SANDWICH FACTORY BOARD GAME WITH MANIPULATIVE GEARS FOR PRIMARY STUDENTS TO CONSTRUCT THE WAYS TO FIND LEAST COMMON MULTIPLES: A CASE STUDY

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Yiyao Liu

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ABSTRACT

Students enjoy playing games, as games bring motivation and active learning. Board games could be used as meaningful learning tasks for mathematics education. For this purpose, this study employed the qualitative method, case study, to examine the Sandwich Factory Board Game with fourth grade students. The board game designed with an educational purpose in mathematics is used to teach least common multiple concept. Therefore, the main goal of this study is to develop an educational board game and explore students' LCM learning process while playing the board game. Nine students were selected from 45 students through the prior knowledge test and were divided into 3 groups (high achievers, middle achievers, and low achievers) to participate in this study eventually. Data was collected using video recording, interview, and questionnaire. And the results found that all students are able to develop efficient methods to find the LCM. The findings also show that students had great interests in the game, especially in gold trading, calculation, and function cards.

KEY WORDS: LEAST COMMON MULTIPLE / MANIPULATIVE / EDUCATIONAL BOARD GAME / CONSTRUCTIVISM / GAME BASED LEARNING

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CHAPTER I
INTRODUCTION

"Math is not a spectator sport. It's not body of knowledge, it's not symbols on a page. It's something you play with, something you do." Keith Devlin (2015). Student's understanding of mathematics always is a paramount issue in educational circles. For primary students, teaching and learning of number sense are considered to be a major topic in international mathematics curricula (Australian Education Council Review Committee, 1991; National Council of Teachers of Mathematics 1989). Many primary graders lack number sense and baffle with these concepts. Because the relations of this knowledge in math are closely related and persistent (Getgood, 2000). For example, LCM (Least Common Multiple) can help students to find a common denominator, so that students will be able to combine various fractions. Besides, student should connect the abstract mathematical concepts to LCM examples in their daily life. For instance, they need to realize the meaning of dynamic events or continuing events, also learning the ways to deal with LCM problems in different type of numbers, such as prime number and composite number, instead of just multiplying the two numbers together (Sinclair, Zazkis, & Liljedahl, 2003). Therefore, it is necessary and significant to figure out a good way to help students to find LCM efficiently. An increasing number of educators show that the game increasing students' performance both in achievements and attitudes. (e.g., Sung & Hwang, 2013; Hwang & Chang, 2011; Bai, Pan, Hirumi, & Kebritchi, 2012 etc). But the most existing games for learning LCM are focused on the practicing the fluency of calculation. The students only can play the games after completed the relevant topics (Moyer 2001). A good educational board game should provide an adaptive and motivating setting, which can meet the learning needs of low as well as high achieving children (Stebler, Vogt, Wolf, Hauser, & Rechsteiner, 2013). In addition, Heddens (1997) argued that using manipulative materials in teaching mathematics will help students to connect real-world situations, also can symbolize the mathematics problems in many different ways.

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Students contact with objects directly and consequent manipulation will facilitate them to think, to formulate hypotheses and to test their hypotheses (Bednar, Cunningham, Duffy, & Perry, 1995; Gärdenfors & Johansson, 2005; Papert & Harel, 1991). Thus, students can better understand and remember the topics, feeling a positive sense of success when the task is completed (Jonassen, 1999; Wing-Mui SO, 2002). In this study, the LCM board game with manipulatives was designed based on constructivist. Mayer (2004) proposes learners should be "cognitively active" during learning and that instructors use "guided practice", which means the teacher should offer some prior knowledge before teaching. In contrast, Kirschner (2006) describe constructivist teaching methods as "unguided methods of instruction." They suggest more structured learning activities for learners with little to no prior knowledge of subject matter. This research will study how the LCM board game help students who don’t not have abundant prior knowledge to construct the ways to find LCM. For that propose, this study will focus on the playing process of specific groups. Video recording and interview are necessary for collecting data to analyze how students construct the ways to find the LCM. And results will be benefit to the educators who want to develop an educational board game for teaching LCM or relevant topics, or teachers who going to teach LCM concept in a mathematics classroom. Therefore, the current study will investigate the LCM board game with manipulative materials by focusing on the following research questions.

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1.1 Purposes of study

The current study will investigate the Sandwich Factory Board Game in a primary school, by focusing on these research objectives:

1) To develop an educational board game for students to learn the least common multiple.

2) To explore students' learning process of least common multiple while playing the board game.

3) To explore the rank of attractiveness of the elements of board game.

1.2 Research questions

Following the research objectives, there are two research questions.

1) How do fourth-grade students construct LCM knowledge while playing sandwich factory board game?

2) What are students' perceptions toward the sandwich factory board game

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CHAPTER II
LITERATURE REVIEW

2.1 The concept of LCM, GCD, and PF in mathematics

2.1.1 Least Common multiple (LCM)

In general and number theory, the least common multiple (LCM) of two integers *a* and *b*, usually denoted by LCM (*a*, *b*), is the smallest positive integer that is divisible by both *a* and *b*. Since the division of integers by zero is undefined, this definition has meaning only if *a* and *b* are both different from zero.

There are 3 ways to solve the LCM problems; 1) "List Method", list the multiples of each number, Find the first Common (same) value. 2) Individual Prime Factoring Method, firstly, finding the prime factorization of each number in the group. Secondly, making a list of all factors, raised to the highest power that appears in any factorization. Thirdly, multiply out to get the LCM. The procedure of operation shows in figure 2. 3) Group-Factoring Method. Using prime numbers as divisors, divide each number in the group, if possible. When a number is not divisible by the prime divisor just bring it down to the next row along with the quotients from the other divisions. Continue dividing by prime divisors until all quotients are ones. Multiply all of the prime divisors and the product is the LCM. The example shows in figure 3.

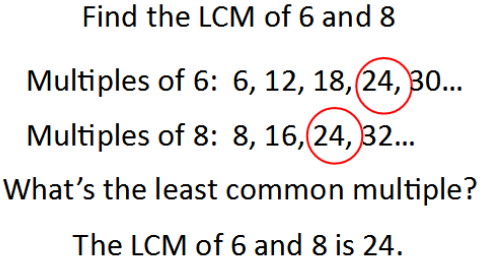

Figure 2.1 "Number List"

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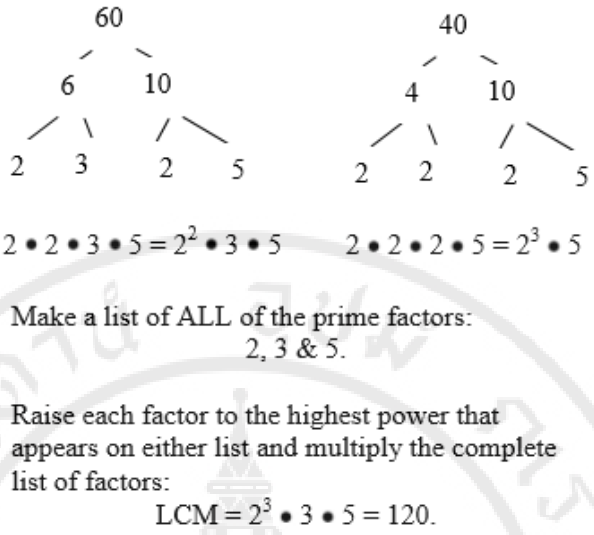

Figure 2.2 Individual Prime Factoring Method

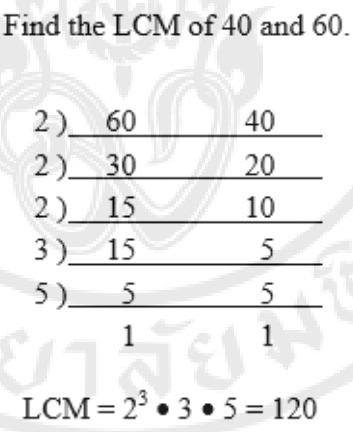

Figure 2.3 Group-Factoring Method

2.1.1 Great Common Divisor (GCD)

The greatest common divisor is also known as the greatest common factor (GCF). The greatest common divisor of two or more integers, which are not all zero, is the largest positive integer that divides each of the integers. For example, the GCD of 8 and 12 is 4. GCD Problems may be asking students to split things into smaller sections. Or equally, distribute 2 or more sets of items into their largest grouping.

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2.2.2 Prime Factorization (PF)

Prime factorization or factoring is the decomposition of a mathematical object (for example, a number, a polynomial, or a matrix) into a product of other objects, or factors, which when multiplied together give the original.

2.2 LCM in Chinese curriculum

According to full-time compulsory education mathematics curriculum standard (2011). Students are required to study the topic of "number and algebra" within the fourth to sixth grades, and the least common multiple is included in this topic.

The curriculum requires students to understand the concept of the common multiple and the least common multiple at first. And the standard requires students to find all the multiples of the natural number within 10 in the natural number from 1 to 100, and know the characteristics of the multiple of 2, 3, 5. Besides, students are also required to find the common multiple and the least common multiple of two natural numbers within 10. In the curriculum, the list method always comes first, then students will learn the individual prime factoring method and group-factoring method. In the Chinese mathematical curriculum. Students usually learn a lot of other concepts before learning LCM, such as factors, prime number, composite number, odd number, and even number.

2.3 The difficulty and misunderstanding of LCM

Primary students had the difficulty in understanding the concept of Least Common Multiple (LCM) and in solving problems related to that concept (Orhun, 2002; Dias, 2005; and Camli&Bintas, 2009). The ideas of least common multiple and greatest common divisor (GCD) are often taught separately. But after students learned these two concepts, they always confused LCM with GCD. Some students have the difficulty in differentiating LCM and GCD as well as understanding the questions. And students have the difficulty when work with various types of numbers at the same time and consider the relationships to each other (e.g. primes, squares and specific sets of multiples).

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2.4 Significance of the least common multiple

Understanding always is a paramount issue in mathematics for young kids, fraction concepts and operations, ratios, proportions and related skills using greatest common factor (GCF), least common multiple (LCM), and prime factorization (PF), as well as ideas about number theory in general. The relations of this knowledge in math are closely related and persistent, Getgood (2000). The ways to solve problems of Least Common Multiple could use the knowledge of greatest common factor and prime factorization, and their associated operations are significant for the upper school levels.

MFD (mixed numbers, fractions, and decimals) also has been built and tested for fifth and sixth graders. Students learn about the addition, subtraction, multiplication, and division of whole numbers, fractions, mixed numbers, and decimals (Beck, Stern, & Woolf, 1997). Fractions, a part of the rational number system, have many applications in daily life, therefore, the ability to handle fractions is necessary and significant in life. Students' rote memorization often make mistakes and interferes with their informal knowledge of fraction, Mack (1993). To add fractions, a student must find the LCM firstly, convert the fractions to equivalent forms using that LCM, add the numerators, simplify the result, and convert the result to a proper form. All these foundations are necessary for learning the sequential concepts of rational expressions and polynomials in algebra (Van de Walle, 1998)

LCM is a member of number sense in primary school. Helping children develop number sense is considered as an important task of mathematics education in the international community (Australian Education Council, 1991; Cockcroft, 1982; Number sense refers to a person’s general understanding of numbers, operations, their relationships, and the ability to handle daily-life situations related to numbers. This ability is used to develop useful, flexible, and efficient strategies (including mental computation and estimation) to handle numerical situations (Reys & Yang, 1998; Huang, 2004).

2.5 Manipulative in mathematics.

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Manipulatives, are physical tools of teaching, engaging students visually and physically with objects such as coins, blocks, puzzles, markers, etc. The use of

manipulatives is constructivist because students are actively engaged in discovery during the learning process.

Educators consider using various strategies to promote students understanding of mathematics. Zoltan Dienes' (1969) work convinced researchers that the use of various representations of a concept, or 'multiple embodiments,' were needed to support students' understanding. Manipulative materials are objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences. Piaget (1952) mentioned that children have no mental maturity to master abstract mathematical concepts in words or symbols, and many experiences need to be learned with concrete materials to help them achieve the goal of learning, such as drawings and playing. Hence, it is necessary to design specific math manipulatives focusing at different mathematical concepts.

Getgood (2000) studied about using Factor Blocks™ as a manipulative for teaching GCF, LCM, and PF, and the experimental group shows a positive response to using Factor Blocks™ for learning these concepts. Research also found that manipulatives provide a way to motivate students to enjoy mathematics and to experience success in understanding mathematics.

Some common misconceptions about manipulative materials are pointed out by Jackson (1979), such as the fact that manipulative does not necessarily simplify the learning of math concepts. The more manipulative students used for a single concept, the better the concept is learned. And manipulative is more helpful in primary grades than in Secondary. And it's more helpful for low achievers than high achievers. In simple terms, using the manipulatives in class is not straightforward, good employment requires careful defining the role of the teacher, aims and the potentials of the tasks involved. Furthermore, students sometimes used manipulatives in a rote manner (Hiebert & Wearne, 1992). Clements and McMillen (1996) claim that student often fails to link their action with manipulatives to describe the actions. Therefore, the combination of game-based learning and manipulative materials might have a miraculous effect on students learning performance.

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2.6 Kolb-Learning style

David Kolb (1984) proposed his learning styles as the Figure 4 shows us. Kolb's experiential learning theory works on two levels: a four-stage cycle of learning and four separate learning styles. In Kolb learning cycle, each style is considered a stage of learning and students learn by cycling. For example, the cycle begins with the student's personal involvement through concrete experience; next, the student reflects on this experience, looking for meaning; then the student applies this meaning to form a logical conclusion; finally, the student experiments with similar problems, which result in new concrete experiences. From here, the learning cycle begins again (Hartman, 1995).

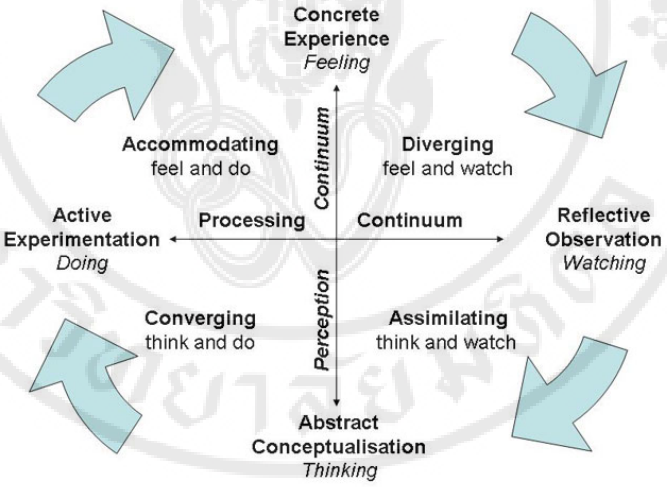

Figure 2.4 Four Stages of Experiential Learning. (Kolb, 1984)

2.7 Social constructivism

Social constructivism states that learns work together to build artifacts. while social constructivism focuses on artifacts created through social interactions of groups. Active participation where learning is created based on collaborative effort. (Prawat & Floden, 1994). Some educators believed social constructivism is a pedagogy stating that, “knowledge is the result of social interaction and language usage, and thus

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is a shared, rather than an individual, experience". So, shared interaction creates common knowledge. People within a group construct group knowledge, then that knowledge become a part of that group. Students can interpret that common knowledge with their own experience.

2.8 Game-based learning

Advances in game-based learning and interactive learning opportunities for students that may enhance student's achievement. Games have been proposed as meaningful educational activities and cognitive tools might improve students' active involvements in the teaching-learning process and encourage their reflections on the concepts. Computer games rising sharply with the technology development, and there a lot of computer games have been proved as a useful and meaningful tool by many educational researchers (e.g., Durmus & Karakirik 2006)

Mathematics is unique, unlike other science subjects which contained many aspects in one lesson (Moyer, 2001). Children vary greatly in the mathematical knowledge. They possess when they enter school. These differences in initial mathematical knowledge appear to have large, long-term consequences. Proficiency in mathematics at the beginning of kindergarten is strongly predictive of mathematics achievement test scores years later, in elementary school, middle school, and even in high school (Stevenson & Newman, 1986). Therefore, the significance of understanding the concept perfectly of primary students looks more urgent. Many evidence suggests that game-based learning can improve engagement and motivation (Perrotta, Featherstone, Aston, & Houghton, 2013). Teachers should have more resources and activities to support game learning. Which means the link between game and knowledge must flexible enough to facilitate students to explore the further step.

Educators have realized advances in game-based learning and interactive learning opportunities for students that may enhance student's achievement. For this reason, there are many teachers are exploring a good method to teach mathematics. Every year, a lot of math games were invented, therefore, the number of existing games is huge. But some activities and games are more focused on the practicing the fluency of calculation, the designing of some games is quite brief. And evidence shows that many games (38 out of 67) have no significant difference between games and

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conventional classroom instruction. Which makes innovation in learning is more significant for students(Randel, Morris, Wetzel, & Whitehill, 1992).

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CHAPTER III
METHODOLOGY

3.1 Board game design.

3.1.1 Concept design.

The idea of game design is based on the initial way (Number List) to deal with the LCM problems. The problem can be solved using Least Common Multiple because we are trying to figure out when the soonest (Least) time will be that as the event of exercising continues (Multiple), it will occur at the same time (Common). Therefore, the main mission of the board game was designed to facilitate students to find LCM by themselves. There are many common multiples of two numbers, but these common multiples also represented the amount of money that they are going to pay. So, the biggest number they found, the more money they paid.

Using the gear system to calculate LCM in mathematics is a useful manipulative material for students, Ura and Stein-barana (2012) describe how to "build" a machine to find the LCM of two numbers as the figure 5 shows. To find the least common multiple, engage the gears with the radial marks lined up. Turn one gear and count how many laps it gives until the marks coincide again. This way of using gear needs two students work together to mark and count, which is time consuming and error prone. In order to match the rule of the game, researcher redesigned the gear as the figure 6 shows.

The role of manipulative gears is determining the correctness of the answers given by the challenger. During game play, if there has an LCM but no one was aware of it. In that circumstances, players can continue playing the game without finding the LCM, so they don't need gears. But if one player realized LCM that other players have not found. This player can give the best answer that he thought to challenge another player. So, if this challenge happened, players can choose to use the manipulative gears to examine the best answer (least common multiple). But the gears only can be used to check the correctness of the challenge number if the challenge occurs.

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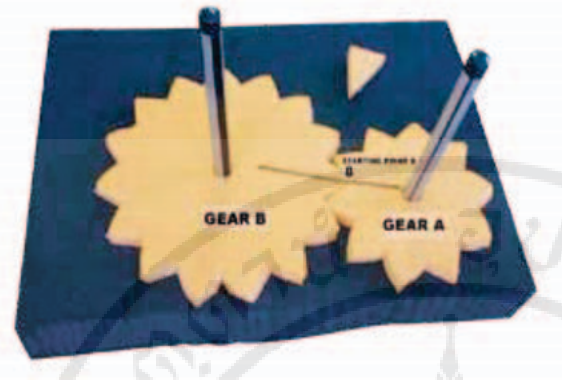

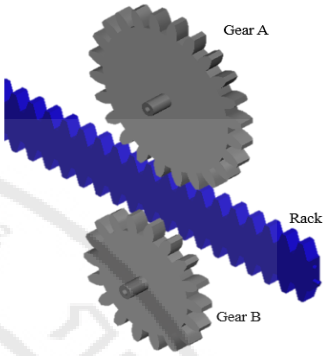

Figure 3.1 The gear system used to calculate LCM **Figure 3.2 The gear and rack**

3.1.2 Game design

The purpose of game design is entertainment, whereas the purpose of instructional design is education. Good game design requires much of players in terms of time, emotion, and problem-solving skills. Hall (1998) suggested some factors that engaged learning should cover some parts as the figure 7 shows. Sandwich factory board game was designed based on this outline and tried to cover these factors as many as possible. Howland (2002) was developed these factors into game design and specified some details. For game design, focused goals should contain narrative (Storyline), the difficulty of the mission and strategic hooks etc.

Engaged Learning

- Focused goals
- Challenging tasks
- Clear & compelling standards
- Protection from adverse consequences for initial failures
- Affirmation of performance
- Affiliation with others
- Novelty & variety
- Choice

Figure 3.3 The elements of engaged learning (Hall, 1998)

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The following illustrates the intent of the design of the significant elements that exist in the game, the function and the way to use these elements in the game shows in appendix A.

3.1.2.1 Board layout: The whole session of the sandwich factory game was designed containing 28 spaces of playing, each side of the board has the 7 spaces which represent 7 days (1 week) and the moving direction follow the clockwise.

3.1.2.2 Narrative/Storyline: Sandwich ingredients packed in a package with a certain amount, as a chef, students need to combine the daily special packages with the weekly special packages to make sandwiches.

3.1.2.3 Character: The different color of cubes represents the individual players.

3.1.2.4 Dice: Six-sided unbiased dice with numbers from one to six.

3.1.2.5 Weekly card: Each side (week) has the corresponding 5 cards. Totally, there are 20 weekly cards for the whole game (4x5=20).

3.1.2.6 Stamp: players pay for the sandwich then get the corresponding stamp (10 kinds of stamps, each kind has 5 cards). Collecting stamps can control student's property within a manageable range through making a balance of the payment and gathering. Collecting also can facilitate students to continue the game.

3.1.2.7 Patent card: Patent was designed to be hard to get but valuable in the game. Possessing a patent can make students feel accomplished which is significant to their confidence, and it also speeds up the process of one round.

3.1.2.8 Earning day: There are 2 earning days in 1 round, which can offer an opportunity to players to exchange the cards or patents for money. To make sure that each player has enough money to continue the game.

3.1.2.9 Challenge card: The significance of the challenge card is to help players paying attention to other players' outcome and to provide a chance to correct other players' mistakes.

3.1.2.10 Daily special card: There are five kinds of sandwich ingredients: Cheese, lettuce, tomato, egg, and beef. And all of these ingredients were packaged into a box as single slice form Figure 8. Sandwiches are required to contain two different ingredients only. There are five daily special cards of each week, so totally are 20 daily special cards. And every kind of ingredients will appear at least once a week.

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3.1.2.11 Weekly special card: Ingredients show in a weekly special card is same as the daily special card, but the number on the card is different, see *the table 1*. The reason for the numbers that design on the cards is that they are not hard for students to calculate. And the number shows in cards are prime number, composite number and multiple (the number shows in one card is the multiple of the number shows in another card). The purpose is letting students notice that there are different solutions to deal with different types of numbers.

Figure 3.4 A brief process of using two cards to make sandwiches

Table 3.1 All possible numbers on daily special card and weekly special card

1st Week	Daily special	6	10	12	18	24
	weekly special	3	4	6	12	18
2nd Week	Daily special	7	9	11	5	10
	weekly special	3	5	7	9	11
3rd Week	Daily special	12	8	16	4	5
	weekly special	3	4	6	8	10
4th Week	Daily special	9	14	16	18	6
	weekly special	4	6	8	9	12

As the table 1 shows us, there are all possibilities of combination of the two numbers. Researchers have divided these combinations into four categories in order to avoid questions being too random and to provide different types of combinations of numbers. Such as one number is a multiple of another number, the combination of prime number and composite number.

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3.2 How to play the game

3.2.1 Game process

The layout of Sandwich Factory board game shows in appendix B.

3.2.1.1 Before starting the game, students assign the sequence of playing depend on age; "*the youngest one from players will move first"*, then the first player rolls the dice and move his/her character as the dice shows. Then the player will get a daily special card first from the board where the character stands. The brief procedure shows in appendix C.

3.2.1.2 Secondly, player draws a card from the weekly special to get another package card. Then students will get two numbers from each card.

3.2.1.3 Thirdly, the player required to combine the daily special and weekly special (two numbers) to make sandwiches. But the player needs to follow the rule of making the sandwich; *"You can use two selected packages as many times as you want. But you need to use all ingredients in all packages with none left over. And each sandwich must be the same with 2 different ingredients."*

3.2.1.4 Finally, the player decides how many sandwiches to make and pay the money according to the menu. Then get the corresponding stamp. Players can use the stamps exchange for money or keep the stamps for future use, the details show in appendix D.

3.2.2 Challenge rule

Players who put the challenge card on the center of board firstly will become a challenger, the challenger only has 5 seconds to speak out his challenge number, and otherwise, he will lose his chance of the challenge. Other players can monitor the 5 seconds challenge time. All possibilities of the challenge results shown in figure 9.

3.2.2.1 If the purposed number meet the rule (The rule of making sandwich, the same below) and it's the least number. Then the answer given by the previous player is the best answer.

3.2.2.2 If the purposed number meet the rule but it's not the least number. Then the other players can challenge him and can give the best answer he though. He will challenge success if the challenge number is lower than the previous number, otherwise will fail.

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3.2.2.3 If the purposed number doesn't meet the rule, the challenger will success if his/her number meet the rule.

3.2.2.4 The challenge still could be possible even other players already got the best answer, because other players might do not realize that answer is the best answer. If this situation happened, the challenger could give a wrong answer.

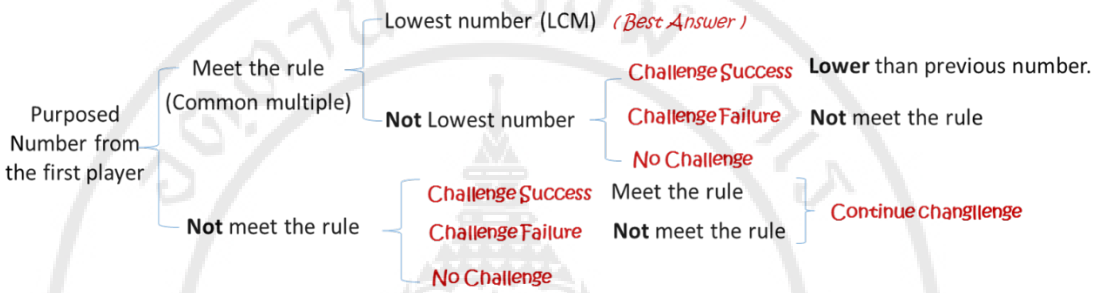

Figure 3.5 The situations of challenge

3.2.3 The way to introduce the gear to players

In this study, the researcher introduced the role of the manipulative gears to players after the introduction of playing. The researcher told the players that the gear is a tool which can determine the correctness of the answers given by the challenger when challenge happened in the game. Then researcher introduced this set of manipulative gears and the way to use them to players as the following example.

There are 14 gears with different number of teeth and 4 racks with the same length. Four racks can be connected together to form a double-sided long rack with 100 teeth. The number of teeth of the gear is the same as the number shows on the card (daily special card and weekly special card), which means each card has a corresponding gear. The demonstration of the gear occurred when the gear is needed for the first in the game. Thus, the first demonstration of each group used different gears. For example, as the figure 10 shows, the gear A has 10 teeth, which represents the weekly special package with 10 slices of tomatoes, and the gear B has 8 teeth representing the daily special package with 8 slices of cheese. The players need to use their hands to rotate two gears along the rack simultaneously. At the same time, the player needs to observe when the green point on each gear meet at the second time, the number of teeth passed on the rack represents the best answer (least common multiple) given by challenger. But students

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can continue to rotate the two gears along the rack until the green point meet again at the third or fourth time. The number of the third or fourth encounter may represent the answer given by the previous player.

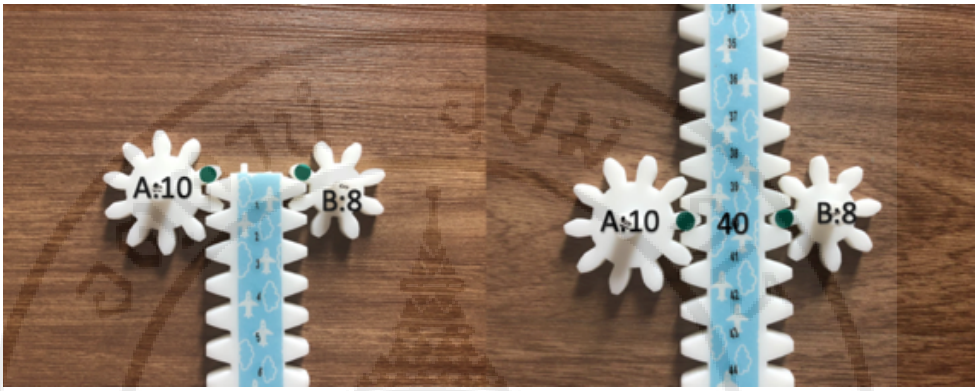

Figure 3.6 The demonstration of using gears

3.2.4 How to win a game.

The player who collects 2000 euro first will become a winner. If no winner come out after one hour, then all players will be required to exchange their cards for money and the player who possesses the most money wins.

3.2.5 The results of the challenge

Challenger will obtain the stamp if his/her challenge is successful. And the previous player needs to pay for him/her according to the right number. Challenger will be fined 100€ if challenge failed, and the previous player obtains this fine.

3.3 Research design

In this study, the research team focused on the learning process of primary students constructed LCM knowledge while playing the board game. Stake (1995) defined the case as a "specific, unique, bounded system in which researchers gather comprehensive, systematic, and in-depth information". This case study offered a rich description of playing process to figure out how learners construct their own interpretations and mental models.

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3.4 Participants

In this study, the aim is not generalizing, but explaining the playing and learning process of specific groups as comprehensively as possible, focusing on specific meanings and practices.

This study adopted the convenience sampling and purposive sampling. For convenience sampling, the school and class were selected because of their convenient accessibility and proximity to the researcher. And the main goal of purposive sampling is to focus on particular characteristics of a population that are of interest, which enable the researcher to answer the research questions.

Firstly, the location of this study was selected in a primary school (Xi'an Xinhua primary school). In the city of Xi'an, Shaanxi Province, China. The characteristics of participants in this study is 4^{th}-grade students who didn't learn the LCM in the formal class yet. To make sure that, the prior knowledge test includes one question to require students to find LCM directly, which is able to select participants who didn't learn this concept. According to Chinese mathematics curriculum, most students will learn the LCM at the beginning of the grade five. If the most of students have learned this concept already, the researcher will choose the fourth graders as the participants.

Secondly, forty-five students (a whole class students) from grade four was required to participant the prior knowledge test first (Appendix F). which is adapted from Sinclair, Zazkis, and Liljedahl (2003). The original test was developed for a computer game called *number worlds,* to measure student's knowledge about divisibility rules, prime and composite numbers, also the multiples and factors. The adapted test has 10 questions with different difficulty which be able to classify students into different groups, and 1 question requires the knowledge of LCM to be solved, which be able to select participants who didn't learn this concept before.

After the prior knowledge, researcher divided the student's grades into five intervals. And students who belongs to first, third, and fifth intervals are high achievers, middle achievers, and low achievers respectively. Group together in three, nine students was selected as the final participates based on their score and voluntary. There are two reasons for this grouping. The first reason is to ensure that the students with similar mathematics skill can play one round of game together. In this way, winning or losing the game will not be so obvious. Students will be more competitive while playing the

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game. Another reason for this grouping is to explore what are the differences among 3 groups of students during the learning activities. But Students was not informed the purpose of the prior knowledge test.

This research focuses on why they think as they do, and what the reasons are behind that thinking, thus participants should be more exploratory. Before implemented the game, all students took orientation session to familiarize them with the gaming environment and train to do think-aloud, then the researcher started to record the participants' learning process of the board game.

3.5 Data collection

This case study approach to qualitative analysis constitutes specific ways of collecting data.

3.5.1 Video recording

In this study, the whole process of playing was occurred in a small classroom due to the number of players, the cameras need to capture student's activities and conversations clearly. The duration of the videos is different from each group based on the game experience when they are playing, but each group is more than an hour.

Three groups of students (high achievers, middle achievers, and low achievers) were played the board game in 3 different days. Interview and questionnaire were proceeded immediately and individually after one group of students finish the game. And the teacher did the debriefing with all participants together.

Two cameras were adopted in this study, which can offer a relatively large capability and flexibility to capture complex field situations. Camera I was placed at the same level with students which can record the playing process in a wide-angle. Camera II will be placed in a position which is close to take a bird's eye view of the process of taking notes.

3.5.2 Interview

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The purpose of interview is to get students' thoughts as well as learning phases in detail while playing the game. In this study, interview was proceeded in a comfortable environment and individually, all 9 students who play the game participate

the interview also. If the answers from students are unclear, researcher will ask an extra open-ended question, such as “can you tell me more about why you think that?” to get more detail information. The interview was conducted based on 9 predefined questions which include six open-ended questions and three problems questions provided in appendix G, the problem questions come from the real-world situations. The interview can get more details about their thinking and to check whether students can adopt the knowledge gained from the game flexibly.

3.5.3 Questionnaire

Prosser and Burke (2011) have explained that *The Fun Sorter* is a tool used to compare a set of related technologies or products. In this research, the questionnaire was adapted based on a repertory grid and made up of 11 columns (11 is the number of items being compared), and 1 row (the most fun), the questionnaire shows in appendix G. The pre-prepared picture cards (Appendix I) representing the different elements of the board game was offered to participants before doing the questionnaire, and all representations of cards was explained clearly to each participants to make sure that they are fully understand the meaning of the cards, then students was required to sort the cards from the most fun to the least fun. The purpose of the questionnaire is to find out the student’s favorite elements of the game, thus providing a general direction for the upgrade of the game and the design of future games with the similar topic.

3.6 Data analysis

The way of analyzing the data is based on the data collected. The important raw data of this study is the video recording which records students’ activities while playing the LCM board game. There are variety of actions and artifacts was activated during the learning activities, such as words (orally or in written form); face expression (confused, confidence). The researcher used the data above to sum up the methods that students used in the game. There is also the date from the interviews and questionnaires.

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3.6.1 Coding

For the video recording, the researcher chose the most significant data and create categories by brining several codes together. Labeled categories and decided

which are the most relevant and how they are connected to each other. After reading the transcriptions of the video recording, the notes were marked the first impressions (label the relevant words, phrases, and sentences). Relevance depends on something which was repeated in several places or similar to the previously published reports. The researcher recorded what happened according to the timeline and see whether the methods that students used were changed as the game goes on.

The interview consists of two parts. The first part serves as a supplementary data to further explore how students construct the ways to find LCM. After interpreted the voice recording of the interview, the researcher searched for themes with broader patterns of meaning. then reviewing themes to make sure they fit the date. For the second part of interview, researcher can judge whether students can transfer the knowledge they have learned in the game to new situations based on their answers.

3.6.2 Sorting

The researcher can sort the elements directly that students ranked on the questionnaire to see which part of the game is their favorite.

3.7 Ethics considerations

Ethical concerns are highly concerned in this research. To ensure the confidential data, participant welfare, voluntary participation, minimized potential harm and discomfort to students' feeling throughout the study.

The formal letter was sent to the school principal where the research was conducted to ask for permission in doing research. The letter contains overview information of this research which describes research purpose, research objective, research methodology, time and duration of data collection, moreover name of research's advisor, researcher affiliation, and contact information was given. The implementation of this study will not occupy the formal class teaching time, which minimized the impact on students learning time. To analyze students' learning process, video recording was offered broad perspectives then allow the researcher to grasp the specific events while playing the game. Such as speech, gestures, inscriptions, which are necessary and significant to this qualitative research.

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For the benefit of students, debriefing was proceeded immediately after the data collecting. Because debriefing is significant for participants to consider what was learned. In this study, debriefing was proceeded as the following steps. Firstly, the researcher focused on having students discussing what happened in the game. Secondly, the facilitator encouraged students to explore how they performed in the game. Thirdly, the teacher summarized the efficient methods from the game and introduced the concept of least common multiple.

All potential participants and their guardians were received an assent form which shows the research title, research objective, research methodology, time and duration of data collection, possible risk and discomfort, participant right or choice during the research implementation. All documents were explained clearly to participants and their guardians to acknowledge and ask for their permissions. During the implementation, all participants have their own choice to quit this study at any time. Alias name and number code was adopted while analyzing process and final reporting. All data was kept in secure online drive only without secondary used and data will be destroyed immediately after publication.

3.8 Summary

Table 3.2 The summary of methodology

Research questions	Data collection	Data analysis
1.How do fourth-grade students construct LCM knowledge while playing sandwich factory board game?	Video recording; Interview (6 Open-ended questions; 3 problem questions)	Coding
2. What are students' perceptions toward the sandwich factory board game?	Questionnaire (Fun sorter)	Sorting

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CHAPTER IV
RESULTS

4.1 Results

The researcher summed up the eight methods that students used to get the LCM according to what they said in the game as well as students' scripts mostly. For example, when players get the two numbers in the game. They will speak out how to multiply the numbers directly if they didn't write down the calculations on the paper. So, students' mental calculations can be present by their words and captured by the video recordings. And students also write down their calculations on the scripts. Then the researcher can see the calculation process directly from their scripts and analyze the methods that students used in the game. There are some examples as the table 3 shows.

Table 4.1 The example of classifying the method

Question	Proof	Description
(10,3)	"just take 3 ingredients from 10"	Student didn't think to multiply the numbers, just take out the 3 ingredients from one package to make sure the equal. We can conclude that student used Method I.
(18,7)	Student blurt out the number 84 quickly, then divide the 84 by 18 and 7 with oral calcuation.	Student found that 84 divided by 7 is 12, but 84 cannot be divisible by 18. Then he noticed that he made the mistakes. And we can judge student used method II
(5,8)	"3 multiply 5 is 15, 2 multiply 8 is 16"	Student speak how to multiply the numbers directly from the multiplication table, then stopped. So, the researcher can conclude that the student used method III.

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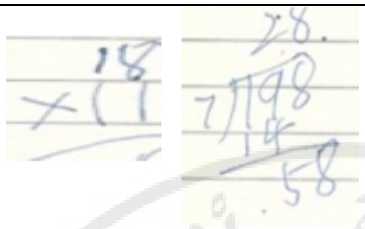
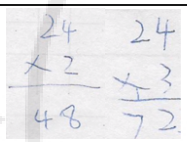
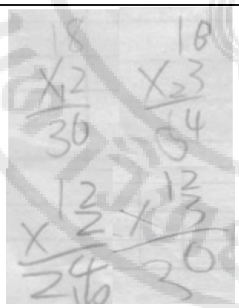
Table 4.1 The example of classifying the method (cont.)

Question	Proof	Description
(18,7)		Student used 18 multiply the number 11 (The random multiplier is bigger than another number) and see whether the product can be divided by 7. So, the researcher can judge this student used method IV.
(12,5)		The method V is just multiplying the two number directly. So, student can blurt out the calculation directly, or just write down the calculation on the paper.
(24,18)		we can see that students tried to calculate the multiples of 24 (start from small multipliers, such as 2 and 3. Then divide 72 by 18 to see whether it can be divisible by 18. Then we can conclude that this student used method VI.
(18,12)		We can see that student calculated the multiples of number 18 and number 12 at the same time, then he found the LCM is 36. So, the researcher can classify that the student used method VII.

4.1.1 Description of Eight methods

The researcher found that no student used all summarized methods while playing the game. Some methods were adopted because of various certain conditions, after students passed that stage, they may not use that methods anymore. So, students will not use Method I to Method VIII strictly in turn. In general, the researcher summarized the following eight methods.

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Table 4.2 Explanations of eight methods that students used in the game

Method	Description
Method I (Rule understanding phase)	Students calculate multiples of smaller number from two selected numbers, then try to find a product which is close to another bigger number.
Method II (Trial and error)	Students blurted out a big number randomly and see whether this big number is a multiple of two numbers they picked from game. (They will use the basic division method to examine whether the numbers given can be divisible by the 2 numbers picked from game)
Method III (Trial and error)	Students calculate the multiples of one number with a random multiplier to get a product. Then try to calculate the multiplies of another number to get a same product.
Method IV (Trying hard phase)	Students calculate the multiplies of one number from two picked numbers. Student will see whether the products are obviously multiples/can be divisible by another number. (But when students calculate the multiples, the multipliers are random. they blurted out a multiplier which could be greater* or less than the two numbers picked, no any pattern)
Method V (Turning point)	Multiply the two numbers selected from the game to get a common multiple.
Method VI (Sorting out phase)	Students calculate the multiplies of one number from two picked numbers. Student will see whether the products are obviously multiples/can be divisible by another number. (1. When students calculate the multiples, the multiplier is less than the two numbers selected from game. 2.The multiplier is ascending from small number.
Method VII (Method generation)	Students calculate the multiplies of two picked number at the same time, and the multiplier starts from the smaller number. (But the multiplier does not have to start from the number one or two, because such a small multiplier is obviously cannot get a common multiple.)
Method VIII (Extra method)	Students give the answers directly without any manuscript or oral voice recording. (This situation happened due to some reasons: 1. Students understand the rule of game. 2.Some numbers are too easy to calculate 3. students' proficiency of the calculation enhanced.)

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**If greater, which means students didn't realized multiplying the two numbers directly will get a common multiple.*

4.1.2 The Relationship of Eight methods

These eights methods are not strictly progressive in terms of method effectiveness from method I to method VIII. As the table 5 shows, Method I to Method IV can be seen as a development. The criteria basically depend on how many numbers that students considered when they are finding the LCM, and how efficient to find the LCM.

Table 4.3 Development and criteria of inefficient method

Method development	The numbers that students considered	Calculation method	Efficiency of finding LCM
Method I (Rule understanding phase)	One number	No calculation	No possibility
Method II (Trial and error)	Two numbers	Guess the CM	Ultra-low possibility
Method III (Trial and error)	Two numbers	Calculate the CM	low possibility
Method IV (Trying hard phase)	Two numbers	Calculate the CM	Takes long time

The method I is the phases that students trying to understand the rule of game. Students considered the only one number of two numbers and cannot find the LCM with this method. The method II and method III are the ways that students do the trial and error, using these two methods, student may get the correct answer with extremely low possibility. The Method IV is more efficient than pervious method but still takes time to reach the correct answer. But the Method IV illustrates students are trying to generate an effective method to get the answer from the game. In this study, all players cannot find the LCM with these four methods while playing the game.

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Table 4.4 Effective methods

Method	Can students find the LCM
Method V (Turning point)	Yes
Method VI (Sorting out phase)	Yes
Method VII (Method generation)	Yes
Method VIII (Extra method)	Yes

The researcher didn't claim that the Method V to Method VIII is a development. Because all of these four methods can help student find the LCM efficiently and correctly. Some methods were used because of the numbers themselves in the questions. So, the researcher regarded these four methods as the effective methods, if students used these four methods, we can conclude that students can figure out a method and be able to find the LCM during the game.

The Method V can be seen as a turning point, because students realized "common multiple" of two numbers. students use this method because they have realized how to get a common multiple.

The method VI and method VII show students are trying to find a least common multiple, due to the rule of game, students think "common multiple" is not "cost-effective". The method VII is called "Number list" in math curriculum, which is a method that student used when learning LCM at the beginning. by using this way, students can get the LCM step by step correctly but not efficiently. So, in most cases, students will skip smaller multiples when using this method. But students didn't write in the formal way of "number list", they write many multiplication equations on the manuscript.

The researcher seen the method VIII as an extra method, on occasion, the LCM is quite obvious to find after the players understand the rule of game, the researcher cannot clearly distinguish which method that students adopted, because students give the LCM quickly without any manuscript or evidence that can be analyzed. So, the researcher needs to classify the method according to the previous method that student used and afterwards also. By doing so, if we still cannot distinguish the method

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that student used, the researcher considered that students use the Method VIII, but we didn't claim that the method VIII is an advanced method than others. The truth is students can get the correct answers quickly, which also means students comprehensive the rules of game and be able to find the LCM in the game.

4.1.3 Summarize

In this research, the researcher regards the Method V as a turning point, if students using the Method V and afterwards, we can draw the conclusion that students have their own methods to get the least common multiple. And we can see how students develop their methods from the figures of method development clearly with different questions. The researcher will judge which methods that students used according to their speaking, manuscripts, and conversations with their peers mostly.

In this game, students were told that they need to compete to each other, and they are allowed to challenge other players to get more coins if he or she has a better answer. So, in each turn, students also pay attention when other players are playing and calculating. But they still talked and negotiated the result with other players of the most time, which made the players influenced by others' thoughts and ideas. So, the researcher will do the individual data report of each students, but also show the interactions among players in each group if it happened.

4.2 Result of Group I (High achievers)

The researcher described the representative cases of each students to show their method development individually. From the videos, we can see clearly whether students pay attention on the questions or not. So, if students focused on that questions and had the activities, the researcher will report it even it' s not his/her turn. Students from Group I are top achievers. They are good at calculation compared to other groups.

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4.2.1 Video interpretation of student A in group I

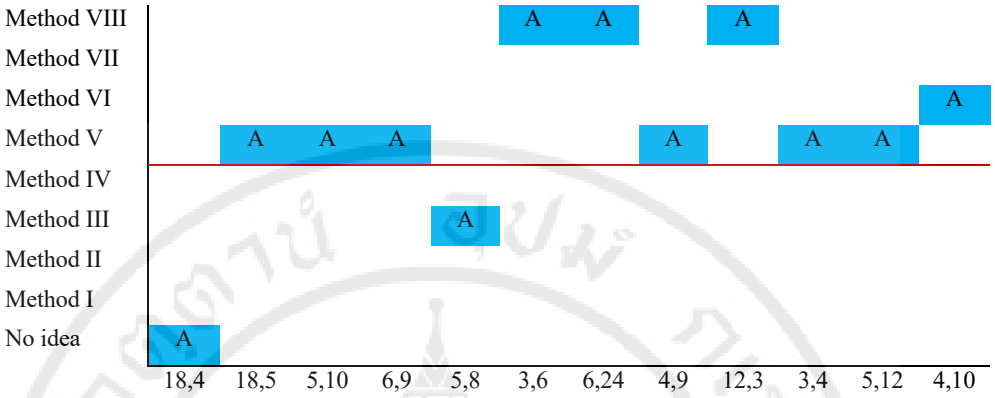

Figure 4.1 Method development of student A in group I

Table 4.5 The report of method development of student A (Group I)

Question	Report
(18,4) (15,5) (5,10) (6,9)	Student A also has no ideas at the beginning. But after she observed other players performance and has more understandings of the rule, she noticed the Method V is a way to get the correct answer. So, she used this method V few times even with the number 5 and 10. But other players showed her a way to find a smaller number. And at the second time, when she met the number 6 and 9, she asks other players directly: "How you get the number 18". And student explained "you need to find a smaller number".
(5,8)	From the videos we can see that student A used the method III to find a smaller answer after she has been challenged by others with the previous questions. So, she used 2 multiply by 8 and get the 16, used 3 multiply by 5 to get the 15, then she stopped. At least she has the awareness of finding a smaller answer now. So, this progress can be seen as a development of student A.
(3,6) (6,24) (4,9) (12,3) (3,4) (5,12)	After got more new experience from the game, students have more understanding of the game, she might have the sense of numbers. She realized which numbers need to be simply multiply together, and which number is the multiple of another number. Then she can give the answer quickly and directly.

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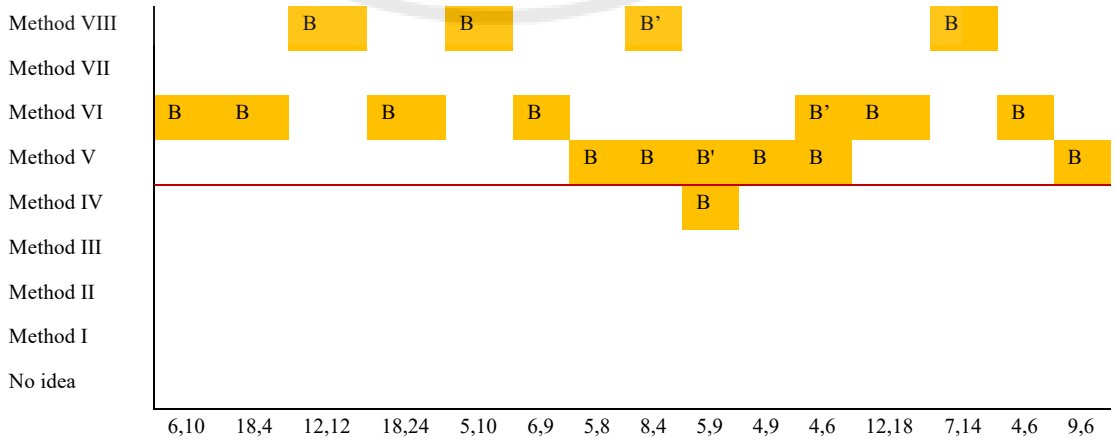
Table 4.5 The report of method development of student A (Group I) (cont.)

Question	Report
(4,10)	For the last question, student A used method VI, we can conclude that student A generates a way to find LCM while playing the game eventually. The reasons that student used this method are the numbers themselves and the development of his thinking.

Student A struggle a lot at the beginning, because there are many questions that she cannot give any answers, she observed and asked other players how they did it. But other players also cannot give a clear explanation to her.

Student A also gave the answers at the beginning, But she just simply multiple the two numbers together. So, student A felt and watched a lot from other player's results than herself attempts. Then student A is able to look the results from different perspectives. With more reflective observations, she had wide-ranging information and organized in a clear logical format. As the figure 11 shows us, student A gave the answers quickly at the second half of game. Researcher considered this period as the stage in which student A is gradually mastering the way to find the LCM. because she has ability to find the LCM quickly with the different type of numbers.

4.2.2 Video interpretation of student B in group I



Note: B prime (B') indicates that the student B has used a new method in the same question.

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Figure 4.2 Method development of student B in group I

Table 4.6 The report of method development of student B (Group I)

Question	Report
(6,10) (18,4)	Student B used method VI at the beginning after she understood the rule of game, she tried to find the multiplier of the bigger number from two selected numbers and see whether the products are obviously the multiples of another number. Student B is good at the calculation and she didn't use the method which below the red line too much during the whole game.
(12,12) (5,10) (8,4) (7,14)	These pairs of numbers are obvious to find the multiple relationship for her. So, student B gives the answers directly. But for the numbers 8 and 4, she multiplies the two numbers first, after that she realized the mistake immediately and gives the correct answer.
(18,24)	Student B also tired the method VI few times to find the answers, but she stopped and didn't find the answers eventually. The researcher thinks this pair of numbers is quite difficult to get the answer due to the difficulty of calculation. Because these two numbers both are two-digit numbers, if student used method VI and calculate the multiples of one number, it's hard to see the relationship of the product with another number. And the calculation takes time.
(5,9)	Student B gives the answer "90" directly, she found a common multiple and blurt out to others, but other students noticed the mistakes and given a smaller answer, then student B also realized the mistakes. So, the researcher found that students also went back the lower method even they know some advanced methods, but they didn't go back and used the lower method frequently, they made mistakes, but they will be realized it immediately.

We can see that student B is a type of player who think and do it. She didn't hesitate with the numbers at the beginning. The figure shows that she used method VI since the beginning of game, she solved problems and used her learning to find the solutions. and she was less concerned with other players during the game. Thus, we can conclude that student B can find the LCM while playing the game from her performance. because she found the LCM at the beginning with her ideas. She also experimented with her new ideas when she got the different numbers, and she also made the mistakes of calculation because of the careless.

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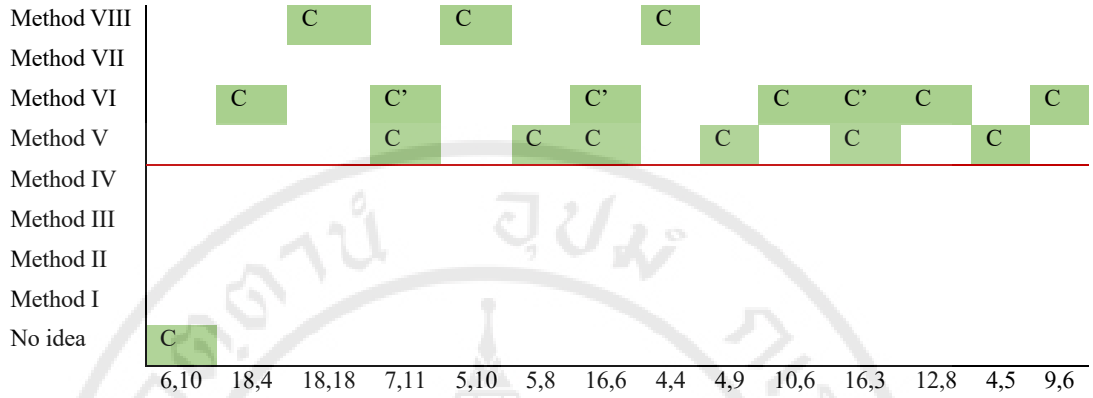
4.2.3 Video interpretation of student C in group I



Note: C prime (C') indicates that the student C has used a new method in the same question.

Figure 4.3 Method development of student C in group I

Table 4.7 The report of method development of student C (Group I)

Question	Report
(6,10)	Student C observed how the student B did it.
(18,18) (5,10) (4,4)	Only these three questions student C gives the answers quickly without any activities, the rest of other questions, student C takes time to think.
(7,11) (16,6) (16,3)	Student C tried to calculate a smaller number after she multiply the numbers together. such as number 16 and 3, she multiplies the numbers first and didn't report that answer, she took few seconds to find a smaller number.
(4,9) (12,8)	Student C takes long time to think about the number 4 and 9, But eventually she given the correct answer. The researcher think she already know the method V, she is trying to find a smaller number which takes her time to give the answer, but she didn't have any activities which can show the way of her thinking.

Student C watched others when she has no ideas at the beginning with the first question. So, she observes how other students did. After that, she always used methods which above the red line.

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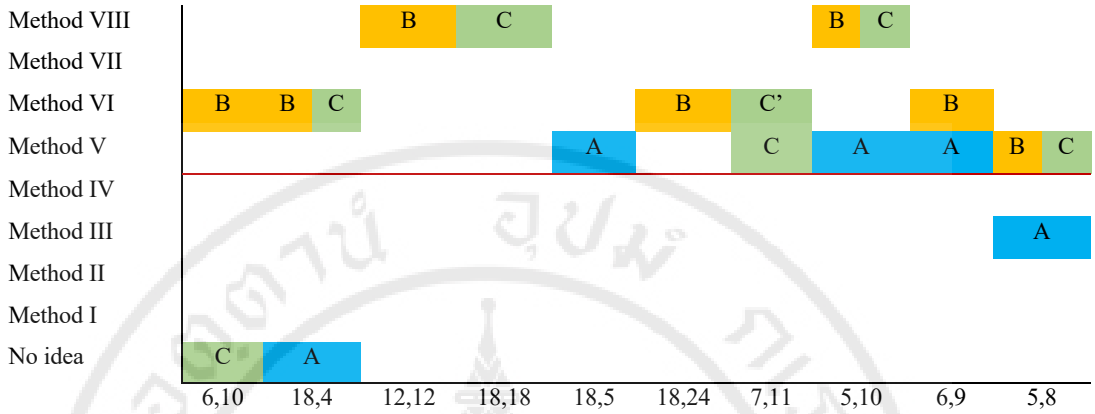
4.2.4 Group interpretation



Note: C prime (C') indicates that the student C has used a new method in the same question.

Figure 4.4 Partial method development of group I

All students in group A are focused on their own problems more than others' problems compared to other groups. If others' problems can be easily calculated mentally, they may have a try. They would ask directly if they don't understand the question or how other players get the correct answers. For the question (6,10), we can see that student C had no ideas about the answer or how to approach the answers. But student B show her ideas with method VI. Thus, student C used the same method with the question (18,4). And for the question (12,12) and question (18,18), student B and student C used the same method also, and student A also observed how they find the answers during this time. So, after few attempts of playing with different methods, students in group A could build a "common knowledge".

The common knowledge was accumulated from the previous game experience. So, student will not make the same mistakes that their peers made before. And due to the different views, students also can get the different reflective observations and new methods from this common knowledge. So, student A used method V with question (18,5) when it is second time of his turn. And we can see that for the easy question (5,10), all players are willing to have a try. But student A seems still didn't find a way to get the LCM, she only multiplies the two numbers with the question (5,10) and question (6,9). But she asked student B how you get the correct answer at question (6,9).

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Student explained “you need to find a smaller number” and showed her how she calculates. So, student A can learn from the players who used the efficient method to find the LCM. Even she went back to use the inefficient method with the question (5,8). But after this question, she always can find the LCM with the efficient method. So, after student A received different reflections, she can conceptualize the abstract content of the game. Then she started the active experimentation with different kinds of questions. But with new questions or situations happened, students can accumulate the new reflections and come out the new methods again. Student B and student C are more active then student A, they have more calculations compared to student A when there is a question come out to the group. So, they can get more reflections when from their own trial and error.

4.2.5 Interview results of group I

Table 4.8 The interview report of group I

Number	Question & Answer
	6 Open-end questions:
(1)	How you make the sandwiches with the daily special package and weekly special packages?
	A1: “Calculate the smallest number”. B1: “Just follow the rule to make sandwiches”. C1: “There are two packages, make it (number) same. Then multiple that number by. 2, then I got the money that I need to pay.”
(2)	What factors seems to affect you to win the game?
	A1: “Maybe I am out of luck” B1: (Silent) C1: “I think I am out of luck, because I pay the money for my peers”. (she needs to pay the money for the owner who has the patent card)
(3)	What do you think is making it hard for you to play this board game?
	A1: “No difficult”. B1: “Oral calculation is difficulty”. C1: “I think the beginning is hard, because I don’t know how to play, and the calculation is hard”.

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Table 4.8 The interview report of group I (cont.)

Number	Question & Answer			
(4)	Do the manipulative gears can help your performance well at next round of the game? Why?			
	A1: “I think the gears can help me to do better next time.” B1: “I think the gears did helped me”. (but don’t know why) C1: “I think the gears can help me”. (without any explanation)			
(5)	What would you have to do to become wealthy in the game? If that, what tricks you used?			
	A1: “Let me think. I think I need to make sandwiches as less as possible, then I will pay a little bit”. B1: “My money comes from the exchange day, I think exchange day makes me wealthy”. C1: “Exchanging day, when near this day, I need to consider my money which is still sufficient or not, if not, I need to use my stamps to exchange the money, otherwise the stamps cannot exchange so much money”			
(6)	Could you try to tell me the definition for the word “Least common multiple” from the experience gained from the game?			
	A1: (Silent) B1: “Don’t know”. C1: “Don’t know”.			
	3 Problem questions*	A1	B1	C1
(7)	12,9 (hotdog)	Correct	Correct	Wrong
(8)	14,8 (gear)	Correct	Wrong	Correct
(9)	40,60,80 (time)	Correct	Correct	Correct

**Please see the Appendix F of the whole 3 problem questions, the table only shows the numbers that need to be calculated in the question.*

The interview was conducted individually after playing the game, and there are 6 open-ended questions and 3 real world LCM questions. All students completed the interview, but the truth is students cannot explain their thoughts clearly during the interview, even the researcher asked many sub questions from each main question. So, the researcher tried to sum up the fragmentary answers with their original sentences from students to report the data of interview.

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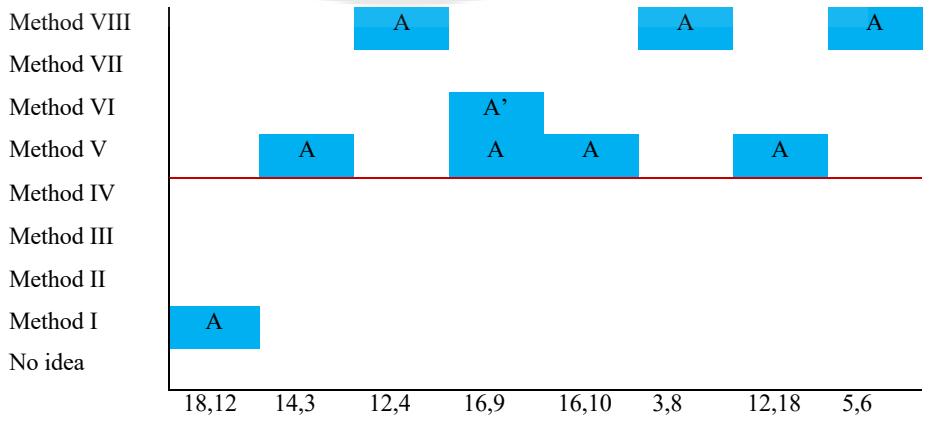
We can see that students talked about the game itself more than the calculation and numbers. we can conclude that all students from group I are mastered the rules of the game from question 1 and their behaviors in the game. But students considering the “luck” is a significant factor which effected their achievements of the game. The researcher believes that what students talked about luck is the difficulty of the problems they got, the value of the numbers, the acquisition of patent cards, and so on. And the interview also shows that students don’t have too much understanding about gears, they cannot explain clearly about the gears. Researcher thinks this might due to the times of using gears.

And from three problem questions, we can see that students can answer the most questions correctly, all of them answer the questions 9 correctly, which means they have the ability to deal with the 3 numbers LCM questions.

4.3 Results of Group II (Middle achievers)

Students from Group B are middle achievers, the researcher selected representative cases which can present the method developments of each students in group B.

4.3.1 Video interpretation of student A in group II



Note: A prime (A’) indicates that the student A has used a new method in the same question.

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Figure 4.5 Method development of student A in group II

Table 4.9 The report of method development of student A (Group II)

Question	Report
(18,12)	Student A want to use only 1 time of each package, then there will be ingredients left. So, it's not obeyed the rule of game. The researcher think student A want to pay less money by making less sandwiches at the beginning, but then he found the left ingredients is not fit the requirement of the rules.
(14,3)	Student A is quite silence and like to observe other players' activities. After few rounds of game, he noticed multiplying the two numbers will get a correct answer at least, even the answer might not be the best solution (LCM), but the method can support him to continue playing the game.
(12,4) (3,8) (5,6)	The easy question (numbers are small), student A multiplies the two numbers together, and for the numbers like 12 and 4, it's also easy for him because 12 is the multiple of 4, which is obviously for student.
(16,10) (12,18)	Student A always prefers to multiply the numbers together without further calculation. But he saw another student's calculation, which pay less money than him
(16,9)	For this pair of number, student A used the Method VI and tried to find a small number, but he failed to find a small one.

We can see that student A didn't have too many activities that can show us, he likes to observe how others deal with the numbers. He used method I at the beginning, which means he is at the phase of understanding the rule. After he understood the rules of game. He prefers to use method V to get a common multiple. But from the performance and interactions among three students. We can see that student A know the method VI, but he didn't used that method frequently. He only used method VI once.

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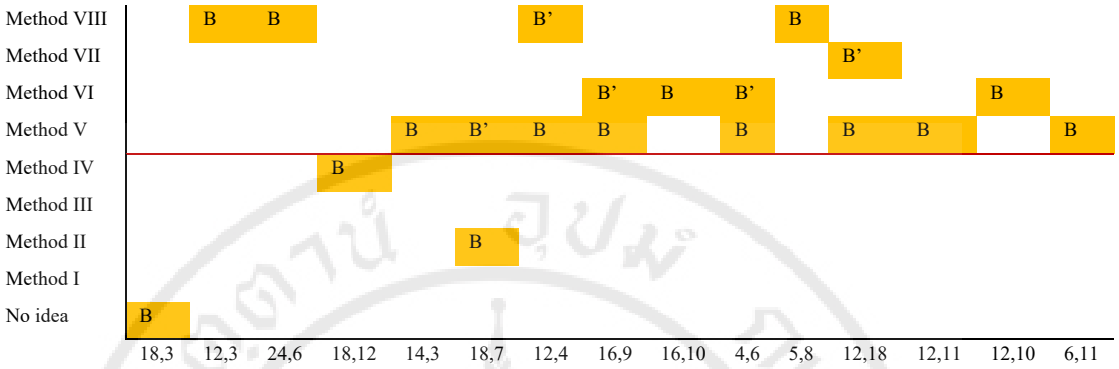
4.3.2 Video interpretation of student B in group II



Note: B prime (B') indicates that the student B has used a new method in the same question.

Figure 4.6 Method development of student B in group II

Table 4.10 The report of method development of student B (Group II)

Question	Report
(18,3)	Student B has no idea what he needs to do at the beginning, he didn't have any activities at beginning, so the research explains the rule again and give a clue to students.
(12,3) (24,6)	For the easy questions, student B gives the answers directly, he gives the answers because of the numbers themselves. The researcher conclude that even students used method VIII to get the correct answers, but he still not generates an efficient way to find the LCM yet. He still under the phase of understanding the rule of game. As the figure shows us, student B still struggle with the questions in the next few rounds.
(18,12)	Students B seems has the answers already from his excited and confident expression. But other players didn't show any calculations, so students B didn't give the answers eventually, but the researcher adopts the gears the examine the answer for all of them, and students might have some ideas from the gears.

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Table 4.10 The report of method development of student B (Group II) (cont.)

Question	Report
(18,7)	Student B blurt out a number 84 firstly, then he tries to use the numbers 84 divide by 7, and he got the answer 12, then he divides 84 by 18, then he realized the mistakes. And he changes to the method V to get the answer. The researcher analyzes that student B blurt out a number first because he wants to get a common multiple of two numbers, he might estimate an approximate common multiple mentally without any calculation. But he will realize that the estimating is not worked and won't use it anymore.
(12,4)	After student A used method V, he is so excited to multiply the two numbers together if the two numbers are not too hard to multiply mentally. And he is excited to yelling the answers to us. but after a few seconds, he noticed the 12 is the multiple of 4 and changed the answer.
(16,9)	Student A multiplies the two numbers directly, then uses the method VI to do the further calculation. because he starts to concern whether the two numbers have a smaller common multiple or not. So, the researcher can conclude that student A didn't realized LCM of the prime numbers.
(16,10) (12,10)	Student B calculates the multiples of number 16 firstly, he plans to get a two-digit number which the ones digit is zero. We can see this phase as student B mastered this method VI. And student used the same method with the number 12 and 10.
(4,6)	As we mentioned before, if the two numbers are too easy to do the mental calculation, student prefers to multiply the two numbers first, then they might start to observe the two numbers and try the method VI.
(12,18)	This is the second time that student met this question (number 12 and 18), three players they all multiply the 18 by 12 first, after they got a big product. all of them trying to find a small number, and students B used the method VII to get the answers.

We can see how student B develop his method step by step, at the beginning, student B is quite struggle with the rules of game, he tried the different methods at the first few times. The figure shows us that student B used method IV once then go back to the method II, the researcher analyze that student used method II because he don't want to calculate, he just guessed a number which might be the common multiple, even the guess is according to his prior knowledge or the mental estimate, the researcher considers the guess is a kind of low level method, but student B only went back and

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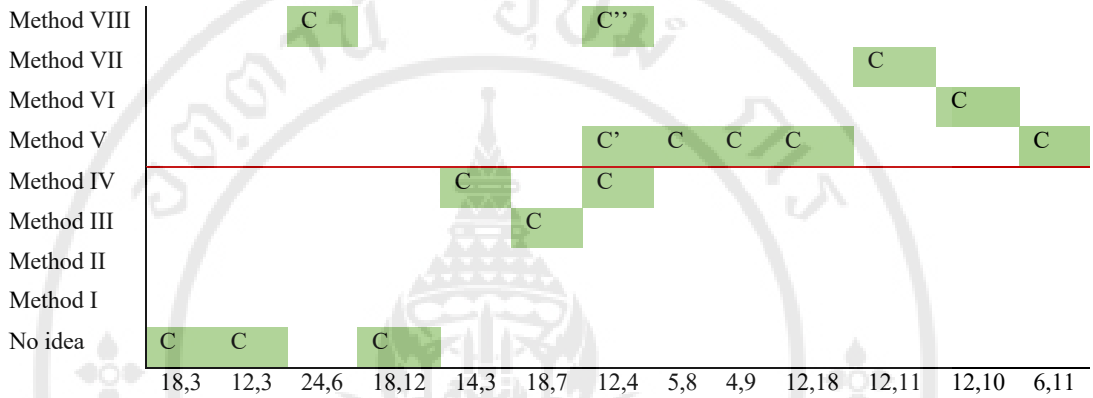
used this method once. After that, student B used the advanced methods all the time. So, we can conclude that student learned how to find the LCM in the game, and we can see how student developed his method from the figure.

4.3.3 Video interpretation of student C in group II



Note: C prime (C') indicates that the student C has used a new method in the same question, and C'' means student C has changed method 2 times from the first method.

Figure 4.7 Method development of student C in group II

Table 4.11 The report of method development of student C (Group II)

Question	Report
(18,3) (12,3)	Student C has no ideas at the beginning, he takes long time to think, even the numbers are not difficult. But other players show him how to deal with these numbers when he didn't have any responds. Student C did not know how to deal with the numbers and struggle with the calculation at first, because he didn't pay too much attention on the rules,
(24,6) (18,12)	Student can use the method VIII to find the LCM because he learned from others and the numbers themselves. It's obvious for him to find the 24 is the multiple of 6. But when he met the number 18 and 12, he still doesn't know how to deal with the numbers which is non-multiple relations. We can conclude that student C still has no ideas about how to find the LCM at this phase, even he got the correct answer with the question: number 24 and 6. He just mechanically copied the methods of others, that's why he didn't know how to deal with the number 12 and 18.

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Table 4.11 The report of method development of student C (Group II) (cont.)

Question	Report
(14,3)	Student C used method IV, he used number 14 multiply by 2, and see whether the number 28 is divisible by 3, then he stopped. but student B gives him a correct example with method V.
(12,4)	We can see how student C develop him method with this question, he multiplies the 12 by 7 firstly, then he got the number 84, after that, he used method V to multiply the numbers together to get 48. Then he suddenly realized 12 is the multiple of 4, then given the answer directly. We can conclude that student C know how to get a common multiple at least at this phase.
(5,8) (4,9)	We can see that student C already master the way to find a common multiple from these two pairs of number. Student C gives the answer quickly and correctly.
(12,18)	After student C multiplies the two numbers, he tried to find a smaller number quietly, there is no writing or speaking can be captured to analyze.
(12,11) (12,10)	Student used method VII to calculate the multiples with smaller multiplier at first, then he tried the bigger multiplier. but he skipped some multipliers until he used 12 multiply by 11. And student C found the LCM of number 12 and 10 with method VI. We can see that student C used his method and got the LCM eventually at the end of game.

Student C is not confident during the games, and he shows more interests in game itself, rather than the calculating the numbers. So, he didn't calculate too much during the game, especially the first half of game, guess is more than calculation, and most questions are too easy for him to get the correct answer with less calculation. But at the second half of game, the numbers are not obvious, and he has to calculate to get a smaller number to pay less money. So, the rule of game motivates him to find the LCM and he did it. we can conclude that student C can find the LCM in the game.

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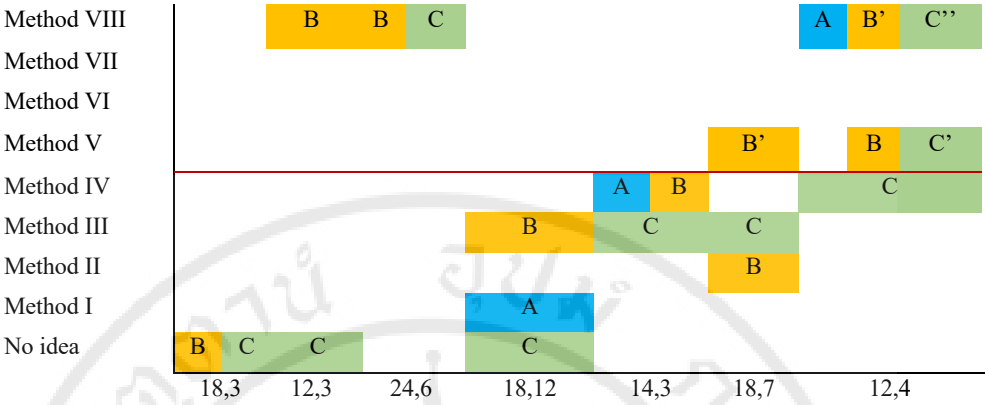
4.3.4 Group interpretation



Note: B prime (B') indicates that the student B has used a new method in the same question, and C'' means student C has changed method 2 times from the first method.

Figure 4.8 Partial method development of group II

When student B got his first question (18,3), Student B and student C are both immersed themselves in thinking. But they didn't figure out a way to solve the question at first. For the question (12,3), student C still had no ideas even it's his turn. But students B speak out how he calculates the two numbers directly and demonstrate the way of playing game. So, the researcher can analyze the method that he used. And for other players. they also can notice the way that student B used. Thus, students can accumulate the reflective observations with different questions.

Then student C adopted the same method that student B used when he met a same type question (24,6). But when they met a difficult question (18,12), student C had no ideas again. But student A and student B tried the different methods, which brought the different consequences. After this, we can see that student C used method III with question (14,3) and question (18,7), the researcher concluded that student C used this method because he learned from both student B and game. With more reflective observations, student C were able to generate the different methods. And for the questions (12,4), we can see that student C developed his method two times until he got the best solution, which is the stage of active experimentations.

We can see that student B is a player who focused on his own attempts more than others. he just feels and do it from his understanding. So, he can learn from his experimentations and develop his methods. Student A is a quiet player, he only

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calculates when it is his turn, but he also learned from the group common knowledge and develop his method.

4.3.5 Interview results of group II

Table 4.12 The interview results of group II

Number	Question & Answer
	6 Open-end questions:
(1)	How you make the sandwiches with the daily special package and weekly special packages?
	A2: “Multiply the numbers from the cards”. B2: “Use the cards”. C2: “I need to see what kind of sandwich I should make first, then I used the two ingredients(numbers), then calculate the numbers”
(2)	What factors seems to affect you to win the game?
	A2: “Calculation” B2: “Calculation” C2: “Money, if I didn’t calculate carefully, I will lose many coins”.
(3)	What do you think is making it hard for you to play this board game?
	A2: “When I nearly go broke”. B2: “Gold is difficulty”. (he means calculate the money about the payment) C2: “When I spend money, I need to calculate the money to need to pay”.
(4)	Do the manipulative gears can help your performance well at next round of the game? Why?
	A2: “I used two times of gear”. (Student cannot explain) B2: “No” C2: “No”.
(5)	What would you have to do to become wealthy in the game? If that, what tricks you used?
	A2: “Calculation, if I calculated a big number, I need to pay a lot, if I calculated a small number, then I pay less”. B2: “Using the stamps to exchange the money”. C2: “Patent card can bring me more coins in the future”.

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Table 4.12 The interview results of group II (cont.)

Number	Question & Answer			
(6)	Could you try to tell me the definition for the word "Least common multiple" from the experience gained from the game?			
	A2: No idea. B2: Cannot say it clearly. C2: Cannot say it clearly.			
	3 Problem questions*	A2	B2	C2
(7)	12,9 (hotdog)	Correct	Correct	Correct
(8)	14,8 (gear)	Wrong	Wrong	Wrong
(9)	40,60,80 (time)	Correct	Correct	Correct

**Please see the Appendix F of the whole 3 problem questions, the table only shows the numbers that need to be calculated in the question.*

In group II, all students considered the calculation as the key factor which affect them to win the game, they also think the calculation is the difficulty part of the game. Because they all have the problem about calculation while playing the game. For the manipulative gears, students in group II also think it doesn't help them and cannot explain the mechanism behind the gears. So, students cannot fix the LCM problems which related to the gear, but they can answer the LCM questions with other situations. and they all answer the question 9 correctly.

4.4 Results of Group III (Low achievers)

The researcher offers a figure below which shows the development of method that students used in the game.

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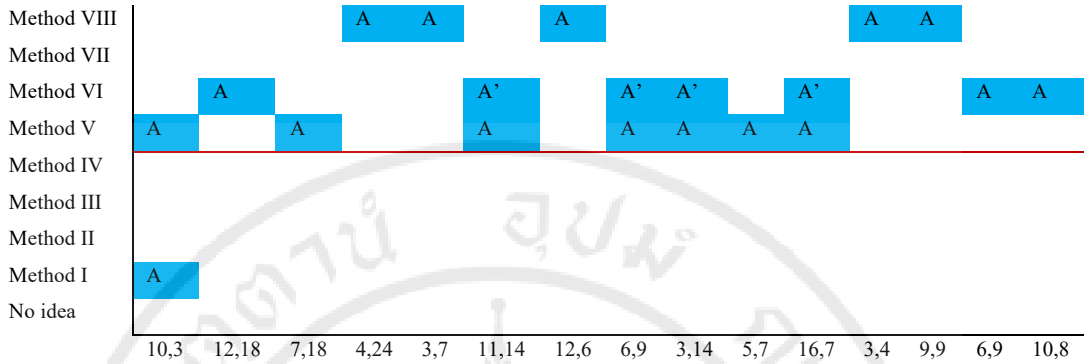
4.4.1 Video interpretation of student A in group III



Note: A prime (A') indicates that the student B has used a new method in the same question,

Figure 4.9 Method development of student A in group III

Table 4.13 The report of method development of student A (Group III)

Number	Report
(10,3)	At the beginning, student A are trying to understand the rule of game. For example, student A trying to use a whole one package which has 3 ingredients and another 3 ingredients from a 10 ingredient-package. then he can make 3 sandwiches. But student C remind him "you cannot waste the left ingredients (no ingredients left in one package)". So, they use the few seconds to think how to deal with it, and the researcher remind them the packages is unlimited when they selected these two cards from the game. After students realized the rule of "no waste" and "unlimited packages", they will try to find the "equal (common)" of two numbers. Then student A said: "use 3 packages with 10 ingredients and 10 packages with 3 ingredients". The researcher found that students may change their methods in one question with the development of understanding. when the answer they given has the conflict to the rule of game, they have to adjust their original method to fit the rule first. And the player will observe and learn from other player's trial and error. Then he might not make this mistake again.

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Table 4.13 The report of method development of student A (Group III) (cont.)

Question	Report
(12,18)	For the question "12 and 18". Student B seems has no clue about this question and be quiet, but student C said "18 multiply by 4" firstly, he only said this, and he seems going to get the common multiple with a" bigger" multiplier, but after the researcher ask them: "can they find a smaller number or not?" Then student A multiped the numbers from the smaller multiplier and then he finds a way which is "18 multiply 2, and 12 multiply 3, the product both are 36. then he got the LCM. Student A already used Method VI after two turns, he multiplied the numbers start from the small multipliers, then he noticed that smaller multiplier can help him to find the LCM. Since then he never goes back and use the low-level method.
(7,18)	All students are focused on this question, and student C calculated on the paper. He trying to find a smaller number which can make him to pay less. He started to calculate the multiples of two numbers with the smaller multipliers like 2, 3, and 4… but when he cannot get the answer. he tried to multiply with the bigger multipliers. For example, he tried multipliers 9 and 10. Student C didn't realize the way to get the "common multiple" which is just multiply two number together. that's why he tries the big multipliers (Method IV). But student A said: "just multiply the two numbers." He noticed the method of multiplying the two numbers directly.
(4,24) (3,7) (12,16) (3,4) (9,9)	Sometimes the answers are quite obvious for students because of the two numbers. For example, when a number is obviously a multiple of another number, or the two numbers are smaller. students will get the correct answers quickly; the researcher cannot distinguish which method they used without any evidence. But we still can see this situation as an extra method that student used in the game to find the LCM. Because the students still follow the rules and give the final answer.
(11,14)	Student A and C got the answer of 11*14 quickly, but the answer they got is quite big and they need to pay a lot of money, so they start to calculate on paper to find a smaller number. student A said each package only used twice, otherwise he will lose a lot of coins. But student C remind him that the rule of "no waste" then A use the 11*14 to get the LCM. Students may have the feeling about prime numbers when finding the LCM, but they always prefer to find a smaller number after they multiply the two numbers, even one of the numbers are prime number, which means they didn't realize the pattern when finding the LCM of prime numbers.

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Table 4.13 The report of method development of student A (Group III) (cont.)

Question	Report
(6,9)	Student A multiply the two numbers directly (Method V), and all of them noticed the common multiple of these two numbers, but student C are trying to find a smaller number using method VI, so both of them are trying to find a smaller number. Student C said there don't have the smaller number (he didn't find it). but student A get a smaller number. But when Student meet this pair of two numbers next time, they will start the Method VI directly.
(10,8)	Student A didn't multiply the two numbers directly, he uses the method VI to get the answer.

As the figure shows us, Student A struggle with the rule at the beginning of game, especially the first question he got. When he got two numbers, he tries to find a "approximately equal" of two numbers at first, with the deepening of understanding to the rules and the try and error. he noticed the importance of "equal" (common) is the key to complete the game.

So, he changed to method V after he cannot find a common multiple with the Method I. After that, student A use the method V, VI, and VIII all the time, he didn't go back and use the previous method which below the red line. When student A become more proficient in mastering games and rules, He tries to find an LCM in the most cases even the number is a prime number, which means he didn't comprehend the nature of prime numbers during the game. But given that the prime number is not the purpose of our research, the researcher only focused on the development of method that students used. So, For the student A, we can conclude that he can find the LCM quickly with his method in the game eventually.

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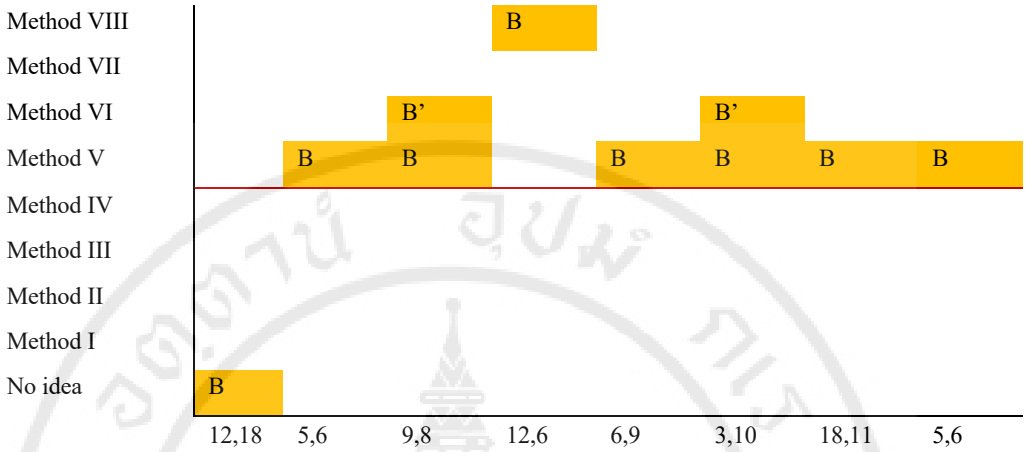
4.4.2 Video interpretation of student B in group III



Note: B prime (B') indicates that the student B has used a new method in the same question.

Figure 4.10 Method development of student A in group III

Table 4.14 The report of method development of student B (Group III)

Question	Report
(12,18)	As the researcher mentioned before in the report of student C, student B has no idea about this question, so he is quite without saying anything. But student C show his thoughts to other students, and other two students all are focused on what he did and speak.
(5,6)	After other Student C and A give the answers quickly, student B didn't follow their answers, he thinks about number 5 and 6 for a few seconds, then give the correct answer. But when the student B encounters this problem for the second time (last turn), he gives the answer quickly.

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Table 4.14 The report of method development of student B (Group III) (cont.)

Question	Report
(9,8)	Student C gives the answer: "8 multiplies 9 equal 72" directly, Student B also noticed that, because he already learned the method V from other students, but he is trying to find a smaller number, he multiplies 8 by 5 to get 40, and then sees if it can be divisible by 9. When the value of the two numbers in the question is very close, they might notice that it's impossible to find a smaller than a product of two numbers, but students still need to try then get a sense of numbers few times. Then next time they encounter the numbers like this, students will give the answer directly without trying to find a small answer.
(12,6)	Student B also gives the answers directly of this question.
(6,9)	Student B noticed the common multiple of these two numbers, but he didn't try to find a smaller number.
(3,10)	Student B said 30 after thinking a while, he is trying to find a smaller number, but he couldn't find it.
(18,11)	Student times the two numbers directly.

As you can see, the figure of student B is quite short, because there is no obvious thinking behavior that can be captured of student B, he is silent for most of time. And other students always give the answers first and challenge him.

After no ideas at his first question, Student B answer the rest of questions correctly. He didn't try to find the LCM in a question but only the common multiple, which is acceptable, then he will have the sense to find a "smaller number" which can let him pay less. He also can learn from other players in the game, especially learnt from the other mistakes. So, the researcher can conclude that student B learned how to find the LCM in the game.

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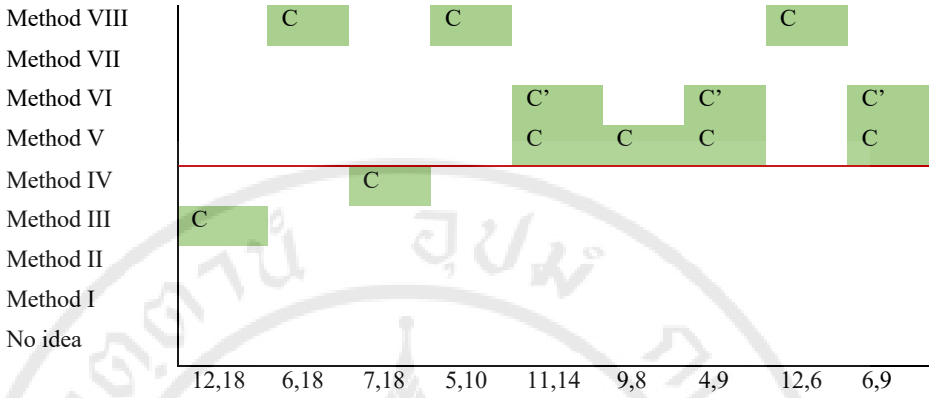
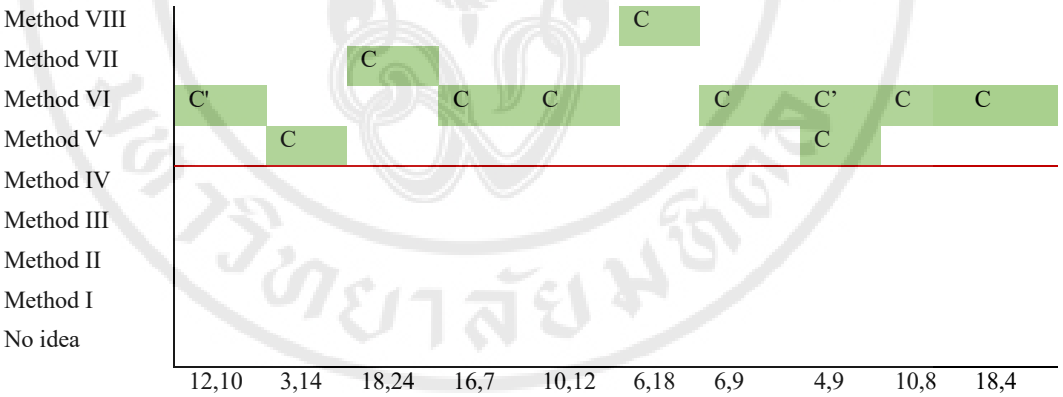
4.4.3 Video interpretation of student C in group III



Note: B prime (B') indicates that the student B has used a new method in the same question.

Figure 4.11 Method development of student C in group III (Part I)



Note: B prime (B') indicates that the student B has used a new method in the same question.

Figure 4.12 Method development of student C in group III (Part II)

Table 4.15 The report of method development of student C (Group III)

Question	Report
(12,18)	Student C use 18 multiply by 4 firstly, which is the only activity he did in the game for this question, so he seems going to get the common multiple with a" bigger" multiplier. Student C get some ideas when other students show their calculation and methods, so he tries to find the common multiple first.

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Table 4.15 The report of method development of student C (Group III) (cont.)

Question	Report
(6,18) (5,10) (12,6) (6,18)	These numbers are obvious (one number is the multiple of another number) for students to give the answers quickly and correctly. Thus, student C also gives the answers directly and correctly without any activities can be captured for further analyzing. But the researcher can conclude that student can learned how to get LCM from the game. If students can get the answers directly from these numbers, the researcher can conclude that the student can get the LCM by themselves. because students give the correct answers based on their understanding of the games rule eventually.
(9,8) (3,14) (6,9)	Student multiply these pair of numbers directly without any further calculation, the number of 9 and 8 are too close so he won't try other methods, but for the number 6 and 9, Student C said there don't have the smaller number (he didn't find it). But after other students find a smaller number, student C feels surprised and he will pay more effort on finding a smaller answer afterwards. So, he used the method VI directly at the second time when he met the numbers of 6 and 9. Student just multiply the two numbers together because of numbers themselves, student C noticed there is no chance to find a smaller number for some questions, so he will not try to use another method to get it. But this kind of number senses is not always correct.
(11,14) (4,9) (6,9)	After student C understands the rule of game, he always multiplies the two numbers first then do the further calculation with Method VI, but this situation only occurs at the mid-previous period of game. When student C realized the way to find a common multiple, he uses the method V to make sure that he can give an answer first, then he will use the method VI, because the rule of game push him to find a smaller number.
(16,7) (10,12) (18,4) (10,8)	Student C already know the method of finding a common multiple, so he will not try to multiply the numbers firstly, instead he will try to use the Method VI to explore a smaller common multiple at the beginning. This explained that student C already comprehensive the method to find an LCM, but he still doesn't know which pair of numbers have a least common multiple or just a common multiple. Student C has a kind of degree of number sense but not so clearly, because they didn't learn about the prime number and factors yet in Chinese math curriculum.

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Table 4.15 The report of method development of student C (Group III) (cont.)

(18,24)	Student C used Method VII with this pair of numbers, the reason he uses this method is depend on numbers themselves, 18 and 24 are two digital numbers, which are harder to calculate, he calculates the multiples of each number then try to find a same product of two numbers, which is more obvious for him to give the answer.

As the figure showing us, Student C is quite active while playing the game, there are many activities of him can be captured. Student C also trying to find the common multiple first after understanding the part of rules, and then he finds out some numbers (prime numbers) just multiplied together you will get it, some numbers need the future calculation to get a smaller answer. But student C only has some basic idea (number sense) about this, he still not clear which kinds of numbers are not just multiplying together. So, they will try to find a smaller number at the most time with the Method VI.

At the beginning, when student C trying to find the answers, he tries the random multiplier or a multiplier which is bigger than two of numbers. Then he finds out this way is not an efficiency way and related to luck, because he needs to do a lot of calculations sometimes. So, he developed his method to Method VI through starting from the smaller multiplier, which can guarantee that he won't missed the correct multiplier at the beginning. And If the numbers are difficult to calculate, he will use the Method VII to calculate step by step to get the answer, by doing this, he can definitely get the LCM. We can see the method development of student C from the figure clearly, and how did he construct step by step to get the LCM from the report. He can find the LCM correctly while playing the game, and more efficiently and quickly at the second half of game. So, the researcher can conclude that student C can find the LCM after playing the game.

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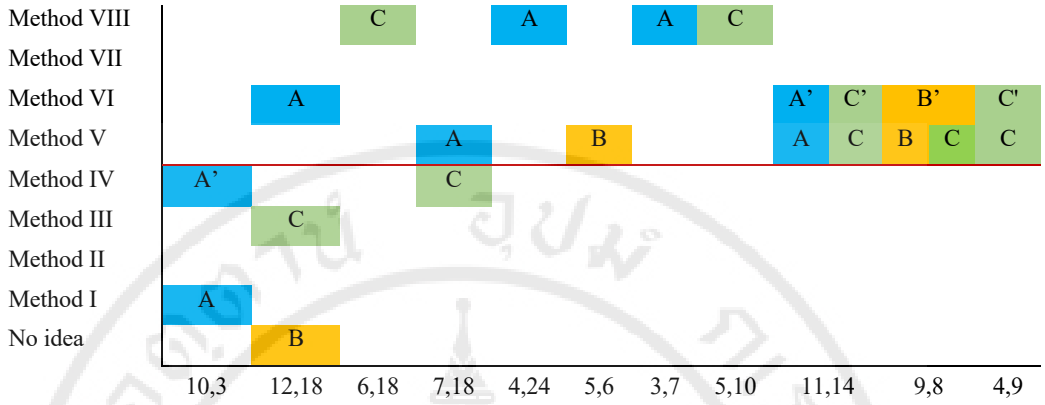
4.4.4 Group interpretation



Note: A prime (A') indicates that the student A has used a new method in the same question.

Figure 4.13 Partial method development of group III

The researcher also shows the partial group method development and analyzed from the social constructivism and collaborative learning. As the figure 4 shows, for the first question (10,3), student A trying to use a whole one package which has 3 ingredients and another 3 ingredients from a 10 ingredient-package. then he can make 3 sandwiches. But student C remind him "you cannot waste the left ingredients". Then student A changed to method IV from method I, because student B remind him that his current method not fit the rule of game. Thus, student A adjusts his method to fit with the rule first. So, the language is the most significant factor to make student A changed his method at the beginning. So, student A is a player who always has his method, he just feels and do it, then he observed the reflections from his attempts even is not his turn. Then he started active experiment with the efficient methods.

From the video and figure, we can see that student B is a player who is not so active while playing the game, because he only calculates when it is necessary (his turn). For the question (12,18), student B had no clue to deal with this question. And student C and student A shown their methods about how to fix this problem. So, student B prefers watch and think, he watched other players attempts rather then his own attempts. Then he also can accumulate reflective observations. So, when he deals with his second question (5,6), he uses the correct method directly. With more observations,

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we can see that student B started active experimentations from the question (8,9), he tried to use the method VI to find a smaller number after he multiply the two numbers.

At the beginning, student C remind student A that his method is not fit with the rule, which means student C will not make the same mistake that student A did before. But it doesn't mean student C can use the efficient method directly. Thus, when student C meet a difficult question (7,18), he still uses the inefficient method. So, he still under the stage that accumulate the reflections from different question. So, the students learn from the interactions of game environment until they build the similar common knowledge from all kinds of trial and error. But each player might have the different understanding of this common knowledge and develop the different methods.

4.4.5 Interview results of group III

Table 4.16 The interview results of group III

Number	Question & Answer
	6 Open-end questions:
(1)	How you make the sandwiches with the daily special package and weekly special packages?
	A3: "We need decrease the cost of daily special package and weekly special package". B3: "Calculate the smallest number". C3: "Calculate the smallest number"
(2)	What factors seems to affect you to win the game?
	A3: "The coins, you will win if you get more coins, and use more patent cards," B3: "At the beginning I had a lot of money, then they calculate fast then me". C3: "Confidence and coins".
(3)	What do you think is making it hard for you to play this board game?
	A3: "Dice, sometimes I cannot go fast". B3: "Two-digit numbers calculation" C3: "If you lost a lot money at the beginning. but you can earn them back".

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Table 4.16 The interview results of group III (cont.)

Number	Question & Answer			
(4)	Do the manipulative gears can help your performance well at next round of the game? Why?			
	A3: No ideas. B3: No ideas. C3: "Because there is a point on the gears".			
(5)	What would you have to do to become wealthy in the game? If that, what tricks you used?			
	A3: "Decrease the cost and get more patent card, and you need to find the smallest answers, otherwise other players will change you, and you need to pay the stamp for him". B3: "Patent card, discount, and free card can help me to save money". C3: "Collect 3 stamps, then you can get a patent card".			
(6)	Could you try to tell me the definition for the word "Least common multiple" from the experience gained from the game?			
	A3: "Two numbers, like the money, we need to find the lowest cost price". B3: Cannot say it clearly. C3: "Calculate the smallest number".			
	3 Problem questions*	A3	B3	C3
(7)	12,9 (hotdog)	Correct	Correct	Correct
(8)	14,8 (gear)	Correct	Correct	Correct
(9)	40,60,80 (time)	Correct	Wrong	Wrong

**Please see the Appendix F of the whole 3 problem questions, the table only shows the numbers that need to be calculated in the question.*

From the interview we can see that all students in group III can understand the rule of game and play the game well, they all know the calculation is the difficult part of game, and student B said the two-digit numbers are harder to calculate. but it's quite surprised that students in group III can answer the question 8 correctly even they cannot explain the function of gears. the researcher thinks they might cannot understand the question well, but they know how to deal with the two numbers in question.

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4.5 Results of fun sorter questionnaire

Table 4.17 Results of fun sorter questionnaire

Group	Student	Sorting*	The reasons of this sorting
Group 1	A1	10,1,8,7,9,5,4,11,6,3	The reason I chosen the patent card as one of my favorites is it can bring me more coins. I don't like the card number 3 (calculation) because it has challenge, I cannot play it well.
	B1	2,6,10,1,8,7,11,5,3,4,9	I think using all kinds of ingredients to make sandwiches is most fun for me. Because I can obtain a stamp after I finish making sandwiches.
	C1	3,6,10,1,2,11,9,4,7,5,8	I think the calculation in the game is most fun, because it has more challenge.
Group 2	A2	3,1,10,5,4,6,7,8,2,11	The calculation is very interesting.
	B2	3,4,9,11,8,7,1,2,6,5,10	Because I like the calculation.
	C2	3,6,1,11,9,7,8,4,2,5	I can learn something.
Group 3	A3	1,8,6,10,9,4	I like the free card, because it can allow you free to pay, and it also can exchange to money when game end,
	B3	8,10,4	I like the exchange day, because I can exchange a lot of coins that day.
	C3	1,8,6,4,10,9	I like the coins, because the one who has the most coins wins.

**Please see the table 20 of representations of each number below.*

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Table 4.18 The representations of each number

Number	Representation	Number	Representation
1	Discount card and free card.	7	Collecting stamps and the function of stamps.
2	Using the ingredients to make the sandwich.	8	Collecting patents and the function of function.
3	Calculation.	9	Challenge.
4	The design of the board.	10	Earning day.
5	Gears.	11	The rule of making sandwich.
6	Gold trading.	12	Other possible elements of board game.

All players completed the fun sorter questionnaire after playing the game. As the table shows us, the sorting and explanations are quite different from each group. But the sorting from the same group is similar. In general, we can see that most students rank the gold trading. calculation, discount card, earning days, and patent cards as the top of fun, and most students sort the challenge as the disliked part.

All students from group I have more interests on earning days, only student C1 thinks the calculation is fun, because she likes the challenge which is the calculation. And student A1 is more interested in the parts which can bring her more coins, such as patent card and discount card. But student A1 also gives the explanations that she doesn't like the calculation is she cannot handle it well. And student B2 is enjoying the process of making the sandwich.

We can see that all students from group II are more interested in calculation. They all put the calculation as the top of fun. Group II students think the calculation can help them to learn the knowledge and it has more challenge.

Group III students consider the discount card and patent card as the favorite parts, which can bring them more coins in the game, and in group III, each student get more patents card and get benefit from the patent card.

The researcher thinks different game experiences bring different results. And whether students like the calculation is not positively related to their calculation performance in the game, which means even students good at the calculation while playing the game, they still don't like the calculation, and vice versa. Students from group I and group II consider the gears as the less fun part, and group III students even

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didn't put it on their questionnaire, the researcher believes that this situation caused by the frequency of the gear use. Because three groups of students didn't used the gears too much. So, they don't have too much judgments of the gears.

4.6 Trustworthiness

In view of this study is qualitative research. To be accepted as trustworthy, the researcher demonstrated that data analysis has been conducted in a precise, consistent, and exhaustive manner to ensure the process is credible.

The researcher triangulated the different collecting modes such as video recording, interview, and manuscripts. Then researcher stored raw data in well-organized archives, and kept records of all data field notes, transcripts. After well-prepared the original data, the researcher started generating initial codes. The researcher interpreted the video recordings into thick descriptions according to the timeline. And there are three members checked the interpreted descriptions to search the themes (All methods that students used in the game). The researcher also took the notes about the development of team meeting of each time. Then the researcher reviewed the themes，and the themes vetted by team members also. So, after the peer debriefing and team consensus on themes, the researcher defined and named the eight methods of finding LCM. Lastly, the researcher produced the report which contained the process of coding and analysis with sufficient details, and the report also checked by other members to ensure the consistent of the results.

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CHAPTER V
CONCLUSION

5.1 Discussion

This research study investigated the question, “How do fourth-grade students construct LCM knowledge while playing sandwich factory board game?”. In addition, the study investigated the question “What is the rank of the elements of the board game that are attractive to use?”. For this qualitative study, there are three interpreters which allowed the researcher to analyze the data from different views to get different perspectives.

The results also provided evidence that using Sandwich Factory Board Game does promote student’s performance on finding the LCM. In this researcher, but there is no significant difference of method development among three groups. All participants have the similar pattern of method development. The researcher also found it’s hard let students clearly conceptualized the composite number and prime number. And it is difficult for students to clearly summarize how the prime number and the composite number are different when looking for LCM. They might have the awareness about this pattern which considered as a part of “number sense”. Besides, the difficult questions (two-digit numbers) can motivate student to develop their method to fit with the new situations. And the difficulty of numbers also affects the methods that students used while playing the game. In this study, the existence of the interview was intended to further explain the student's learning process. But the results that researcher got from the open-ended questions are consistent with analysis from the videos, there is no additional discoveries. Three problem questions show that all students have the ability to adopt the method which used in the game into other situations to find the LCM. And most students can find the LCM with the 3 numbers correctly.

The rakings of elements of the game are quite different from each group, but the opinions of the students in the same group are similar. And in general, students show the great interest on the trading of coins, calculations, and all function cards in the game.

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5.1.1 Discussion of game theory and students' perceptions toward game

Researcher adopted the game theory Hall (1998) when develop the game. And sorting the elements of the game was intended to find the most engaged part of the game. The results will be benefit to the researcher to refine and improve the LCM game and the educators who has the intention to design a game to teach the relevant topics.

The result shows that students sorted the parts that can bring them more fortune as the most fun elements, because the rule is that the player who obtain the most coins wins, so students classified the elements related to winning as the most interesting elements. So, earning money from making sandwiches (focused goal) is significant for the game design. The difficult question (challenging tasks) also facilitated students to develop the method in this game. But students who had been challenged and lost money from it considering the challenge as the not fun. Which means the sorting of elements was based on the student's personal game experience to some extend. Some students might don't like it, but it is also important for the game. The biggest difference between this board game and other LCM games is the integrated mechanism with manipulative gears, which can lead students to find the LCM and develop their methods. It is suggested that the future game with similar topic to focused on the goal of game, and it is recommended to add the game elements in the goal of game. The challenge is a significant element in the game. Even the results shows that some students don't like it, we can make some adjustments between the game's gains and losses, or simplify the challenge rules instead of deleting the challenge part directly.

5.1.2 How Sandwich factory board game facilitate students to find the LCM

We can see that the mechanism of game was constructed successfully from the performance of students during the whole game. The game was designed based on the real situations of making sandwich. Students got two numbers from the two packages, and the rules of making sandwich are the key for facilitating student to develop their method. Because the rule required no ingredients left over. This means when they are using a package, they have to use all ingredients in that package. And they can use a package as many as possible if they needed in the game. But in the real-world situations, students are allowed to have the remaining. So, this mandatory setting of the game is slightly conflict with the student's existing experience. So, this

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inconsistence caused the new experience for students, then they will adjust their methods to fit with the rule first. In this game, the numbers they calculated represent the amount of coins that they need to pay. Thus, in order to win the game, students must find the smallest number which fit with rule also. And the gears can be used to examine the answer from the challenger if the challenge happened.

5.1.3 The method development of finding LCM

The researcher found that the students' method development is support by Kolb's learning style (1984), which include concrete experience, reflective observation, abstract conceptualization, and active experimentation. And Kolb concluded that the effective learning is seen when a person progresses through a cycle of four stages. But is possible to enter at any stage and follow it through its logical sequence. After students go through the cycle and get the new concreate experience, the learning cycle may begin again.

In this game, the cycle begins with students' concrete experience. When students encountered a new satiation (making sandwich to find the LCM), they interpret the rule of game with their existing experience at first. And students have the different initial ideas (different methods) of making sandwich, which due to students' different experience. So, at the beginning, students who just feel and do will come out the low efficient methods at the first round of experiential cycle. And the students who have more thinking (abstract conceptualization) could come out an efficient method directly. For example, students who come out the method I and find the "approximately equal" of two numbers, and they noticed there will be some ingredients left. This consequence will become their new experience, then the students start a new round of cycles with the new concrete experience. Due to the game is an interactive process, students are allowed to challenge and talk to each other while playing the game. And students need to pay attention to other players attempts, so, the process of mutual supervision can also be seen as a way of collaborative learning. Thus, students observed different methods from other players, then they will notice that different methods bring the different consequence (Reflective observation). Students will find the inconsistencies between their previous experience and understanding. And this reflection gives rise to a new idea, or modification of an existing abstract concept (abstract conceptualization). Then students apply the new method in the game to see what results. At this round of

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experiential learning cycle, some students developed method III and method IV, which means they realized the importance of "common" in the game, students noticed by using the random multiplier, or using a multiplier which is bigger than two of numbers are affected the efficiency of finding LCM. But some students might come out the efficient method which above the red line also. After the few rounds of experimental learning cycle, all students will come out the efficient method eventually.

In this game, every participant didn't used all methods while playing the game. because various factors influenced students learning style, for example, social environmental, educational experiences, or the basic cognitive structure of the individual. And the researcher found that even students have generated a good method, they still went back and used the previous undeveloped method sometimes.

5.1.4 Social constructivism

Vygotsky (1978) posits that since learning is socially situated, learners should therefore learn from the real world. The diversity of experiences that learners bring to the learning environment is acknowledged; it is through reflection and thoughts on these experiences that learners construct meaning.

Game is an interactive and dynamic process, students were not only facilitated by the game to develop the methods to find the LCM. There will be many interactions among students which indeed helped students to develop their method to find the LCM, not only the game itself. The interaction in this game is the conversation mostly, which caused by students' questions directly. The conversation happened in the group can be captured by any other players. So, the interactions in one group can offer the common knowledge for players, Thus, students within one group construct group knowledge, that group knowledge then become part of that group.

And some students are sensitive, they prefer to watch rather than do, this caused they will learn from other players attempts more than himself. And some students focus on their own attempts and use their learning to find the solutions. So, the individual learning behavior affected students' learning style. And students of different learning style focused on the different learning resources during the game. But the method development of students still is similar. Since the focus of this study is not on the analysis of the student's learning style, the researcher did not go too far on it.

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However, it is recommended to the further educators who interest in learning styles affecting the board game learning to do the further studies.

5.1.5 Manipulative gears

The researcher found that the manipulative gears have minor effect on helping students to find the LCM during the game. The researcher want student to be able to link their actions with manipulatives to describe the actions in the game. But Students cannot link the gears to the game clearly, and student said the gears are amazing because the gears can show the correct answers to them. Researchers believe that the reason for this is that students do not use the manipulative frequently, they only use the gears two times during the whole game due to the rule of using. The manipulatives should be used as much as possible during the game, which support the suggestions from Piaget (1952). The researcher didn't consider that the gears are useless for this game. Instead, researcher believes that students can obtain some ideas when we enhance the frequency of using gears, student will see how the LCM comes out through the rotating gears along the rack, which can help them to link the gears with the game.

5.2 Limitation and suggestion

As with any study involving game-based learning, there were problems and limitations over which the researcher had no control.

5.2.1 The rule of challenge

The researcher designs the rule of challenge which want all students can focus on every calculation and motivate each player to develop their method if they have been challenged. But in the process of playing, the students did not strictly follow the rules of the challenge. Sometimes they challenge each other and sometimes help each other. But scholars believe that the mechanism of challenge can help students learned and developed their method effectively. The researcher suggests that the future studies or the educators who will apply this game in the teaching should strictly implement the rule of challenge.

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5.2.2 Manipulative gears

In this research, due to the real situation of challenges in the game, students do not have much opportunity to use gear. So, it's suggested that the future study can focus on the manipulatives and use the gears as much as possible.

5.2.3 Time duration of the game

Due to various factors, the duration of the whole game is less as expected. So, the researcher suggests that the future studies extend the game time to two hours at least. And see whether students can summarize the pattern of finding the LCM of composite number and prime number or develop other efficient methods.

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APPENDICES

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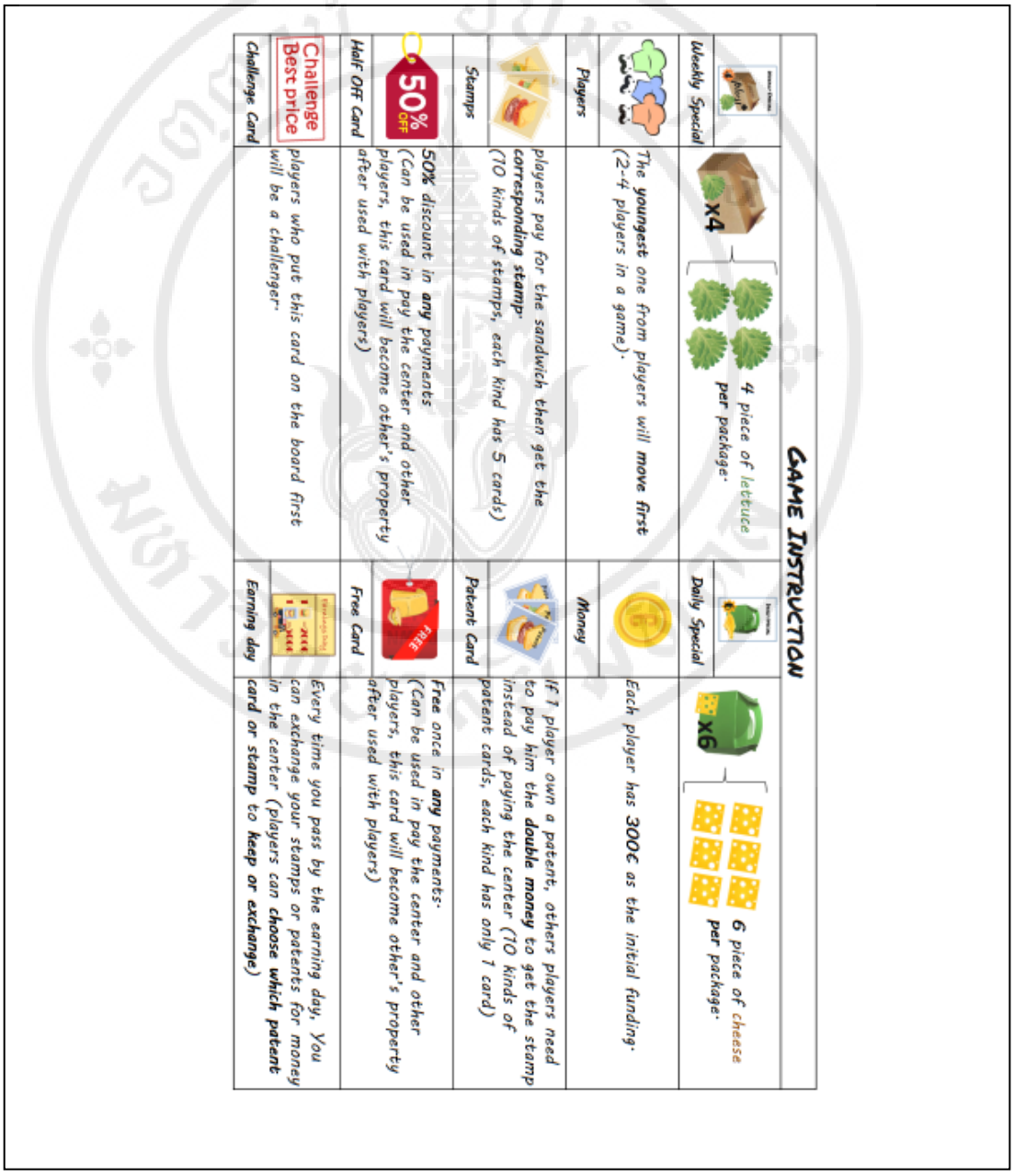
APPENDIX A
GAME INSTRUCTION



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APPENDIX B
THE LAYOUT OF THE BOARD GAME

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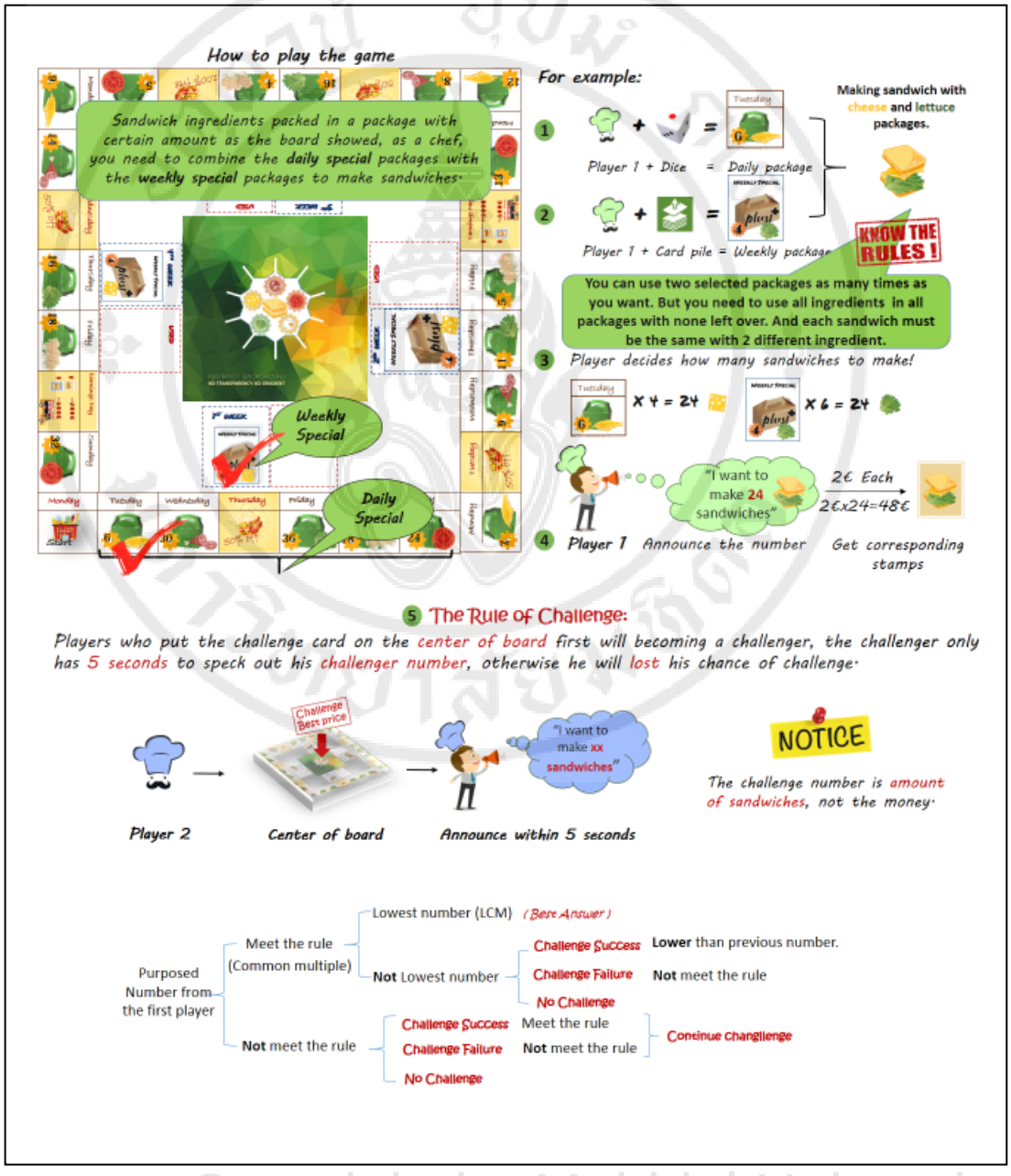
APPENDIX C

THE BRIEF PROCEDURE OF THE GAME



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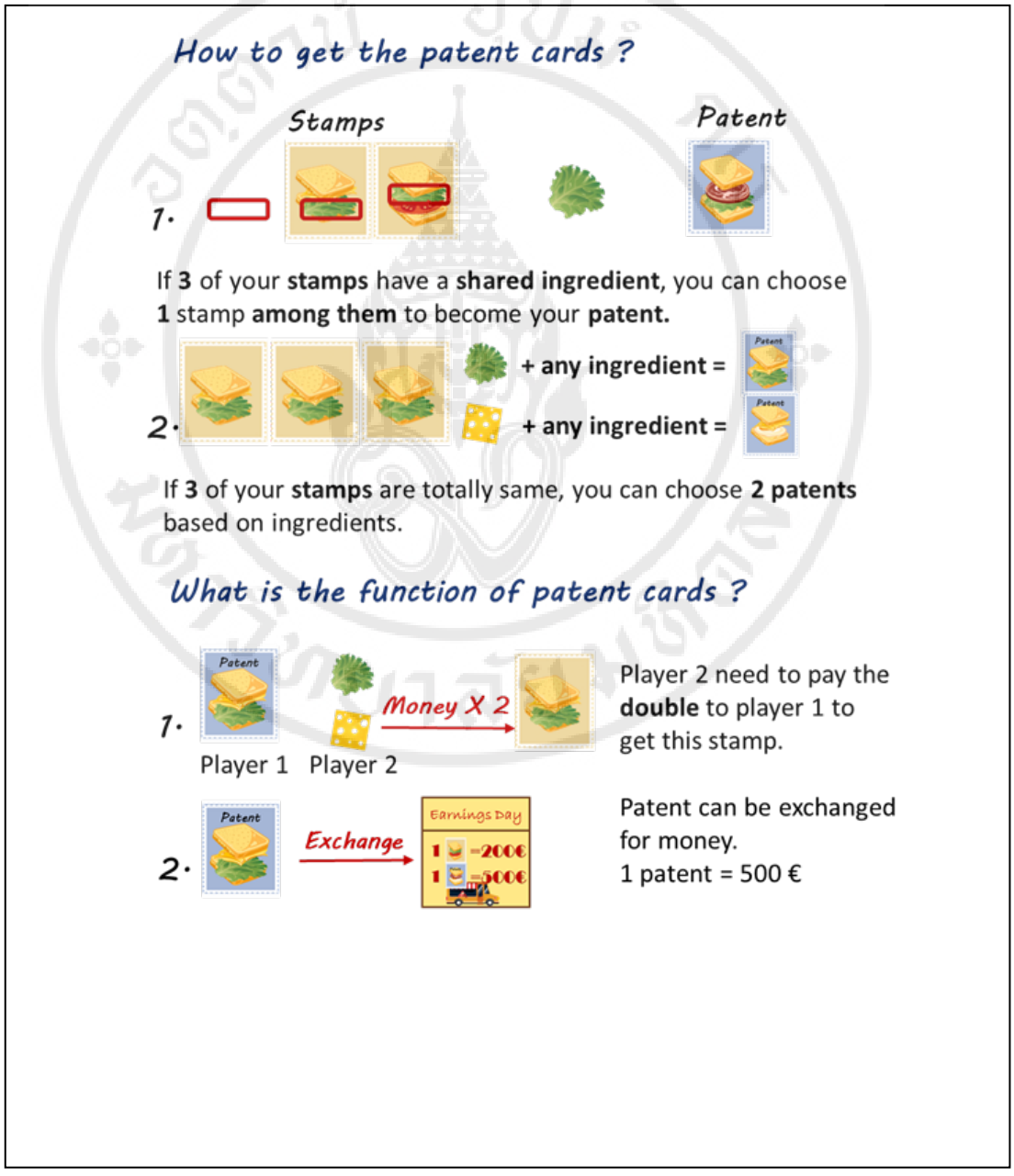
APPENDIX D
THE STAMPS



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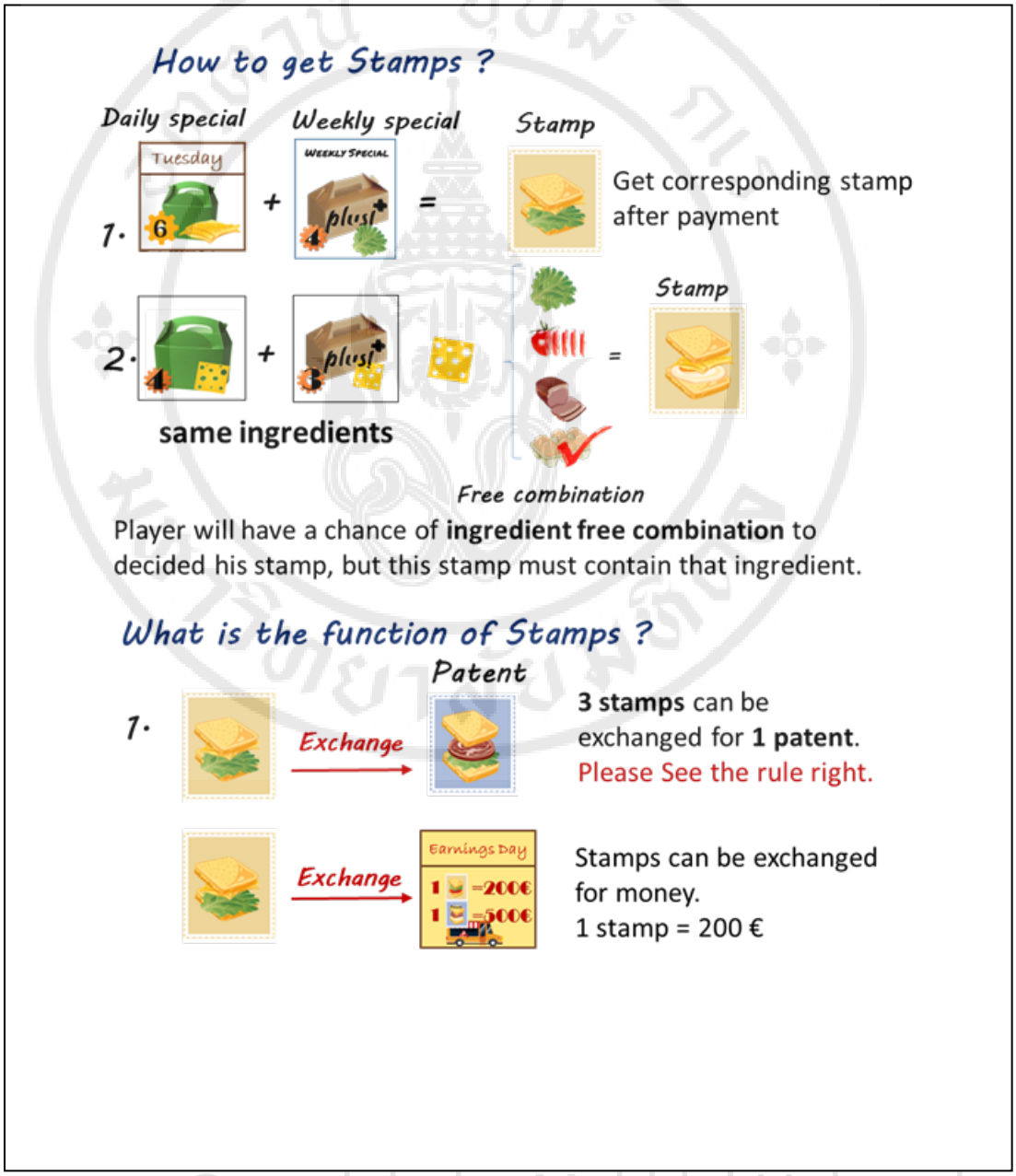
APPENDIX E
THE PATENTS



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APPENDIX F
PRIOR KNOWLEDGE TEST

Name:_____Age:_____Gender:_____Grade:_____

Note: 1 is a factor of all non-zero natural numbers.

(1)	1	2	3	4	5	6	7	8	9	10	11	12
(2)	13	14	15	16	17	18	19	20	21	22	23	24
(3)	25	26	27	28	29	30	31	32	33	34	35	36
(4)	37	38	39	40	41	42	43	44	45	46	47	48
(5)	49	50	51	52	53	54	55	56	57	58	59	60
(6)	61	62	63	64	65	66	67	68	69	70	71	72
(7)	73	74	75	76	77	78	79	80	81	82	83	84
(8)	85	86	87	88	89	90	91	92	93	94	95	96
(9)	97	98	99	100	101	102	103	104	105	106	107	108
(10)	109	110	111	112	113	114	115	116	117	118	119	120

(1). Please circle the **even number** in the **9th row.**

(2) Please circle the number which is **multiple of 11** in the **6th row.**

(3) Please circle the number which **can be divisible by 3** in the **4th row.**

(4) Please circle the **prime numbers** in the **10st row**.

(5) Please circle the numbers which are **factors of 12**.

(6) Can you give 3 examples of number that have **exactly 2 factors**?

_____

(7) Can you give 3 examples of number that have **exactly 3 factors?**

_____

(8) Can you give a number **smaller than 45** but **has more factors than 45.**

_____

(9) Can you give a number **greater than 500 that has exactly 2 factors**?

_____

(10) Please circle the number which is **multiple of both 11 and 4** in this table.

(11) Please write the least common multiple **of 12, 3, and 15.**

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APPENDIX G
INTERVIEW QUESTIONS

6 Open-end questions:

(1) How you make the sandwiches with the daily special package and weekly special packages?

(2) What factors seems to affect you to win the game?

(3) What do you think is making it hard for you to play this board game?

(4) Do the manipulative gears can help your performance well at next round of the game? Why?

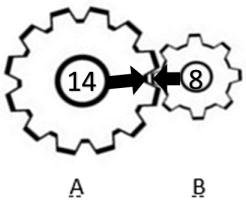
(5) What would you have to do to become wealthy in the game? If that, what tricks you used?

(6) Could you try to tell me the definition for the word "Least common multiple" from the experience gained from the game?

3 problem questions:

(7) Chief Li needs to make hotdogs for his customers tomorrow, he went to the market and found that there are 12 sausages in a box and 9 breads in a package. He doesn't want to leave any ingredients after cooking, so how many sausage boxes and bread packages he need to buy and how many hotdogs he can make?

(8) As the picture shows, gear A has 14 teeth and gear B has 8 teeth, for each pair of teeth from the first contact to the second contact, how many rounds did each gear rotated at least? How many teeth are each gear rotated at the second contact?



(9) Salaya town has a small train station with 3 tracks, the trains on first track pass through the station every 40 minutes. The trains on second track pass through the station every 60 minutes, and the trains on third track pass through the station every 80 minutes.
Ming saw the three trains passing by the Salaya town's railway station at 1:00 pm, so when will he see this situation happen again?

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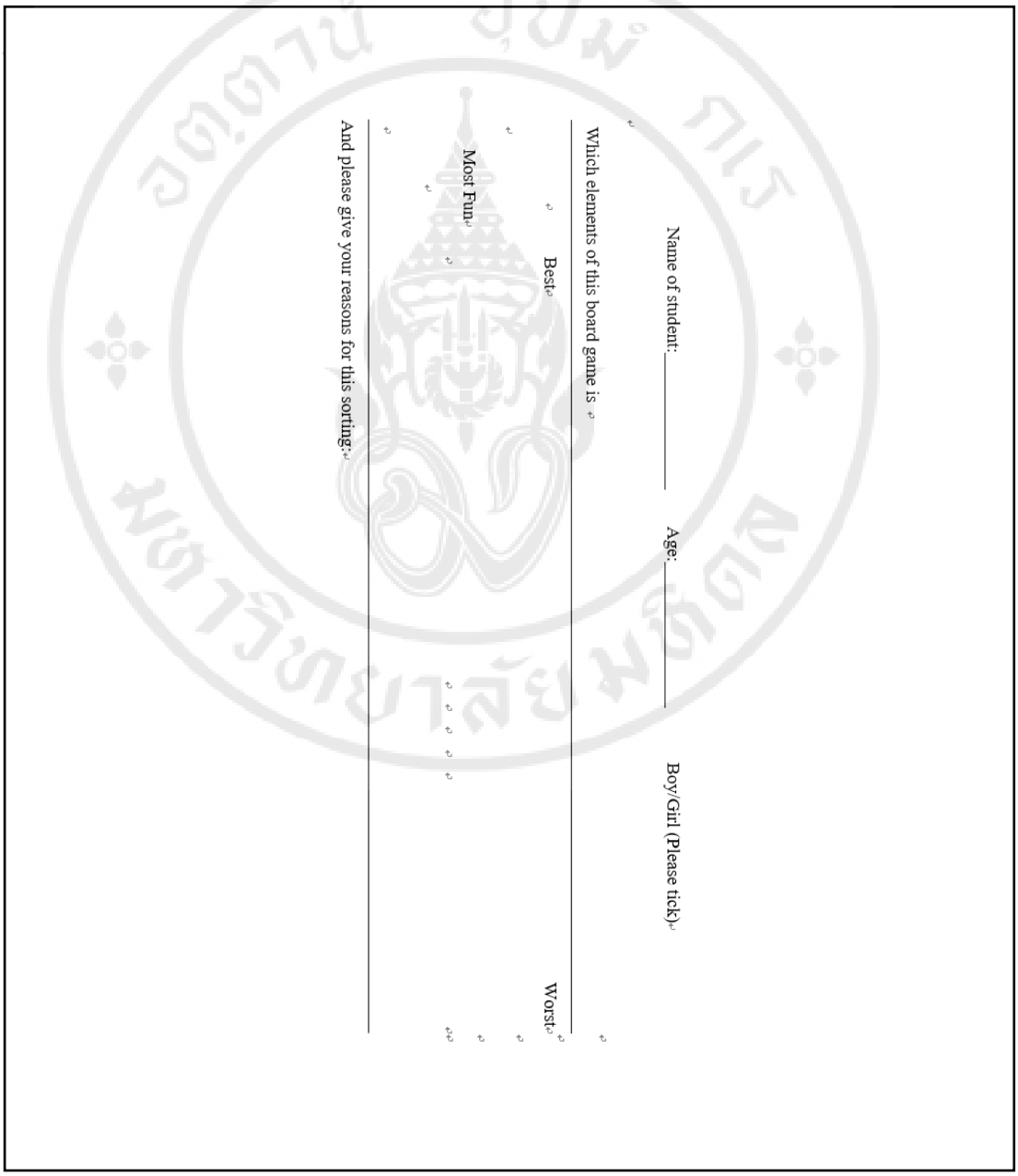
APPENDIX H
FUN SORTERT QUESTIONNAIRE



Name of student:_____ Age:_____ Boy/Girl (Please tick)

Which elements of this board game is

Best

Worst

Most Fun

And please give your reasons for this sorting:

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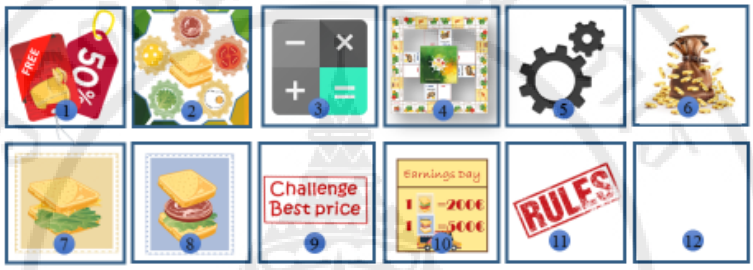
APPENDIX I

THE ELEMENTS OF THE GAME



1.Discount&free card.
2.Using the ingredients to make sandwich.
3.Calculation.
4.The design of the board.
5. Gears.
6. Gold trading
7.Collecting stamps and the function of stamps.
8.Collecting patents and the function of stamps.
9.Challenge.
10.Earning day.
11.The rule of making sandwich.
12.Other possible elements of the board game.

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