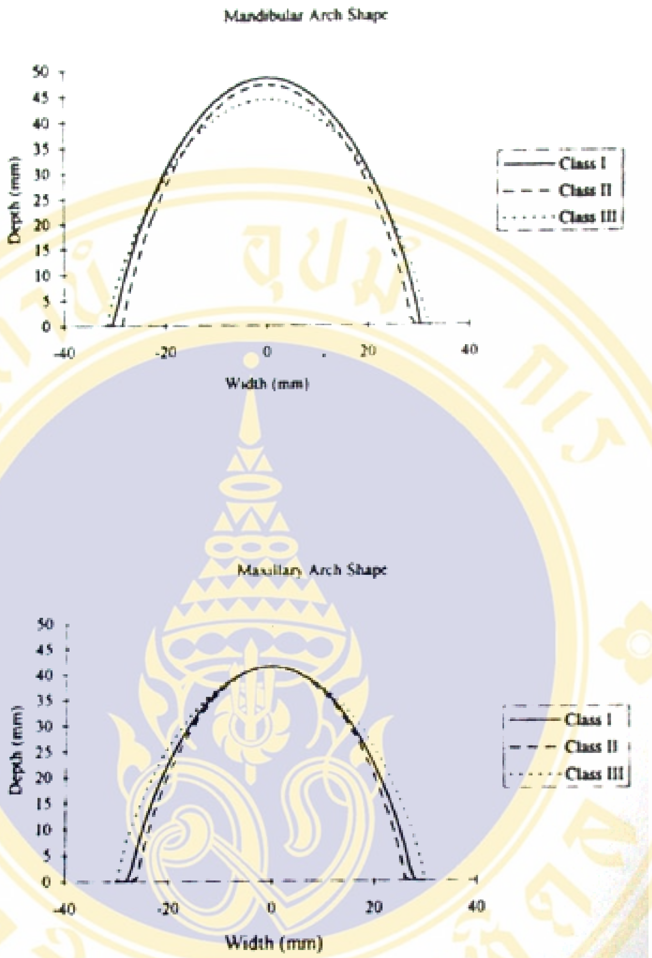

Figure 14: Average (typical) maxillary and mandibular arch shapes for Class I, II, and III occlusion in Braun et.al.'s study.

Braun and Hnat (22) studied the dynamic relationships of the mandibular anterior segment. Twenty-one mandibular casts of untreated patients that showed to minimal anterior segment irregularities were selected. Measurements were made at each canine cusp tip, the center of each incisor incisal edge, and the normal contact between the first bicuspids and canines.

An analytical equation of the anterior segment shape was required to describe the relationships between the intercanine width, anterior segment depth, anterior arch perimeter, and incisor angulation.

Thus the generalized equation of the mandibular anterior segment shape may be expressed as

$$Y = -\cos h \{x/ b \cos h^{-1} (h+1)\} +1.0+h$$

b represents ½ the cross arch distance between the normal distal contacts of the right and left canines, as illustrated in figure 15.

h represents the distance measured along a line form the contacts of the central incisors perpendicular to a line connecting the distal contacts of the canine, as illustrated in figure 15.

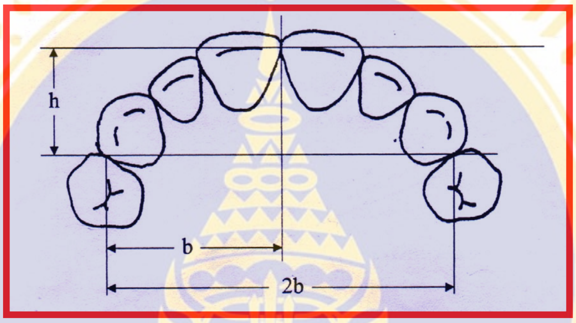

Figure 15: The parameters (h) and (b) used in Braun et.al.'s study.

The arch perimeter may then be calculated using

$$L = \int_{-w}^{+w} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

Where W represents half the crossarch distance from the normal canine/first premolar contact on one side to the opposite side in milimeters, and L is the anterior segment arch perimeter from the canine/first premolar normal contact along the arch to the opposite side in millimeters.

The related angular change for a typical mandibular incisor was given by the formula

$$\Delta\theta = \tan^{-1} \frac{\Delta D}{14.2}$$

Where $\triangle$ D was the change in depth of the anterior segment in millimeter.

The above relationships were of value in treatment planning. The outcome in anterior arch perimeter, depth, and incisor angular change can be forecast with accuacy (r = 0.951) without resorting to trail and error or doing a wax-up.

The variation in human arch form

Over the years the great majority of authors on the subject of arch form have recognized that there is variability in the size and shape of human arch form. Angle (5), for example, stated that the arch form varies within the limits of the normal, according to race, type, temperament etc., of the individual. Because the form of the arches was considered dependent on these variables, Angle did not consider Hawley's method of arch predetermination useful for more than a general approximation as a true line of occlusion. Over the years, however, the majority of edgewise orthodontists used the Bonwill-Hawley arch form as a beginning template for the construction of the edgewise archwire. These construction methods were described by Angle (5), Chuck (7), Boone (8) and others.

Hellman (24) investigated the skulls of apes and human beings, and found no relation between the size of teeth and the form and shape of the dental arches. Therefore, he did not accept the theories of arch predetermenation based on measurement of certain teeth. He concluded that mathematical methods for dealing with the question of arch form were unsatisfactory.

Stanton (25) carried out extensive investigations on arch form and pointed out the error in the Bonwill-Hawley method of arch predetermination. He utilized "arthrographic map makers" to study occlusions, and concluded that arch forms are open and closed, that is ellipse, parabolar, and other kindred curves. Izard (26) based his method of arch predetermination on ratios between arch width and facial depth. He concluded that approximately 75% of arch forms were represented by an ellipse, 25% by a parabola and 5% by a U shape. While Wheeler (27) observed that dental arches generally conform to the shapes of parabolic curves, he stated that nothing anatomic could be reduced to the mathematical exactitudes of geometric terms.

Sicher (28) wrote that the shapes of the dental arches vary considerably, cut that the upper arch generally took on the appearance of an ellipse, and the lower arch a parabola. Remsen (29) studied various arch predetermination methods by comparing them with the arches of a sample of “normal” occlusion. He observed that the parabola best represented the anterior curvature of the dental arch, but stated that an arch which fits a precise pattern was the exception rather than the rule.

White (30) reviewed the accuracy of various standardized arch designs to 24 untreated ideal adult occlusions. His findings were as follows:

1. The Bonwill-Hawley arch form had a good fit with 8.33% of the cases, a moderately good fit with 39.58% of the case, and a poor fit with 52.08% of the cases.
2. The Brader arch form had a good fit with 12.50% of the cases, a moderately good fit was 43.74% of the cases, and a poor fit was 43.75% of the cases.
3. The Cantinary curve showed a good fit with 27.08% of the cases, a moderately good fit in 45.82% of the cases, and a poor fit with 27.05% of the cases.
4. The Rocky Mountain data computer derived arch design which is based on measurements of inter-molar width, inter-cuspid width and arch depth, showed a good fit with 8.3% of the cases, and a moderately good fit with 81.57% of the cases. No cases showed a poor fit.

White also pointed out that most theories on arch form consider arch form to be symmetrical. He observed that there was a great deal of asymmetry in the arches and felt that should be considered in arch design. Because of the variability that White found in arch form, he suggested that an occlusion “map-maker” be used to construct the shape of the arch for the individual and used throughout orthodontic treatment.

In order to determine whether a particular ideal orthodontic arch form could be identified, Felton (31) et.al., studied the mandibular casts of 30 untreated normal cases, 30 Class I non-extraction cases, and 30 Class II non-extraction. After comuterized digitizing and the use of a mathematical function called a polynomial of the fourth degree, they determined that no particular arch form predominated in any of the three samples. They therefore stated that customizing arch forms appeared to be necessary in many cases to obtain optimum long term stability, because of the great variability in arch form observed in the study.

Nojima and Mclaughlin (32) studied the difference of arch shape in Caucasian and Japanese mandibular in different type of occlusion. The results of this study was show in table 1. The Caucasian arch tended to be narrower and deeper than the

Japanese arch, and they found the difference of arch form in different type of occlusion.

Table 1. The distribution of dental arch shape according to the type of occlusion in Caucasian and Japanese.

Caucasian		Japanese	
Class I	90% ovoid and tapered arch	Class I	90% ovoid and square (a square arch form was observed in more than 50%)
Class II	60% tapered arch form (most of the remainder showing an ovoid arch form)	Class II	Over 50% was ovoid arch form with square and tapered shapes for approximately 25% each
Class III	3 arch forms were almost distributed with square arch form found at the highest frequency of over 40%	Class III	Arch form were of either square or ovoid shape, the former accounting for over 50%

Yamada A. (33) studied the relationship between the capacity of oral cavity proper and skeletal malocclusions, result were showed.

1. Class III cases were statistically larger than class I and II in the capacity of the oral cavity proper but they were no significant difference between those of class I and class II.

2. There were no significant difference among class I, II or III in the maxillary capacity of the oral cavity proper. Class III, however, was significant larger than class I and II in the mandibular capacity of the oral cavity proper.

3. There was no significant difference among the three types of palatal morphology.

4. There were difference for class I, II and III as to the relationship between both the dental arch length and width, and the capacity of the oral cavity proper. The

general indication was that width was more related to the size of the capacity of the oral cavity proper than length.

Ferrario et.al. (20) had already investigated gender differences in dental arch shape in a group of young adults with sound dentitions by Euclidean - distance matrix analysis demonstrated no significant gender difference in the shape of arches. A mathematical interpolation (fourth - order polynomials, semi -ellipses and parabolas) of the same arches produced similar mean curves regardless of gerder, showing more a size discrepancy than a shape difference.

Ferrario et.al. (34) studied maxillary versus mandibular arch form differences in the human permanent dentition assessed by Euclidean – distance matrix analysis applied to the selected 57 women and 61 men aged 20-27years. They found that maxillary and mandibular arches had a significantly different shape and size was also different: the upper arch was approximately 12% bigger than the lower arch. The examination of the form – difference matrix allowed separation of teeth into groups that could account for the morphological differences between arch: anterior teeth (incisors and canine) appeared to behave differently from molars and premolar. This finding gives support to the recommendation that anatomical and anthropological description should treat these two sectors separately (20), and that clinical evaluations and treatments should take these differences into account (35,36).

Summary of review

Contribution of variation in Arch size and shape is defined in the fetus as genetic influence, variability in eruption paths of the teeth, growth of the supporting bone and movement of the teeth after emergence due to habits and unbalanced muscular pressure. The result of genetic and environment factor make the difference of arch shape in the differences in ethnic and the difference type of occlusion.

General method to define arch form in geometric terms:

1. In all models the selective dental landmarks were individualized and marked with a pencil.
2. General study took photography or scanning from dental model: frame 1:1.
3. Coordinated of cusp tips and of two points on each axis of symmetry were obtained using a semi-automatic analyser.

4. For all mathematic function: the coefficients were estimated by using a computer program.

According to the gerneral method, the error of measurement was happen in the process of transfering three-dimensions of dental casts to two-dimensions. In the present day, a highly accurate measuring device (CMM) used in the machine tool industry can be applied to record specific landmarks directly on dental casts (41). The dental landmarks are reported in the corresponding of X, Y, Z. Therefore the processing of collecting and recording the data times were reduced.

The coefficients of the cubil spline and the polynomial equation were difficult to interpret. While the parabola, eliptical equation and hyperbolic cosine function were merely described parts of the whole dental arch curve. The beta function with the feasibility coefficient proved to have a high correlation with the dental arch (21). The data were obtained by recording the coordinates of the center of each incisal edge, at the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar.

CHAPTER 3

MATERIALS AND METHODS

Twenty male and 20 female subjects who have never undergone orthodontic treatment were select from the Department of Orthodontics, Mahidol University, private dental clinics, and from second year students of the Armed Forces Acedemies Preparatory School.

The following criteria were used in the selection:

1. Exhibition Angle class I molar without molar shift caused by early loss of primary teeth.
2. Absence of cuspal attrition, fracture of teeth, restorations extending to contact areas, cusp tips, or incisal edges.
3. Permanent dentition including second molars with normal tooth size and shape.
4. Arch length discrepancy were less than 3 mm.

Dental arches of the 40 subjects were reproduced using an irreversible hydrocolloid material and transfered to dental models in orthodontic stone (the typical dental casts was show in figure 16). In all models, the mesial and distal point of each incisal edge of incisors, the cusp tip of the canines and prmolars, and at the mesiobuccal and distobuccal cusp tips of each molar were identified as landmarks. Twenty-two points were labeled on each dental cast (figure 17).

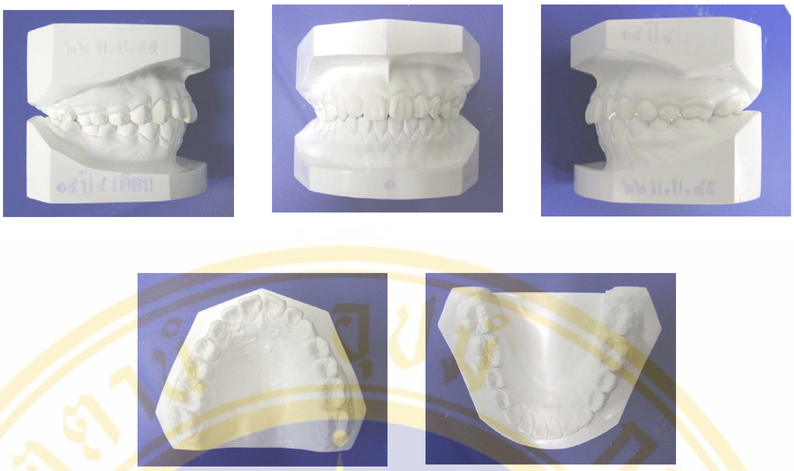

Figure 16: Typical dental models used in this study

Figure 17: The dental landmarks used in this study. Twenty-two points were marked on each dental model.

During the measurement of dental models using Coordinate Measuring Machine (LK G 90C), each cast was oriented in the Coordinate Measuring Machine (CMM). The device was used extensively in the precise machine tool in the Faculty of Engineering, Mahidol university (figure 18). A frictionless air bearing probe recorded the coordinates of a point in space in each of the three orthogonal axes to 10^{-6} meters. The casts were secured to a fixed plane. The touch trigger probe was used to identify each of the following measurement points respectively from the

Maxillary dental models:

1. distobuccal cusp tip of right second molar
2. mesiobuccal cusp tip of right second molar
3. distobuccal cusp tip of right first molar
4. mesiobuccal cusp tip of right first molar
5. buccal cusp tip of right second premolar
6. buccal cusp tip of right first premolar
7. cusp tip of right canine
8. distal point of incisal edge of right lateral incisor
9. mesial point of incisal edge of right lateral incisor
10. distal point of incisal edge of right central incisor
11. mesial point of incisal edge of right central incisor
12. mesial point of incisal edge of left central incisor
13. distal point of incisal edge of left central incisor
14. mesial point of incisal edge of left lateral incisor
15. distal point of incisal edge of left lateral incisor
16. cusp tip of left canine
17. buccal cusp tip of left first premolar
18. buccal cusp tip of left second premolar
19. mesiobuccal cusp tip of left first molar
20. distobuccal cusp tip of left first molar
21. mesiobuccal cusp tip of left second molar
22. distobuccal cusp tip of left second molar

Mandibular dental models:

1. distobuccal cusp tip of left second molar
2. mosiobuccal cusp tip of left second molar
3. distobuccal cusp tip of left first molar
4. mosiobuccal cusp tip of left first molar
5. buccal cusp tip of left second premolar
6. buccal cusp tip of left first premolar
7. cusp tip of left canine
8. distal point of incisal edge of left lateral incisor
9. mesial point of incisal edge of left lateral incisor
10. distal point of incisal edge of left central incisor
11. mesial point of incisal edge of left central incisor
12. mesial point of incisal edge of right central incisor
13. distal point of incisal edge of right central incisor
14. mesial point of incisal edge of right lateral incisor
15. distal point of incisal edge of right lateral incisor
16. cusp tip of right canine
17. buccal cusp tip of right first premolar
18. buccal cusp tip of right second premolar
19. mesiobuccal cusp tip of right first molar
20. distobuccal cusp tip of right first molar
21. mesiobuccal cusp tip of right second molar
22. distobuccal cusp tip of right second molar

Figure 18: The coordinate measuring machine: A) CMM (LK G-90C), B) Touch trigger probe, C) A hand stick was used to control the direction of trigger probe.

Twenty-two points were recorded in each dental model while the X-and Y-coordinates of each landmark were projected to the Z-plane. The coordinates of the landmarks in three dimensional space for each of the 40 sets of dental casts were recorded in the corresponding X-, Y-, Z-coordinates automatically by using the LK Camio suit 5.5 program (figure 22).

Before recording the dental landmarks, the datum (X, Y, and Z plane) was set at the distobuccal cusp tip of the right second molar on each maxillary dental model and at the distobuccal cusp tip of the left second molar on each mandibular model. The coordinates (X and Y) at these points were defined as zero (figure 20). A planar projection of each cast was subsequently obtained by setting the Z-plane from three-point contact between glass slide and occlusal surface of each dental model. The Z-axis was then perpendicular to this plane (figure 19-20).

Measurements of dental casts using CMM in this study was done by an engineer who was trained to use CMM.

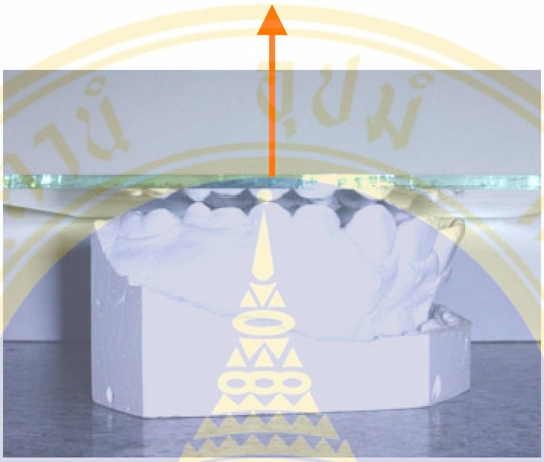

Figure 19: The construction of the Z-axis: the Z- axis was set from the three–point contact between glass slide and occlusal surface of each dental model. The Z axis was perpendicular to this plane.

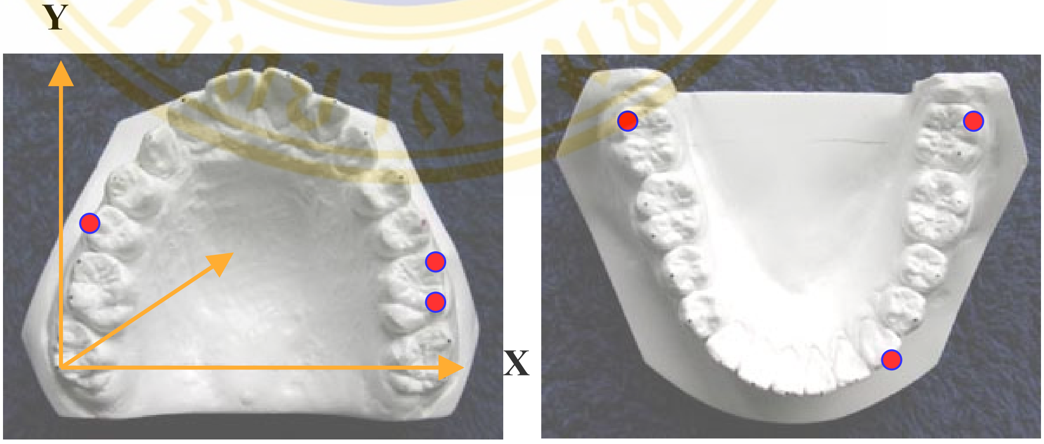

Figure 20: Three points contacted used in constructed of the Z-axis. A planar projection of the Z plane was constructed from the three points (in red) which contacted the glass slide on the occlusal surface of the model.

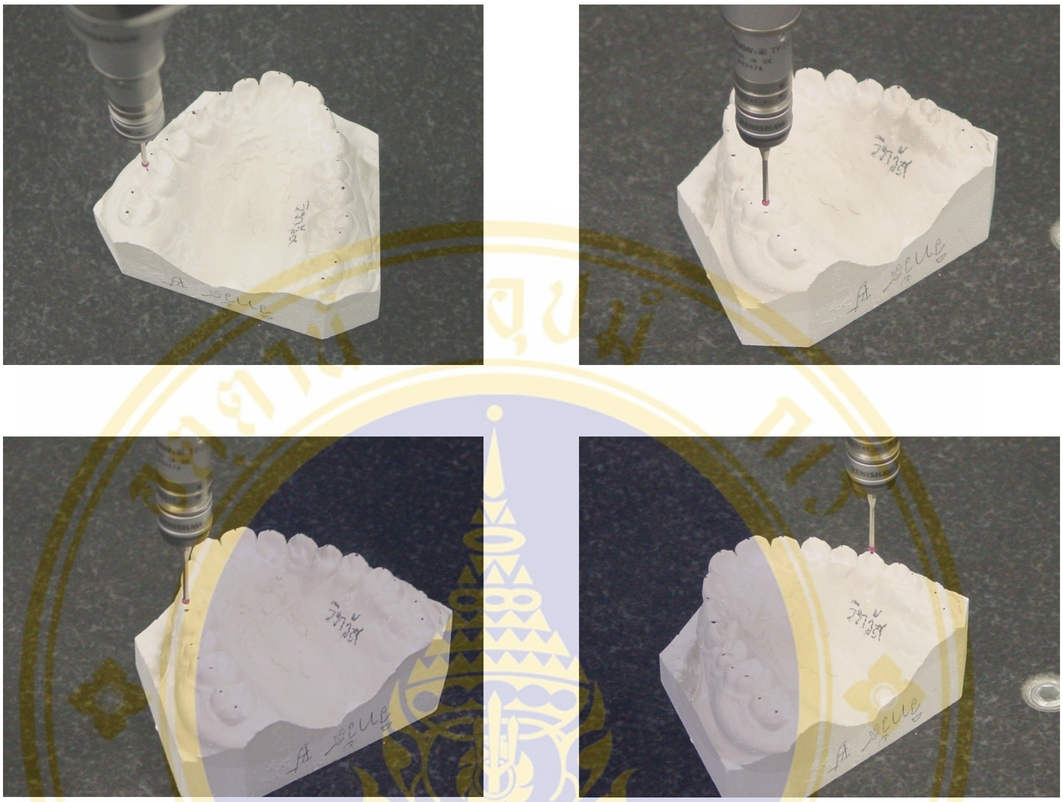

Figure 21: The processing of measuring the dental casts: dental casts were placed on a fixed plane during the recording of the dental landmarks with the Coordinate Measuring Machine.

Point	X	Y	Z
1	-0.035	-0.02	0
2	0.79	5.004	0
3	2.329	10.637	0
4	3.467	15.96	0
5	5.505	21.904	0
6	8.058	27.895	0
7	11.912	36.129	0
8	16.277	40.492	0
9	22.256	41.052	0
10	23.384	44.714	0
11	30.671	44.927	0
12	32.416	45.272	0
13	39.895	44.531	0
14	41.363	42.556	0
15	46.934	40.014	0
16	51.154	36.438	0
17	55.681	28.502	0
18	58.872	21.419	0
19	60.727	15.587	0
20	61.589	10.091	0
21	61.659	4.951	0
22	62.327	0.029	0

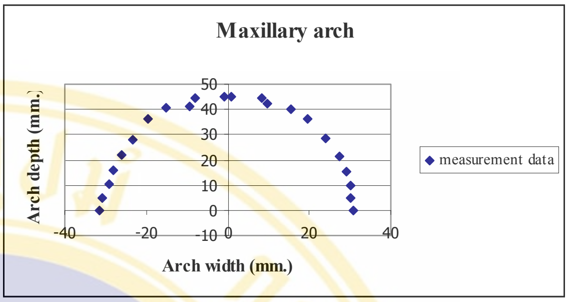

Figure 22: The data of individual landmarks from the CMM measurement: was reported in the X-Y-Z coordinates and a graph was plotted in only two axes from the coordinates X-, and Y-.

Error of measurement

Replicate measurement by the same single examiner were made on 6 sets of randomly selected dental casts after a 1 week interval. The X-, and Y- coordinates of each landmark were compared between two separate of measurements.

The method errors for each of the categories of measurement were calculated according to Dahlberg’s equation.

$$\text{Method error (ME)} = \sqrt{\frac{\sum d^2}{2(n-1)}}$$

d = difference between two successive measurements

n = number of double determinations

Arch interpolation

An analytical equation of the dental arch shape was neccessary to describe the relationships between the arch width (X-coordinate) and arch depth (Y-coordinate). Many mathematical functions were investigated as to fit. The beta function most closely represents the dental arch shape (21). Two measurements (molar width and arch depth) were required to generate the dental arch shape. The beta function representing the dental arch shape is given by the general formula (21).

$$Y = \frac{a\left(\frac{X-b+cm}{c}\right)^{2}\left(1-\frac{X-b+cm}{c}\right)}{m^{d-1}n^{e-1}}$$

$$m = \frac{d-1}{d+e-2};\ n = \frac{e-1}{d+e-2}$$

Where Y is the arch depth at arch width (X).

Parameter (a) represents the arch depth measured by the perpendicular distance from the most anterior point between two central incisors to the molar cross-arch dimension (figure 23).

The parameter (b) is a offset and is set to zero so the arch centerline is at zero, our reference position.

The parameter (c) is the arch width measured by the cross-arch distance between the second molar distobuccal cusp tips. The parameters (d) and (e) are factors the control the symmetry (figure 23).

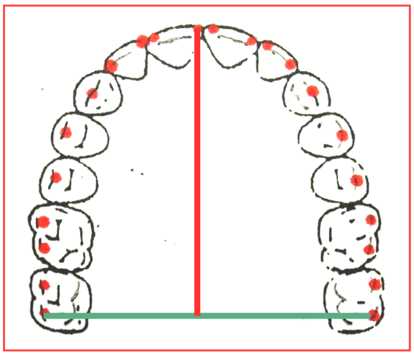

Figure 23: The parameter of beta function: diagram showed the parameter, a; Arch depth (red line), c; Arch width (green line).

If parameter b = 0, and d = e, the beta function becomes symmetrical about the centerline of the teeth. The beta function then become

$$m = \frac{d-1}{d+e-2} = \frac{d-1}{d+d-2} = \frac{d-1}{2(d-1)} = \frac{1}{2}$$

$$n = \frac{e-1}{d+e-2} = \frac{d-1}{d+d-2} = \frac{d-1}{2(d-1)} = \frac{1}{2}$$

$$Y = \frac{a\left(\dfrac{X+\frac{c}{2}}{c}\right)^{d-1}\left(1-\dfrac{X+\frac{c}{2}}{c}\right)^{d-1}}{\left(\frac{1}{2}\right)^{d-1}\left(\frac{1}{2}\right)^{d-1}}$$

$$= \frac{a\left(\frac{X}{c}+\frac{1}{2}\right)^{d-1}\left(\frac{1}{2}-\frac{X}{c}\right)^{d-1}}{\left(\frac{1}{4}\right)^{d-1}}$$

$$= a\left(4\left(\frac{1}{4}-\frac{X^2}{c^2}\right)\right)^{d-1}$$

$$Y = a\left(1-\frac{4X^2}{c^2}\right)^{d-1}$$

For the beta function, the coefficients were estimated using a dediccated computer program, (the computer program was created by Department of Mathematics, Faculty of science, Mahidol university) based on the Differential Algorithm. These also fit standard beta function equations to the data points for each of the eighty dental arches. The curves were fit by least square error methods. That is, the computer was instructed to approximate the coefficients of the beta function of each such that:

$$\sum_{i=0}^{n}(Y_i - \hat{Y}_i)^2 = 0$$

Where n is the number of data points, Y_i is the Y-coordinates of the data point (X_i, Y_i) on the common line of occlusion and $\hat{Y}_i$ is the computed Y-coordinate of the point on the beta function curve corresponding to the original X_i and the sum of squared distances from the cusp tip coordinates to the fitted curve in the Y direction.

$$\sum_{i=0}^{n}(Y_i - \hat{Y}_i)^2 = 0 \quad \text{is minimal}$$

Following this process, each parameter (a, c, d and e) of the beta function were reported for the individual dental arch curvature.

Statistical analysis

1. The mathematics function and coefficient of determination were used to describe the relationship between the recorded and calculated independent variable.
2. Student t-test was used to describe the mean difference of the parameters (a, c, d and e) among the sex and the difference in arch size.
3. Paired t-test was used to describe the mean difference of each coordinate landmark between the first and the second measurements.

CHAPTER 4

RESULTS

Using the least squares method, the curve-fitting program was used for all calculations of the parameters. The results were showed in the table 2 and 3. The a-parameter represented the arch depth, while the c-parameter was the arch width at the distobuccal cusp tip of the second molar. The b-parameter was automatically set to zero by the curve fitting program, the d- and e-parameters were equal. The beta function became symmetrical at about the centerline of the teeth.

The maxillary arch table (table2) shows the values of each parameter in each individual maxillary dental arch according to the beta function. The mean value of the arch depth (a-parameter) was 46.23 mm. with a standard deviation of ± 2.07 mm. The maximum dimension was 50.75 mm. and the minimum dimension was 42.71 mm. The mean value of the arch width (c-parameter) was 61.48 mm. with a standard deviation of ± 2.65 mm. The maximum width was 66.46 mm. and the minimum width was 54.56 mm. The mean value of the d- and e-parameters were 1.67 with a standard deviation of ± 0.009. The maximum value was 1.89 and the minimum value was 1.55.

The mandibular arch table (table3) shows the values of each parameter in the individual mandibular dental arch according to the beta function. The mean value of the arch depth (a-parameter) was 42.02 mm. with a standard deviation of ± 2.08 mm. The maximum dimension was 46.05 mm. and the minimum dimension was 38.00 mm. The mean value of the arch width (c-parameter) was 55.46 mm. with a standard deviation of ± 2.97 mm. The maximum width was 59.56 mm. and the minimum width was 46.54 mm. The mean value of the d- and e-parameters were 1.77 with a standard deviation of ± 0.11 The maximum value was 2.03 and the minimum value was1.63.

Tables 4 and 5 show the goodness of fit between the measured values and calculated values of maxillary and mandibular dental arches respectively according to the beta function. In all instances, the maxillary and mandibular arches were judged to fit the data well, as also verified by the coefficients of determination from 0.93-0.99 in the maxillary arch and 0.94-0.99 in the mandibular arch.

Table 2. The values of each parameter in individual maxillary dental arch according to the beta function.

Sample	Maxillary Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
1	46.26	59.79	1.57	1.57
2	46.66	63.10	1.64	1.64
3	46.23	54.59	1.59	1.59
4	50.75	61.58	1.78	1.78
5	43.77	64.36	1.89	1.89
6	45.26	61.85	1.84	1.84
7	48.56	63.25	1.62	1.62
8	46.28	60.57	1.64	1.64
9	44.57	55.86	1.64	1.64
10	43.50	60.03	1.55	1.55
11	50.02	63.53	1.68	1.68
12	43.59	63.63	1.61	1.61
13	44.45	64.89	1.67	1.67
14	45.34	61.97	1.63	1.63
15	49.06	66.46	1.73	1.73
16	45.91	58.60	1.62	1.62
17	45.04	59.81	1.56	1.56
18	46.00	62.48	1.69	1.69
19	42.76	62.21	1.55	1.55
20	44.80	62.50	1.75	1.75

Table 2. The values of each parameter in individual maxillary dental arch according to the beta function (Continued).

Sample	Maxillary Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
21	44.45	63.32	1.79	1.79
22	48.39	62.51	1.71	1.71
23	47.48	60.40	1.75	1.75
24	44.23	62.15	1.79	1.79
25	42.71	60.19	1.75	1.75
26	47.57	63.85	1.56	1.56
27	47.18	60.35	1.69	1.69
28	46.64	60.30	1.71	1.71
29	51.82	63.75	1.82	1.82
30	46.26	56.92	1.48	1.48
31	43.73	62.53	1.64	1.64
32	48.34	60.89	1.67	1.67
33	45.71	64.49	1.79	1.79
34	48.07	64.74	1.79	1.79
35	45.96	54.56	1.59	1.59
36	46.69	62.45	1.67	1.67
38	46.60	62.58	1.61	1.61
39	45.65	59.83	1.55	1.55
40	46.31	60.57	1.58	1.58
Mean	46.23	61.48	1.67	1.67
SD	2.07	2.65	0.009	0.009

Table 3. The values of each parameter in individual mandibular dental arch according to the beta function.

Sample	Mandibular Parameter			
	a-parameter	c-parameter	d-parameter	e-parameter
1	41.35	54.61	1.77	1.77
2	42.87	57.99	1.84	1.84
3	40.43	49.44	1.75	1.75
4	44.21	53.25	1.75	1.75
5	40.43	58.61	1.98	1.98
6	40.13	54.54	1.98	1.98
7	44.13	56.99	1.74	1.74
8	42.15	53.77	1.63	1.63
9	42.72	48.69	1.65	1.65
10	38.52	53.38	1.67	1.67
11	45.83	58.08	1.88	1.88
12	39.17	58.01	1.74	1.74
13	43.71	57.94	1.72	1.72
14	45.80	56.39	1.78	1.78
15	44.00	58.70	1.85	1.85
16	41.88	54.28	1.74	1.74
17	41.44	52.48	1.68	1.68
18	42.56	56.32	1.95	1.95
19	38.00	58.02	1.74	1.74
20	40.74	56.02	1.92	1.92
21	39.75	58.24	1.96	1.96
22	42.23	55.42	1.75	1.75
23	42.07	56.14	1.78	1.78
24	39.65	55.41	1.83	1.83
25	38.91	57.23	2.03	2.03
26	43.72	59.56	1.69	1.69
27	43.14	53.35	1.63	1.63
28	40.48	52.23	1.67	1.67
29	46.05	58.45	1.85	1.85
30	41.23	50.43	1.65	1.65
31	40.38	58.05	1.78	1.78
32	44.77	54.92	1.71	1.71
33	43.46	56.93	1.74	1.74
34	42.18	56.50	1.80	1.80
35	39.77	46.54	1.57	1.57
36	44.68	56.79	1.96	1.96
37	43.63	56.71	1.76	1.76
38	40.14	57.68	1.70	1.70
39	41.46	56.98	1.69	1.69
40	43.06	53.30	1.67	1.67
Mean	42.02	55.46	1.77	1.77
SD	2.08	2.97	0.11	0.11

Table 4. The measured values, calculated value, and the coefficient of determinant of maxillary arches; in all instances of maxillary arch are judged to fit the data well.

Sample	Measured values (mm)		Calculated values (mm)		R^2
	Depth	Width	Depth	Width	
Maxillary arch					
1	45.53	57.57	46.49	61.38	0.93
2	45.1	62.36	46.85	63.26	0.97
3	44.02	53.89	46.62	54.73	0.96
4	48.72	60.07	49.95	61.22	0.96
5	41.82	62.85	41.75	62.85	0.97
6	42.32	61.85	43.99	61.85	0.96
7	48.22	61.45	48.76	63.71	0.97
8	46.48	59.44	46.73	60.65	0.97
9	43.9	54.76	44.56	56.05	0.96
10	41.33	59.23	43.50	60.72	0.96
11	47.83	63.19	49.93	63.41	0.98
12	42.72	62.85	43.69	63.63	0.98
13	42.74	63.79	44.43	64.87	0.97
14	42.55	61.97	45.41	62.07	0.97
15	48.45	66.47	48.40	66.46	0.98
16	44.13	57.26	46.24	59.02	0.96
17	44.8	59.81	46.03	59.18	0.98
18	45.86	61.6	45.91	62.41	0.98
19	41.33	62.21	43.45	62.73	0.93
20	43.29	61.65	44.49	62.43	0.97
21	42.88	61.94	43.52	62.22	0.95
22	45.6	59.52	48.18	62.02	0.95
23	45.64	60.40	46.57	60.39	0.97
24	43.1	61.44	43.08	61.43	0.98
25	42.12	59.20	42.51	59.65	0.97
26	46.16	63.85	48.41	64.71	0.98
27	45.71	59.94	46.93	60.25	0.96
28	45.12	57.62	46.21	59.81	0.97
29	48.81	62.68	49.94	63.70	0.96
30	44.28	55.98	47.68	58.22	0.94
31	43.43	62.56	44.16	62.53	0.99
32	48.00	61.04	48.34	60.89	0.99
33	44.28	63.44	44.95	64.06	0.94
34	46.50	63.85	46.91	64.36	0.97
35	44.02	54.56	46.80	54.56	0.98
36	45.08	62.50	46.08	62.45	0.98
37	46.86	61.68	47.04	62.31	0.98
38	45.38	61.50	46.99	63.15	0.97
39	44.73	58.68	46.85	61.07	0.95
40	45.81	58.13	46.60	61.02	0.95
Average	44.87	60.62	46.12	61.53	0.96
SD	2.07	2.78	2.05	2.53	0.05

Table 5. The measured values, calculated value, and the coefficient of determinant of mandibular arches; in all instances of mandibular arch are judged to fit the data well.

Sample	Measured values (mm)		Calculated values (mm)		R^2
	Depth	Width	Depth	Width	
Mandibular arch					
1	40.39	53.80	41.42	54.71	0.99
2	40.96	57.98	42.67	57.98	0.96
3	39.28	47.30	40.70	49.52	0.94
4	42.39	53.25	44.65	53.24	0.98
5	38.6	57.57	39.28	57.57	0.98
6	36.99	53.43	39.32	53.45	0.94
7	43.50	56.98	44.37	57.24	0.97
8	42.07	53.78	42.31	53.92	0.97
9	40.76	48.70	43.42	49.20	0.97
10	36.66	53.40	39.78	54.15	0.98
11	43.36	58.05	44.52	58.05	0.98
12	38.37	58.03	39.54	58.00	0.98
13	42.55	57.92	44.28	58.94	0.98
14	44.06	56.25	45.75	56.32	0.98
15	43.17	58.71	43.63	58.69	0.98
16	41.28	54.30	42.25	54.37	0.98
17	40.10	52.13	42.14	53.31	0.97
18	42.2	56.32	42.06	56.31	0.98
19	36.55	57.98	38.03	58.02	0.98
20	38.91	55.98	40.15	59.99	0.98
21	38.81	57.70	39.37	57.71	0.97
22	41.35	54.78	43.13	55.33	0.98
23	41.32	54.36	42.04	55.44	0.95
24	38.33	55.42	38.99	55.40	0.97
25	37.47	56.33	38.97	56.38	0.97
26	42.44	59.52	44.22	60.57	0.98
27	42.06	52.63	43.93	54.40	0.96
28	39.88	52.22	41.08	52.23	0.98
29	44.03	58.45	45.46	58.45	0.98
30	39.91	50.45	41.81	51.03	0.94
31	39.16	58.05	40.30	58.05	0.99
32	43.38	54.92	45.42	55.25	0.99
33	41.70	54.85	43.65	57.10	0.94
34	41.37	56.16	42.03	56.41	0.99
35	38.86	46.36	41.08	48.20	0.96
36	43.12	56.68	43.49	56.57	0.98
37	43.50	56.71	43.63	56.70	0.98
38	39.35	57.69	40.95	57.68	0.99
39	39.90	59.98	42.01	58.93	0.97
40	41.41	53.30	43.68	53.30	0.96
Average	40.73	55.21	42.13	55.70	0.97
SD	2.12	3.17	2.05	2.90	0.01

A sample of the beta function calculated with measured data points for maxillary and mandibular arches are shown in figures 24 and 25 respectively. Measured arch width was based on the coordinate distance between the second molar distobuccal cusp tips, which as can be seen from the curve fit analysis, underestimated the arch width at the second molars by approximately 1.5 mm. in the maxillary and 0.5mm. in the mandibular arches (table 4 and 5). The measured arch depth underestimated the curve fit values by approximately 1.5 mm both in the maxillary and mandibular arches.

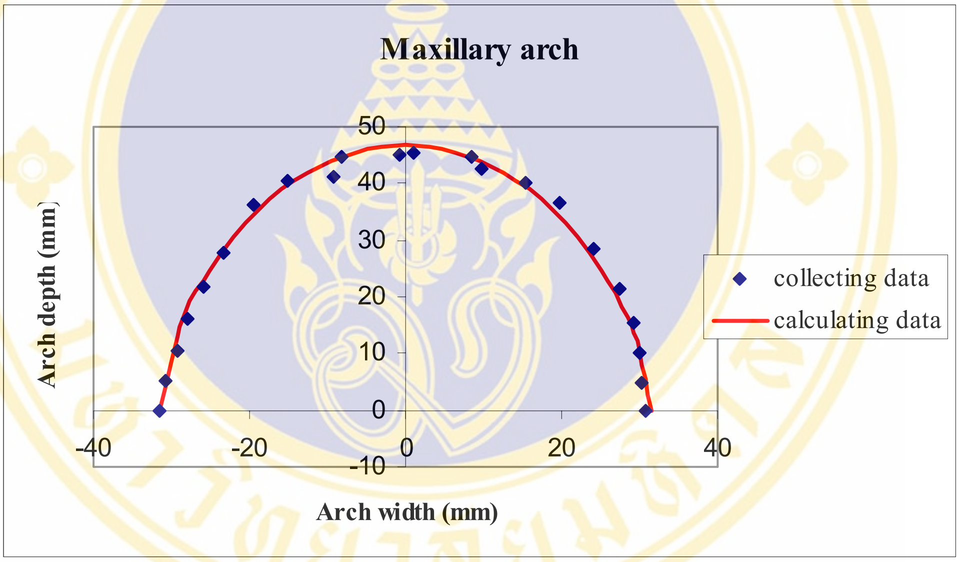

Figure 24: Typical recorded landmarks of maxillary dental arch were collected in this study. The superimposition of the collecting data (in blue point) and the calculating data from the beta function (red line) are showing in this figure.

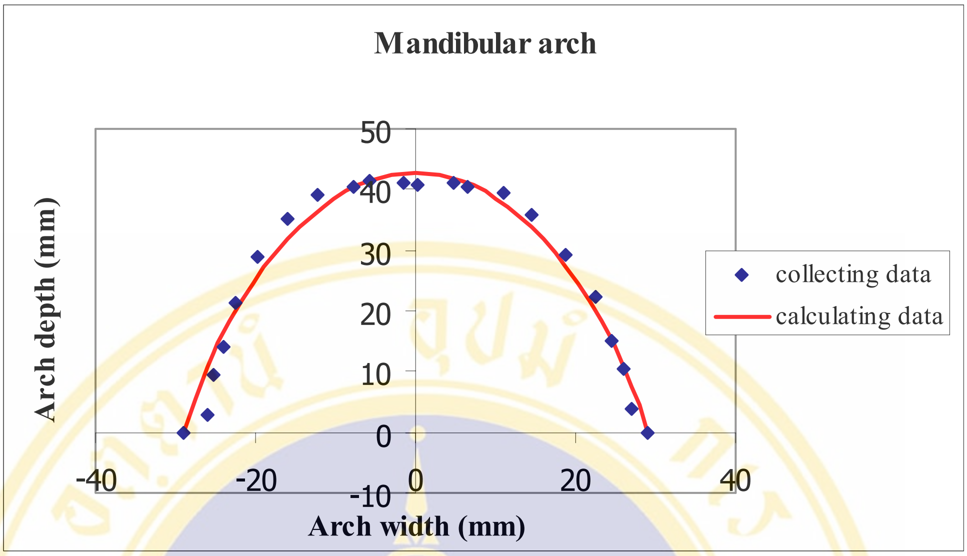

Figure 25: Typical recorded landmarks of mandibular dental arch were collected in this study. The superimposition of the collecting data (in blue point) and the calculating data from the beta function (red line) are showing in this figure.

Table 6. Descriptive statistics (Means and SD) of each parameter both in the maxillary and mandibular arches between males and females.

Parameter	Female (n=20)		Male (n=20)	
	Mean (mm)	SD	Mean (mm)	SD
Maxilla				
a-parameter	44.73	2.11	45.00	2.06
c-parameter	59.36	2.84	61.88	2.12
d-parameter	1.66	0.01	1.67	0.01
Mandibular				
a-parameter	40.48	2.18	41.00	2.07
c-parameter	53.45	3.31	56.96	1.78
d-parameter	1.75	0.12	1.79	0.10

Table 6 was show the descriptive statistics (Means and SD) of each parameter both in the maxillary and mandibular arches between males and females. All of the parameters showed the small difference in values between the sex dimorprism. The comparison of each instance between males and females are shown in table 7. No statistically significant difference of the a- and d-parameters were found between males and females at p-value > 0.05 both in the maxillary and mandibular arches. It was found that the arch width in both arches from males were significantly greater than females at p-value < 0.01. The differences were 2.52 mm. in the upper arch and 3.51 mm. in the lower arch.

Table 7. Statistical difference of each instance between males and females.

Parameter	Mean diff (mm)	t	p-value
Maxillary			
A	0.28	-0.42	0.67
C	2.52	-3.17**	0.003
D	0.004	-0.13	0.89
Mandibular arch			
A	0.52	-0.77	0.44
C	3.51	-4.17***	0.00
D	0.05	-1.43	0.16

** Significant at p-value < 0.01, *** Significant at p-value < 0.001

Because there is a wide range of arch widths in the samples both the maxillary and mandibular arches, it was appropriate to categorize the arch size into small, medium, and large according to the arch width.

Table 8 shows the distribution of the sample size according to the arch size (small, medium, and large) in the maxillary arches according to the difference in the arch width. Five of the dental casts which had small maxillary arch size, had the dimension of molar width ranging from 52.00-57.50 mm. While twenty-four of dental casts which had medium maxillary arch size, the minimum molar width was 57.50mm. and the maximum molar width was 62.50 mm. Eleven of dental casts which had large maxillary arch size, had the dimension of molar width ranging from 62.50-67.50 mm.

Table 9 shows the distribution of the sample size according to the arch size (small, medium, and large) in the mandibular arches according to the difference in the arch width. Four of the dental casts which had small mandibular arch size, had the dimension of molar width ranging from 45.50-50.50 mm. While fifteen of dental casts which had medium mandibular arch size, the minimum molar width was 50.50 mm. and the maximum molar width was 55.50 mm. Twenty-one of dental casts which had large mandibular arch size, had the dimension of molar width ranging from 55.50-60.50 mm.

Table 8. The distribution of the samples size according to the arch size (small, medium, and large) in the maxillary arch.

Maxillary arch size	**Sample**
Small Molar width (52.50-57.5)	5
Medium Molar width (57.50-62.50)	24
Large Molar width (62.50-67.50)	11
Total	40

Table 9. The distribution of the samples size according to the arch size (small, medium, and large) in the mandibular arch.

Mandibular arch size	**Sample**
Small Molar width (45.50-50.50)	4
Medium Molar width (50.50-55.50)	15
Large Molar width (55.50-60.50)	21
Total	40

Table 10. The distribution of the samples size according to the size of maxillary and mandibular arches.

Maxillary arch size	**Mandibular arch size**			**Total**
	Small	**Medium**	**Large**	
Small	4	1	0	5
Medium	0	13	11	24
Large	0	1	10	11
Total	4	15	21	40

Table 10 shows the distribution of the sample size according to the size of maxillary and mandibular arches. It was not nescessary that the sample's maxillary arch size would be the same as the mandibular arch. In five of the dental casts which had small upper arch size, almost all had small mandibular arch size while only one was medium size. Of the twenty-four dental casts which had medium maxillary arch size, more than half had medium size mandibular arch, while eleven had large mandibular arch size. Of the eleven dental casts which had large maxillary arch size, almost all of these had large size mandibular arch while only one had a medium arch size.

Table 11. The discriptive statistics (Mean, SD) of arch width according to arch size (small, medium, and large size) both in the maxillary and mandibular arches.

Arch size	N	Mean (mm)	SD
Maxilla			
Small	5	55.29	1.33
Medium	24	60.43	1.49
Large	11	63.45	1.12
Mandibular			
Small	4	48.20	1.78
Medium	15	53.77	1.0
Large	21	57.57	1.1

Table 12. The statistical difference of arch widths according to the arch sizes (small, medium, and large size) both in the maxillary and mandibular arches.

Comparison of arch size	**Mean different of arch width (mm)**	**t**	**p-value**
Maxilla			
Small-Medium	5.14	-7.09***	.000
Medium-Large	3.03	-5.96***	.000
Mandibular			
Small-Medium	5.57	-5.09***	.000
Medium-Large	3.8	-4.87***	.000

*** Significant at p-value < 0.001

Table 11 shows the discriptive statistics (Mean, SD) of arch widths according to the arch size (small, medium, and large size) both in the maxillary and mandibular arches. The arch width of the medium size was greater than the small size by approximately 5.0 mm. both in the upper and lower arches. While the arch width of the large size was greater than the medium size by approximately 3.0 mm. in the maxillary and 5.0 mm. in the mandibular arches.

Table 12 shows the statistical difference of arch widths according to the arch sizes (small, medium, and large size) both in the maxillary and mandibular arches. It was found that the arch width from the medium arch size both in the maxillary and mandibular arches were greater than the small size. The differences were 5.14 mm. and 5.57 mm. respectively. These were significant at p-value < 0.001. Similarly, it was found that the arch width of the large arch size both in the maxillary and mandibular arches were significantly greater than the medium size (at p-value < 0.001). The differences were 3.03 mm. and 3.80 mm. respectively.

Tables 13 and 14 show the discriptive statistics (Mean, SD), and the statistical difference of arch depth according to the arch size (small, medium, and large size) both in the maxillary and mandibular arches respectively. All of each arch size showed the small differences in values between the arch sizes. No statistically significant difference of the arch depth were found between arch sizes (at p-value > 0.05) both in the maxillary and mandibular arches.

Table 13. The discriptive statistics (Mean, SD) of arch depth according to the arch size (small, medium, and large size) both in the maxillary and mandibular.

Arch size	N	Mean of arch depth (mm)	SD
Maxilla			
Small	5	44.07	0.14
Medium	24	44.85	2.09
Large	11	45.25	2.46
Mandibular			
Small	4	39.70	0.82
Medium	15	40.62	1.94
Large	21	41.02	2.38

Table 14. The statistical difference of arch depth according to the arch size (small, medium, and large size) both in the maxillary and mandibular.

Comparison of arch size	**Mean different of arch depth (mm)**	**t**	**p-value**
Maxilla			
Small-Medium	0.78	-1.81	0.08
Medium-Large	0.40	-0.5	0.62
Mandibular			
Small-Medium	0.92	-0.90	0.38
Medium-Large	0.39	-0.53	0.60

Brader (14) suggested to select of the proper arch form was based on arch width at the second molar. From these samples, there were no statistically significant difference of a- (arch depth) and d-parameter between males and females both in the maxillary and mandibular arches. While there were statistically significant difference of c-parameter (arch width) between males and females in both arches. Furthermore, there were statistically significant difference of arch width according to the arch size difference. Therefore, the curves were generated by substituting the mean arch widths of each category arch size (table 11), the mean arch depth (table 4 and 5) and the mean of d-parameter (table 2 and 3) of the samples into the equation as following:

$$Y = a\left(1-\frac{4X^2}{c^2}\right)^{d-1}$$

The maxillary and mandibular dental arch equations for each arch are shown below:

The maxillary arch

Small arch size

$$Y = 44.87\left(1-\frac{4X^2}{(55.29)^2}\right)^{0.67}$$

Medium arch size

$$Y = 44.87\left(1-\frac{4X^2}{(60.43)^2}\right)^{0.67}$$

Large arch size

$$Y = 44.87\left(1-\frac{4X^2}{(63.45)^2}\right)^{0.67}$$

The mandibular arch

Small arch size

$$Y = 40.73\left(1-\frac{4X^2}{(48.20)^2}\right)^{0.77}$$

Medium arch size

$$Y = 40.73\left(1-\frac{4X^2}{(53.77)^2}\right)^{0.77}$$

Large arch size

$$Y = 40.73\left(1-\frac{4X^2}{(57.57)^2}\right)^{0.77}$$

The form of dental arch, derived from the equations, in different sizes according to the arch width both in the maxillary and mandibular arches are show in figure 26 and 27 respectively.

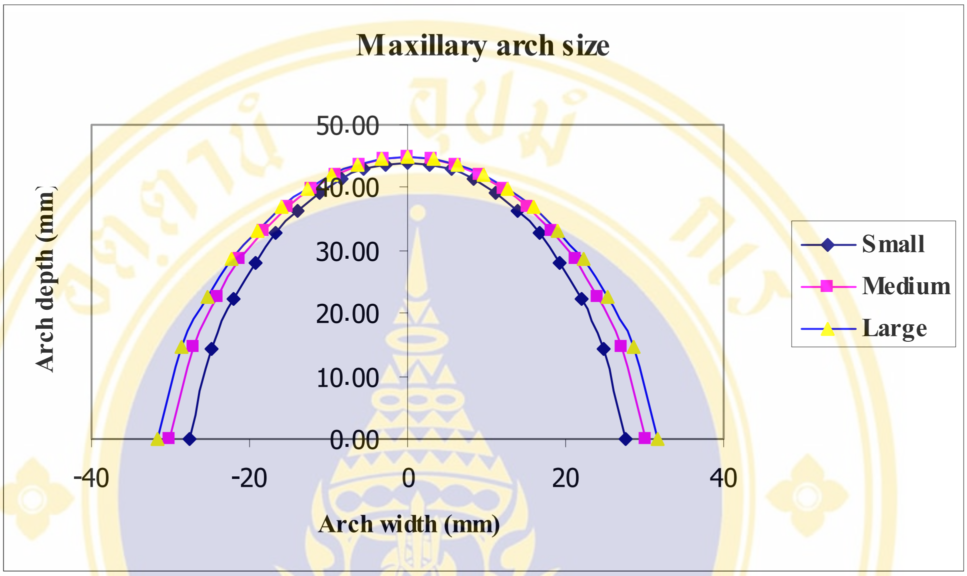

Figure 26: The maxillary dental arch curvature in different sizes (▬ = Small, ▬ = Medium, ▬ = Large).

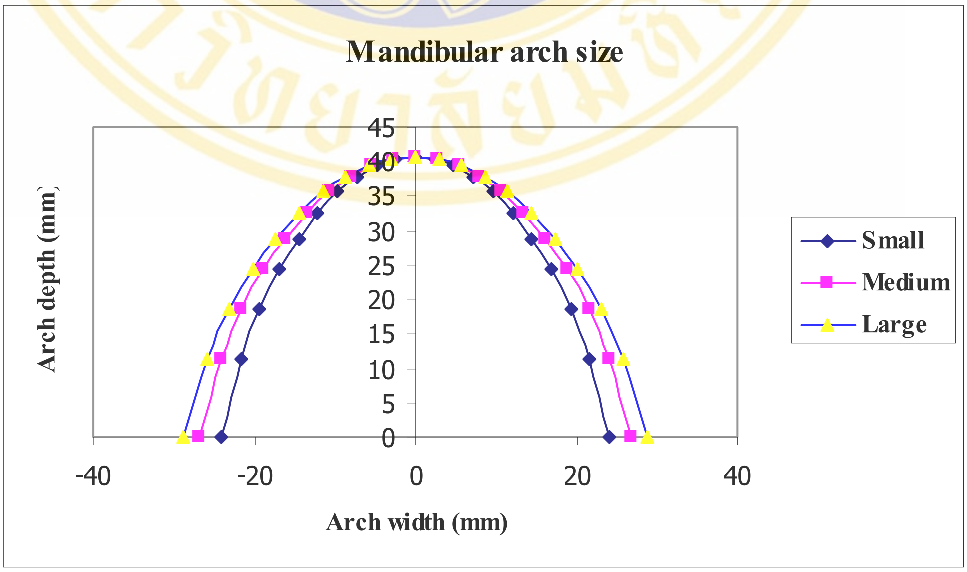

Figure 27: The mandibular dental arch curvature in different sizes (▬ = Small, ▬ = Medium, ▬ = Large).

Error of measurement

The method error was calculated by repeating the registration of dental landmark randomly on 6 sets of study models approximately one week afterwards. Double determination reveal errors in the measurements, including mean and standard deviations. These are summarized in tables 15 to 18. No statistically significant difference (at p-value < 0.01) was found between the two series of recording. Table 19 shows the errors of measurement which were smaller and were acceptable.

Table 15. The statistical differences of the X-coordinate from maxillary dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Maxillary arch X-coordinate						
X1	-0.00	0.00	0.00	0.00	0.00	0.17
X2	0.12	0.46	0.11	0.49	0.01	0.79
X3	2.37	0.68	2.34	0.61	0.03	0.47
X4	3.41	0.90	3.43	0.79	-0.01	0.79
X5	5.57	0.67	5.49	0.69	0.07	0.52
X6	8.08	0.45	8.12	0.40	-0.04	0.57
X7	12.33	0.60	12.32	0.69	0.008	0.90
X8	15.87	0.42	15.75	0.54	0.11	0.27
X9	20.56	0.89	20.45	0.87	0.10	0.19
X10	22.81	0.49	22.71	0.60	0.09	0.20
X11	29.64	0.74	29.56	0.69	0.07	0.36
X12	31.67	0.58	31.66	0.61	0.01	0.86
X13	38.35	1.02	38.24	1.01	0.10	0.05
X14	40.43	0.84	40.40	0.75	0.03	0.74
X15	45.22	1.31	45.19	1.25	0.03	0.72
X16	49.03	1.59	49.00	1.67	0.03	0.51
X17	53.30	1.67	53.39	1.64	0.08	0.35
X18	56.38	1.54	56.31	1.45	0.07	0.18
X19	58.89	1.15	58.82	1.18	0.07	0.10
X20	59.88	1.21	59.86	1.20	0.04	0.74
X21	61.77	1.08	61.64	1.13	0.14	0.23
X22	61.67	1.25	61.61	1.19	0.05	0.58

Table 16. The statistical differences of the Y-coordinate from maxillary dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Maxillary arch Y-coordinate						
Y1	0.00	0.02	0.01	0.01	0.01	0.07
Y2	5.02	0.40	4.95	0.49	0.07	0.35
Y3	10.54	0.62	10.52	0.58	0.02	0.73
Y4	15.80	0.86	15.63	0.76	0.17	0.06
Y5	21.68	1.22	21.48	1.12	0.20	0.02
Y6	28.34	1.79	28.21	1.63	0.14	0.37
Y7	35.78	2.13	35.66	1.92	0.12	0.31
Y8	40.17	1.91	39.95	1.88	0.22	0.04
Y9	42.55	1.88	42.38	1.85	0.16	0.02
Y10	44.23	2.07	44.12	2.10	0.11	0.41
Y11	45.04	1.95	44.88	2.02	0.15	0.10
Y12	45.25	1.84	45.01	1.68	0.25	0.06
Y13	44.33	1.96	44.22	1.83	0.11	0.34
Y14	42.91	1.35	42.87	1.38	0.04	0.71
Y15	40.65	1.34	40.60	1.35	0.05	0.52
Y16	36.45	1.81	36.36	1.87	0.09	0.37
Y17	28.85	1.90	28.33	2.49	0.52	0.38
Y18	21.99	1.57	21.57	1.38	0.76	0.36
Y19	16.28	1.15	15.98	1.15	0.29	0.02
Y20	10.72	0.71	10.64	0.78	0.08	0.45
Y21	5.22	0.51	5.01	0.48	0.20	0.03
Y22	0.01	0.06	0.01	0.02	0.02	0.35

Table 17. The statistical differences of the X-coordinate from mandibular dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Mandibular arch X-coordinate						
X1	0.00	0.00	0.00	0.01	0.00	0.43
X2	1.62	0.62	1.51	0.61	0.11	0.04
X3	2.82	0.82	2.72	0.76	0.09	0.07
X4	4.77	1.03	4.65	1.00	0.12	0.06
X5	7.18	1.23	7.11	1.25	0.06	0.5
X6	10.06	0.95	9.85	1.02	0.21	0.20
X7	14.03	1.23	13.99	1.11	0.03	0.77
X8	17.61	0.94	17.47	1.09	0.14	0.35
X9	21.50	0.74	21.52	0.98	0.02	0.92
X10	23.08	0.68	23.15	0.87	0.07	0.63
X11	27.27	1.02	27.24	1.16	0.02	0.87
X12	28.65	0.96	28.70	1.11	0.04	0.84
X13	32.76	1.18	32.75	1.43	0.01	0.95
X14	34.26	1.25	34.25	1.10	0.007	0.96
X15	38.33	1.38	38.23	1.30	0.09	0.60
X16	41.87	1.55	41.92	1.73	0.05	0.74
X17	46.35	1.52	46.48	1.63	0.12	0.35
X18	49.06	1.59	49.03	1.80	0.03	0.82
X19	51.84	1.43	51.79	1.44	0.04	0.50
X20	53.39	1.75	53.32	1.69	0.06	0.26
X21	55.27	1.36	55.62	1.31	0.34	0.36
X22	56.85	1.68	56.48	1.53	0.37	0.04

Table 18. The statistical differences of the Y-coordinate from mandibular dental landmarks between two measurements.

CMM-recorded	First measure (n=6)	SD	Second measure (n=6)	SD	Mean diff (mm)	p-value
Mandibular arch Y-coordinate						
Y1	0.01	0.01	0.01	0.02	0.02	0.12
Y2	3.93	0.76	3.89	0.69	0.04	0.46
Y3	9.30	3.47	10.92	0.93	1.62	0.36
Y4	15.25	1.05	15.13	0.99	0.12	0.24
Y5	21.74	1.52	21.84	1.62	0.10	0.46
Y6	28.91	1.62	28.86	1.56	0.26	0.36
Y7	35.33	1.67	35.06	1.39	0.27	0.17
Y8	38.27	1.55	37.93	1.31	0.34	0.17
Y9	39.94	2.01	40.02	2.05	0.07	0.48
Y10	40.62	1.55	40.42	1.71	0.21	0.24
Y11	40.54	1.77	40.37	1.79	0.16	0.04
Y12	40.61	1.99	40.45	1.96	0.16	0.32
Y13	40.60	1.89	40.58	1.98	0.01	0.89
Y14	40.24	2.06	40.04	2.11	0.20	0.09
Y15	38.43	2.06	38.67	2.08	0.08	0.36
Y16	36.02	1.91	35.69	1.99	0.32	0.11
Y17	31.27	3.44	29.57	1.90	1.68	0.35
Y18	22.59	1.52	22.42	1.43	0.17	0.41
Y19	16.07	1.21	16.36	1.37	0.28	0.62
Y20	11.90	1.11	11.75	1.24	0.15	0.58
Y21	4.78	0.76	4.56	0.68	0.22	0.36
Y22	0.08	0.07	0.05	0.07	0.03	0.12

Table 19. The measurement errors calculated using Dahlberge's equation.

Dental landmark	Maxillary arch		Mandibular arch	
	X-coordinate (mm)	Y-coordinate (mm)	X-coordinate (mm)	Y-coordinate (mm)
1	0.018	0.019	0.109	0.027
2	0.068	0.144	0.107	0.099
3	0.07	0.114	0.107	0.31
4	0.089	0.182	0.125	0.184
5	0.193	0.188	0.07	0.234
6	0.12	0.267	0.292	0.164
7	0.117	0.211	0.19	0.371
8	0.179	0.22	0.265	0.456
9	0.148	0.157	0.367	0.186
10	0.14	0.232	0.244	0.321
11	0.146	0.181	0.294	0.166
12	0.143	0.265	0.383	0.289
13	0.113	0.203	0.426	0.158
14	0.171	0.184	0.248	0.232
15	0.15	0.138	0.309	0.30
16	0.097	0.176	0.275	0.38
17	0.155	0.10	0.226	0.31
18	0.098	0.283	0.218	0.36
19	0.084	0.153	0.123	0.10
20	0.121	0.183	0.099	0.46
21	0.207	0.199	0.35	0.41
22	0.161	0.051	0.374	0.04

CHAPTER 5

DISCUSSION

According to several authors, the stability of the form and dimension of the mandibular dental arch is a factor in the stability of the therapeutic results. Technological advances in materials, with the use of new arch wires and appliances, have certainly enabled to obtain achievement of rapid results during phases of alignment and leveling between arches. However, the choice of the form of the dental arches has become essential when using these elastic arch wires.

Today, normally in Thailand, the clinician always uses preform arch wires which are based on Caucasian arch forms chosen to closely match to the patient's arch form. Based on the previous studies on stability and relapse (1, 37), it's agreed that preservation of the mandibular intercanine width and original arch form during orthodontic treatment will result in good post-treatment occlusal stability. In the study of Nojima and McLaughlin (32), it appears that the frequency of a particular arch form varies among ethnic groups. Benjamin et.al. (38) found that ethnic differences (American Blacks and Whites) also seem to correlate with arch form differences. Therefore it may prove problematic when the inappropriate superelasticity arch wire is used on the patients.

Since several studies have been performed associating a geometric curve form with the dental arch, many studies have sighted to predetermine a mathematical and geometric arch form on the basis of landmarks recorded on systems of coordinates (20-23). Some authors tend to individualize the arch form to respect the original mandibular intercanine width during treatment, by using arch guides (see Raberin(39) and references therein) or by a computer-assisted determination of an ideal dental arch form.

The present work aims at defining the mathematical equation of the shape of dental arch that best fit to Thais.

The samples in this study were collected from the subjects who were patients in the Orthodontic Department, Faculty of Dentistry, Mahidol University, from private dental clinics, and the Armed Forces Academies Preparatory School. The selection of 40 sets of good occlusion study casts (20 males, and 20 females) was aimed to be used for the study of dental arch shapes in normal occlusion subjects that are related to arch wires form. The models included the full permanent teeth (excluded the third molar).

In this study, the measurement being done by using the CMM (Coordinate Measuring Machine) to record the landmarks on dental model and report them in the corresponding X-,Y-,and Z- coordinates. CMM is used extensively in the precision machinetool industry. As previously described, Braun et al (21) and Qiong et al (40) used CMM to record the X-,Y- coordinates of dental landmarks for the study of human dental arch forms and the comparison of intermaxillary tooth size discrepancies among different malocclusion groups. The study of the reliability of CMM in measuring dental models (41) had conclude that, the Coordinate Measuring Machine can be applied to measuring dental casts in three dimensions with highly accurate, reliable and reproducible results.

Replicated measurements by the same single examiner were made on six sets of study casts after a time lapse of one week. The paired t-test used to assess any systematic error of the machine was reliable and reproducible. The method error of the measurements of the coordinates of dental landmarks were calculated according to Dahlberg's equation. The results of measurement showed a very small range of error (0.019-0.46 mm.). Intraexamination confirmed that the results were accurate and well-calibrated.

Coordinates of the dental landmarks were recorded at the distal and mesial points of each incisor incisal edge, at the cusp tips of the canines and premolars, and at the mesiobuccal and distobuccal cusp tips of each molar. Twenty-two points were recorded in each dental arch.

In fact, there appeared to be at least a question that what points should be used to represent the human dental arch. Herren (42) stated, most authors define the normal dental arch as an abstract curve lying on the occlusal plane, the size and shape of which was determined by the position of the buccal cusps of the molars and bicuspids, the canine tips, and the edges of the incisors. MacConaill and Scher (9) fitted catenary functions to a “curve of common occlusion” defined by occlusal contact points on the upper and lower arches considered together on the upper arch. They selected the buccal cusps of the molars and premolars, and the incisal edges of the canine and incisor teeth: and, on the lower arch, the central fossae of the molar teeth, the occlusal fissures of the premolars, and the incisal fossae of the canines and incisor. Currier (43) compared the fits of parabolas and ellipses to three different sets of loci: the labial landmarks of teeth, the middle landmarks and the lingual landmarks. For the middle landmarks however, he found that parabolas fit best, a statistically unenable conclusion since, as noted above, parabolas were just limiting case of ellipses.

Since the description of shape clearly depends on the choice of data points (43), one must carefully justify the set of points used in a particular analysis. For example, different sets of coordinate landmarks might be appropriate for studying shapes related to arch wires and for studying the shape of the underlying alveolar process.

The size and shape of the human dental arch have been studies for over a century. However, the studies have been handicapped by the lack of good geometric methods for describing or modeling biologic shape and by the lack of a statistical model permitting the investigation of concepts of “ average shape”. The past work in the field of morphometrics, the measurement of shapes, the variation and change provides valuable new tools for the discussion of biologic shape.

Lu (16) used the multiple regression and correlation and the coefficient of determination to define the goodness of fit of the fourth order polynomial equation in the analysis of the form, symmetry and asymmetry of the dental arch. In his situation, however, the observed values were (except for small errors of measurement) the true, while the calculated values were only approximations to the truth, resulting from his efforts to obtain an effective representation, neither can properly be regarded as a random variable, which loosed their more time. While Bookstein’s algorithm for fitting conic sections to data, or Sampson’s modification of that algorithm (see Sampson (44) and the references therein) was well-suited to modeling the shape of the dental arch. Previous approaches to modeling the outline of the dental arch have used fitting algorithms which depend on the exact specification of a coordinate system.

Many of the algorithms are classic least-squares methods, minimizing the sum of squared distances from data points to a fitted curve in the direction parallel to an arbitrarity chosen Y-axis.

In the present, we used the evolution algorithm, a good optimization method (45), to calculate the parameters of the beta function from the collected data of each dental arch. It was based on the minimized sum of squared errors of the calculated values to the measurement values (details about the estimation of its parameters from a sample of data were given by Department of Mathemetics, Faculty of Science, Mahidol university). The goal was to obtain the measure of the closeness of agreement between the true arch form and the approximation. The total sums of squares accounted for by the regression was a measure of closeness of an approximation. The ratio

$$R^2 = 1 - \frac{\sum_{i=1}^{n} e_i^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2} = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

(where y_i is the Y-coordinate of the data point and $\hat{y}_i$ is the computed Y-coordinate of the point on the beta function curve corresponding to the original X_i)
was equivalent to the coefficient of determination in the multiple-correlation sense. As a rule, the beta function will fit the arch form quit nicely, the investigation may require that $R^2 \geq a$; for $0 < a < 1$ to suit this purpose (16).

In this study, we show the goodnesss of fit of the beta function to represent each of the 40 maxillary and mandibular arches, as also verified by the high coefficient of determination (R^2), ranging from 0.93-0.99 (tables 4 and 5). This is in agreement with Braun et.al. (21), who has been the first to offer the beta function to describe the dental arch, with an average correlation coefficient of 0.98 with a standard deviation of 0.02.

According to the geographic mathematic, the beta function describing the human arch form, the general formula (the relationship between the width and depth of the dental arch) can be expressed as:

$$Y = a\left(1 - \frac{4X^2}{c^2}\right)^{d-1}$$

The parameter was described previously. If the a-parameter (arch depth) and c-parameter (arch width) were set to 40 mm. and 55 mm. respectively while the d-parameter was substituted with different values, the resulting curve would express different arch forms, as illustrated in figure 28.

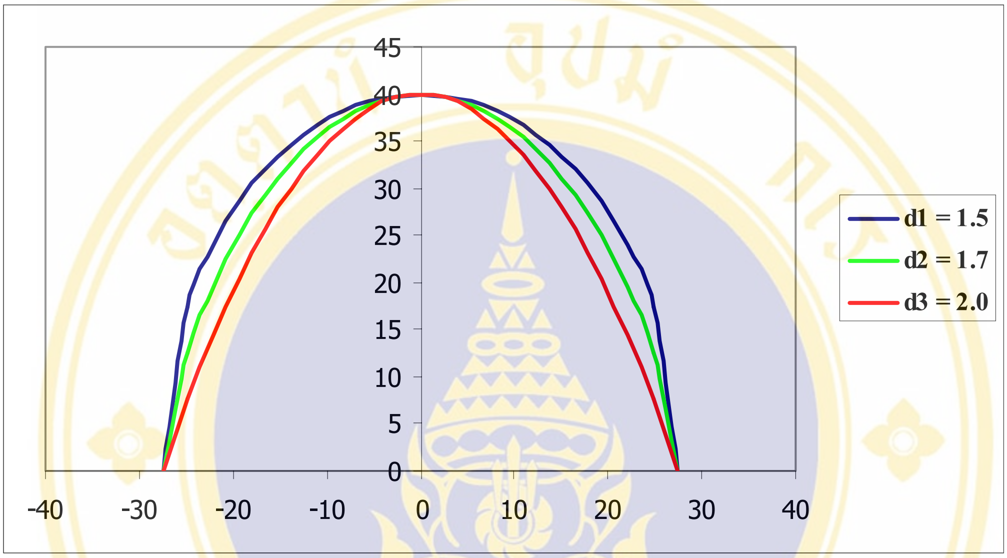

Figure 28: The curves expressed different arch forms according to the difference in d-values.

Since the d-parameter measures the quadratic portion, the larger the magnitude of d-parameter, the more the arch would assume the form of a taper; roughly, we may say it is the measuring of "taperedness". The consequence of small d-parameter in physical terms would tend to give the arch a more square-like look. In this description, we may say that d-parameter is a description of the form of dental arch. The more the values of the d-parmeter, the wuder the arch form is to be. The less the values of the d-parameter , the narrower the arch form is to be.

According to the d-parameter, we found that the arch form of the normal Angle's classification subjects have a more wider-like look (the maxilla d-parameter = 1.67, mandibular d-parameter = 1.77) than the caucasian normal Angle's classification I in the study of Braun with d-parameter equal to 1.8 in both maxillary and mandibular arches (21). This is in agreement with Nojima and MaLaughlin (32) who compared the

Caucasian and Japanese mandibular clinical arch forms. Their study had concluded that the Caucasian population had a statistically significant decreased arch width and increased arch depth compared with the Japanese population, when the subjects were regrouped by arch form, no statistically significant difference in arch dimension was observed between the two ethnic groups in any of the arch form samples. They results suggested that there was no single arch form specific to any of the Angle's classifications or ethnic groups.

The difference of the arch form and the arch depth (d and a-parameters, respectively) between the men and women (tables 6and 7) was not observed. Opposite to the arch width, there had been a statistically significant difference between males and females. The male arches proved to be slightly bigger than female arches (table 6). This observation was the same as Ferrario et.al. (20, 34) who defined the mathematical formulas of the shape of dental arches in humen permanent healty dentitions. The maxillary arch were wider than the mandibular arch regardless of gender. Gender differences were found especially in the maxillary arch, where they reflected more of a size discrepancy than a shape difference. Gender seemed to have some influence on arch sized differences; with men showing larger differences than women. The same conclusion has been drawn by Raberin et.al. (39) who studied the dimensions of the dental arches according to sex, and showed that all transverse measurements were on average smaller in female subjects. Among the sagittal dimensions, arch sizes were significantly smaller in women.

Clinical application

According to the mean of d-parameter, the general formula which represent the maxillary arch are following

$$Y = a\left(1 - \frac{4X^2}{c^2}\right)^{0.67}$$

The general formula which represent the mandibular arch are following

$$Y = a\left(1 - \frac{4X^2}{c^2}\right)^{0.77}$$

The a-(arch depth) and c-(arch width) parameter are varies depending on the arch dimension of the individual. If patient's maxillary arch depth is 42 mm. and arch width is 60 mm., we can predict the arch form of these patient by substituted the a-parameter (42 mm.), and the c-parameter (60 mm.) in the equation. Furthermore, we can predict the arch form after arch expansion, comparing to the pre-treatment arch form.

From the results, we generated the "average" arch shape in the sense that it represents the most likely shape in the sample. We were concerned with the shape of the arch that is average, good-looking, well-developed, and free from any conspicuous deformity. One can note a greater relative dispersal of the arch width values of the subjects (tables 4 and 5).

Unfortunately, attempts to define an average shape have generally been limited to examinations of the average of tooth coordinates, a dubious procedure, even if the different sizes of the dental arches are taken into account. Some have referred to "average shapes" without defining precisely what an average shape is (14, 43).

Therefore, the distributions with homogeneous arch sizes were examined. Then the number of groups was distinctly defined into three categories (tables 8and 9) both in the maxillary and mandibular arches according to the difference in arch width. The arch depth values from the samples were statistically equal. Therefore, the equation to represent the maxillary and mandibular dental arches for Angle's classification I for Thais are given by the formulas:

The maxillary arch

Small arch size

$$Y = 44.87\left(1 - \frac{4X^2}{(55.29)^2}\right)^{0.67}$$

Medium arch size

$$Y = 44.87\left(1 - \frac{4X^2}{(60.43)^2}\right)^{0.67}$$

Large arch size

$$Y = 44.87\left(1 - \frac{4X^2}{(63.45)^2}\right)^{0.67}$$

The mandibular arch

Small arch size

$$Y = 40.73\left(1-\frac{4X^2}{(48.20)^2}\right)^{0.77}$$

Medium arch size

$$Y = 40.73\left(1-\frac{4X^2}{(53.77)^2}\right)^{0.77}$$

Large arch size

$$Y = 40.73\left(1-\frac{4X^2}{(57.57)^2}\right)^{0.77}$$

Limitations and Suggestions

To find out whether an ideal orthodontic arch form could be identified, Felton (31) studied the mandibular casts of 30 untreated normal cases, 30 Class I non-extraction cases, and 30 Class II non-extraction cases. The overall message from these clinical observations and research papers were:

1. There are extensive variations in human arch forms.
2. Because of these variations there does not seem to be any single arch form that can be used for all orthodontic cases.
3. If the patient's original arch form is changed during treatment, there is a strong tendency (in as much as 70% 0f cases) for the arch form to return to its original shape after appliances are removed.

Because of the variation of human arch form, studies needed to be done to fit curves of beta function to dental arches to examine the variation from average shapes in a population.

There are important differences between "Research arch form" and "Clinical arch form". The present study reported the normal Angle's classification I arch form in Thais could be represented by a complex mathematical formula, known as the beta function. This was calculated by entering measurements of dental landmarks on orthodontic models into a computer curve-fitting program. We measured "the mesial and distal point of incisal edge of incisors, the cusp tips of the canines and premolars, and the mesiobuccal and distobuccal cusptips of the molars". The resulting arch form can be surprisingly narrow, but this is a "Research arch form". It is not useful for the clinician, and it is not appropriate for the direct construction of arch wires.

In contrast, the clinician's arch wire shape must be based on the points where the wire will lie in the bracket slots of correctly positioned brackets. This "clinical arch form" relates to the mid-points on the labial surface of the clinical crowns of the teeth, and should include an estimation for the in-out which is built in to the bracket system. The "Clinical arch form" is not related to the incisal edge or the cusp tips which are used to establish the "Research arch form". Two different arch forms are used for the same dental model.

The beta function has been shown to be an accurate representation of Thai dental arches in the Angle's classification I. The precision of these equations to represent the dental arch of the population depend on the form of those dental arches. The more the arch shape resembles that of the study samples, the higher the precision of the beta function.

Because of the small sample sizes in this study, the additional studies need to be done to fit the curves of the beta function of dental arches to examine the variability from average shapes in a population.

Furthermore, earlier studies (20, 21, 46) have focused on the variation of the form of dental arches among the different types of Angle's classification. Additional studies to evaluate arch form in Angle'sclassification II, and III in Thais would be desirable, to achieve better understanding of the Thai dental arch forms.

CHAPTER 6

CONCLUSIONS

The samples comprised of fourty sets of dental casts (20 males and 20 females) which expressed normal occlusion. The results revealed that:

1. From the present study, the beta function accurately described Thai dental arch forms with the high coefficient of determination, ranging from 0.93-0.99. The general formula are generated as following:

The maxillary arch

$$Y = a\left(1 - \frac{4X^2}{c^2}\right)^{0.67}$$

The mandibular arch

$$Y = a\left(1 - \frac{4X^2}{c^2}\right)^{0.77}$$

The equations represent the maxillary and mandibular arches according to the difference in arch width, are following:

The maxillary arch

Small arch size

$$Y = 44.87\left(1 - \frac{4X^2}{(55.29)^2}\right)^{0.67}$$

Medium arch size

$$Y = 44.87\left(1 - \frac{4X^2}{(60.43)^2}\right)^{0.67}$$

Large arch size

$$Y = 44.87\left(1 - \frac{4X^2}{(63.45)^2}\right)^{0.67}$$

The mandibular arch

Small arch size

$$Y = 40.73\left(1 - \frac{4X^2}{(48.20)^2}\right)^{0.77}$$

Medium arch size

$$Y = 40.73\left(1 - \frac{4X^2}{(53.77)^2}\right)^{0.77}$$

Large arch size

$$Y = 40.73\left(1 - \frac{4X^2}{(57.57)^2}\right)^{0.77}$$

2. The d-parameter of beta function is a description of the form of dental arch. The higher the values of d-parameter, the more taperness the arch form is to be, the less the values of d-parameter, the more squared the arch form is to be.
3. According to the d-parameter, we found that the Thai dental arch forms have a a wider-look like (maxillary d-parameter = 1.67, mandibular d-parameter = 1.77) than the Caucasian arch form in Braun's study with d-parameter equal to 1.8 in both maxillary and mandibular arches.
4. Gender seemed to have some influence on arch size differences, with men showing larger significant difference than women.
5. There are important differences between "Research arch form" and "Clinical arch form". The resulting arch form (Research arch form) can be narrow and it is not appropriate to use this shape directly for the construction of arch wires.
6. Further studies need to focus on the clinical arch wire shapes. These must be based on the points where the wire will lie in the bracket slots of correctly positioned brackets, and should include an estimation for the in-out which is built into the bracket system.

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