ENHANCING HIGHSCHOOL STUDENTS' ACHIEVEMENT IN PERMUTATIONS AND COMBINATIONS THROUGH NONTRADITIONAL WORD PROBLEMS, SPORT PROBLEMS, AND PROBABILISTIC GAMES

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entitled

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Nutjira Busadee

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ABSTRACT

Probability concepts in combination and permutation are difficult to grasp, especially when students are taught by formal mathematical presentation without having gone through sufficient real-life situations. Besides, some of the common textbook problems are not interesting to students, being more of math teasers rather than based on real-life situations.

This study tested the newly invented real-life probability problems involving permutations and combinations, and simple problems requiring obtaining the probabilities of events by common sense. Personal relevance, sport situations, and games are the main features of these new problems. The participants were four classes of high-school students in a rural setting. One class served as the control, with the traditional problem unit (TU). The other three classes were given different units on non traditional word problems (NTU), sport problems (SU), and probabilistic games (GU). The results demonstrated that all units enhanced students' learning. However, students learning by NTU outperformed those in the other classes. GU and SU were the less successful ones in enhancing students' achievement. The students showed a positive attitude toward learning through authentic problems. Some unexpected results, especially from sport and game problems, are discussed.

KEY WORDS: STUDENTS' ACHIEVEMENT / PERMUTATIONS AND COMBINATIONS / WORD PROBLEM / SPORT PROBLEM / PROBABILISTIC GAME

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การพัฒนาผลสัมฤทธิ์ทางการเรียนของนักเรียนระดับมัธยมศึกษาตอนปลายเรื่องการจัดเรียงสับเปลี่ยนและการจัดหมู่โดยใช้โจทย์ปัญหาพิเศษ, โจทย์กีฬาและเกมความน่าจะเป็น

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บทคัดย่อ

ความคิดรวบยอดต่างๆ ในเรื่องความน่าจะเป็น โดยเฉพาะเรื่องการจัดเรียงสับเปลี่ยนและการจัดหมู่ถือเป็นเรื่องยากต่อการทำความเข้าใจโดยเฉพาะอย่างยิ่งสำหรับนักเรียนที่ได้รับการจัดกิจกรรมการเรียนการสอนแบบดั้งเดิมซึ่งขาดการนำสถานการณ์ที่เกี่ยวข้องกับชีวิตประจำวันมาประยุกต์ใช้ นอกจากนี้โจทย์ปัญหาต่างๆ ที่บรรจุอยู่ในตำราเรียนไม่สามารถดึงดูดความสนใจหรือกระตุ้นให้นักเรียนสนใจได้เมื่อเทียบกับการใช้ปัญหาที่เกี่ยวข้องกับสถานการณ์ในชีวิตประจำวัน

ในการวิจัยครั้งนี้ โจทย์ปัญหาที่เกี่ยวข้องกับชีวิตประจำวันได้แก่ โจทย์ปัญหาพิเศษ (ต่างจากโจทย์ปัญหาปกติที่ปรากฏอยู่ในแบบเรียนทั่วไป) โจทย์กีฬา และเกมความน่าจะเป็น ได้ถูกสร้างขึ้นมาเพื่อใช้ในการสอนเรื่องการจัดเรียงสับเปลี่ยนและการจัดหมู่ นักเรียนระดับมัธยมศึกษาตอนปลายในชนบทจำนวน 4 ห้องเรียน ถูกแบ่งโดยการสุ่มเป็นกลุ่มควบคุม 1 กลุ่ม และกลุ่มทดลอง 3 กลุ่ม กลุ่มควบคุมเรียนโดยใช้ชุดการสอนปกติ (TU) กลุ่มทดลองกลุ่มที่หนึ่งใช้ชุดการสอนที่เป็นโจทย์ปัญหาพิเศษ (NTU) กลุ่มทดลองกลุ่มที่สองใช้ชุดการสอนที่เป็นโจทย์กีฬา (SU) และกลุ่มทดลองกลุ่มสุดท้ายใช้ชุดการสอนที่เป็นเกม (GU) ผลการทดลองปรากฏว่าทุกชุดการสอนสามารถพัฒนาการเรียนรู้ของนักเรียนได้ดี โดยเฉพาะกลุ่มทดลอง NTU มีค่าผลสัมฤทธิ์ทางการเรียนดีกว่าชุดอื่นๆ ในขณะที่กลุ่ม GU และ SU ให้ผลสัมฤทธิ์ต่ำกว่าสองชุดที่เหลือ อย่างไรก็ตามนักเรียนทุกคนมีเจตคติที่ดีต่อการเรียนรู้ผ่านปัญหาที่เกี่ยวข้องกับชีวิตประจำวัน

คำสำคัญ: ผลสัมฤทธิ์ทางการเรียน/ การจัดเรียงสับเปลี่ยนและการจัดหมู่/ โจทย์ปัญหาพิเศษ/ โจทย์กีฬา/ เกมความน่าจะเป็น

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CHAPTER I
INTRODUCTION

1.1 Background of the Research Study

In their attempts to improve, develop and reform mathematics education, many research works pointed out that permutations and combinations are difficult topics for students to understand and to solve without having clear conceptual understanding (Novick and Hmelo, 1994; Lee 2006; Ben-Hur, 2006). Students are usually given the formal mathematics representation without thinking through the problem matter-of-factly. For example, most students are often unable to analyze a problem as to whether it involves permutation or combination. They think that identical objects are distinguishable or different objects are indistinguishable. For simple combinatoric problems, many students are unable to solve them because they do not know even the meaning of the variables in the formula. In other words, they cannot figure out the solutions because they have no basic ideas about the factors involved and how they come about in a formula.

Research on the difficulties in teaching and learning of probability has been extensive. Starting with the work by Piaget and Inhelder (1951), first translated by Leake, Burrell, and Fishbein (1975), other researchers built on their work, most notably Fischbein (1975), Kahneman and Tversky (1972), and Tversky and Kahneman (1974). This research classified misconceptions in probability according to the intellectual development and the strategies used to make probability estimates. These researchers laid the groundwork upon which research on probability education could be built. A volume by Jones (2005) provided a good perspective on this research.

For more effective teaching and learning, mathematics educators have employed more daily-life problems in promoting students understanding. For example, Johnson (1991), Abramovich & Pieper (1996), Bay-Williams & Martinie (2004), Sriraman (2004), Nunokawa (2005), and Lee (2006) used nontraditional word problems to promote students' learning in probability. Sports problems have also been

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employed to facilitate learning of probability. However, most researchers geared more toward a college-level statistics course rather than a highschool-level probabilistic one (Albert (2002), Cochran (2005), Gelman and Nolan (2002), Kahan & Wyberg (2003), and Smeeton (2003)). Probabilistic games have been exploited to facilitate students' learning at the middle-school level rather than at the highschool level (Amit (1998), Krulik, Rudnick & Milou (2002), Baker (2003), Batanero, Godino, & Roa (2004)). Nevertheless, these studies did not provide enough variety of problems. Also some of them did not investigate students' achievement or compare the achievement in the concepts about permutations and combinations. Some used the problems as an introduction and the students solved the usual problems afterward.

Today's students demand that their lessons be real, interesting, relevant and manageable. Mathematics is one subject that eludes many students partly because its traditional presentation lacks those elements that encourage students to learn. With easy accessibility via the electronic media, people are exposed to a variety of sports more frequently than before. Thus generally students are quite savvy about sports, their specific rules and regulations, and scores. We think that a variety of sports offer opportunities for teaching the topic of probability in the classroom. However, from our experience, teaching probability via permutation and combination formulas has generally very little effect on learning. Many students use these formulas in a very mechanical way with very little understanding. Worse, not so long after their perfunctory learning in the class, they forget the formulas and do not know how to solve even the simplest general probability problems because they cannot recall how to handle the factorials. The fact is they could still solve these simple problems if they would just tackle them by thinking, a possibility they have denied themselves probably because the way mathematics is often formally taught makes them think they have to be totally dependent on such formalism.

Many research studies suggested that inquiry-based approaches helped students learn the concepts better than only traditional teaching strategies. In addition, inquiry could help students not only to investigate and explore, but to solve problems by themselves. Teachers should use an inquiry-based strategy to encourage students' learning in the class with their own experiences in real-life situations (Hofstein & Lunetta, 2003). This study, based on the inquiry strategy, designed instructional units

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containing many new problems related to self, to popular sports and games, and to everyday life to make learning of probability more realistic and interesting.

1.2 Rationale for Investigation

In addition to the formalism, the students were asked to think through a problem deliberately informally so that those who normally fear mathematics formulas would still be confident enough to work out the probabilities of events without the psychological hindrance, especially later in life. To decrease students' tendency to mechanically apply formulas and to facilitate students' learning of probability, highschool students should be encouraged to think about the authentic problems such as those of favorite sports and to think about various possibilities for certain situations. This should be done before introducing mathematical formalism and the use of factorials. Students undergoing this sequence of presentations (real possibilities before formal probability calculation) should have gained a better understanding of the material and have retained it for a longer period of time.

Thus more real-life problems should be developed to engage and motivate student learning in permutations and combinations especially in the four concepts on linear permutation, permutation of similar things, circular permutations and combinations. The problems to be used should be popular to them such as authentic nontraditional word problems, and selected sport and game problems. The instructional units on nontraditional word problems, sport problems and probabilistic games related to permutations and combinations are expected to have a positive impact on students' enhanced conceptual understanding. The researcher therefore was interested to find out whether these learning units exploiting nontraditional word, sport and game problems as engagement would work for students with special emphasis on those in the rural area. The effectiveness of these learning units with more authentic problems was compared to that based on traditional problems.

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1.3 Aims of the Research Study

This research is an experimental research. The aims of this study are as follows:

1. To develop authentic problems in permutations and combinations to be used in mathematics instructional units.

2. To investigate authentic problems to inquiry-based approach.

3. To investigate whether the developed mathematical instructional units can enhance students' conceptual understanding

1.4 Research Questions

1. Do the newly developed learning units on permutations and combinations based on authentic problems enhance students learning?

2. What are the attitudes of students who learned via the newly developed learning units?

1.5 Definitions of Terms

Students' conceptual understanding of permutations and combinations means students construct their fundamental understanding in permutations and combinations which comprises four concepts: linear permutation, permutation of similar things, circular permutation, and combination.

Inquiry-based learning and teaching means teaching and learning strategy wherein the teacher's role is being a facilitator and the learning unit focuses on central concepts. It emphasizes that the learners should be able to authentically investigate problems, raise notions, interpret the results by themselves, and plead their own statements to show their conceptual understanding in the process of inquiry. In this study, problem solving is central to instructional procedure with minimal guidance.

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Guided-inquiry activity means a set of inquiry-based activities in which students perform investigations to answer the questions posted by the teacher. The main responsibility of the students is figuring out how to answer the given questions.

Traditional instructional unit means the unit composed of the exercises, examples, practices and problems in the IPST's textbook for highschool students including the problems which the researcher collected from other Thai and English textbooks, exercise books, and educational websites which are used for normal mathematics classes.

Nontraditional word problem instructional unit means the unit composed of the probability problems created from the unusual word problems, interesting situations, or real-life word problems. Most of these problems are created by the researcher (and some members of the committee), including interesting problems from the papers in the literature review.

Sport problem instructional unit means the unit composed of the probability problems created from popular sport situations. All of these problems are created by the researcher (and some members of the committee) such as the football problem, the ping pong problem, the golf problem, etc.

Probabilistic game instructional unit means the unit composed of new probabilistic games. All of them are created by the researcher (and some members of the committee) such as the icosahedron game, the wheel game, the card game, etc.

1.6 Organization of the Thesis

The thesis comprises six chapters:

Chapter one provides an introduction along with the justification for the work, states the research questions which motivates the study and specifies the objectives of the study including describing the definition of terms.

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Chapter two provides a literature review relevant to the topic of this study. It includes the details of the mathematical review involved with teaching and learning of mathematics contents, especially permutations and combinations. The educational review describes an inquiry-based teaching and learning in mathematics education, mathematics problem solving, and students' achievement. Theoretical framework of this study is also described in this chapter.

Chapter three includes the detailed development the instructional units in permutations and combinations. The participants in this study are mentioned. The details of the research methodology are explained including the pilot studies and the implementation of instructional units for highschool students. Finally, the research instruments for this study and the data analysis are described.

Chapter four describes the results on the effectiveness of the four guided-inquiry instructional units on student conceptual understanding. In addition, the effects of the four instructional units on students' perception are described in this chapter.

Chapter five offers a discussion of the findings of the study with emphasis on the impact of the instructional units on student outcomes. The effectiveness of the instructional unit and students' perspective toward the instructional units are discussed. Moreover, the implication of the instructional units and limitations of the study are also mentioned.

Chapter six concludes the overall findings of the research study. The recommendations are included in this chapter.

1.7 Summary of the Chapter

This chapter describes the background of this study and the rationale for investigation. The aims of the research study, the research questions, the definitions of terms, and the organization of the thesis are also presented.

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CHAPTER II
LITERATURE REVIEW

Mathematical Review

2.1 Permutations and Combinations and Traditional Teaching and Learning

Probability is an important concept which both secondary and undergraduate students have to learn. Changes in school mathematics curricula in the last few decades have brought along an increase on the importance placed on probability (NCEE, 1983; NCTM, 2000). However, most students have problems in learning this concept, especially permutations and combinations. Permutations and combinations are branches of probability and they are important in high school mathematics curriculum.

Probability Standard for Highschool Level

National Council of Teachers of Mathematics (NCTM, 2000) indicated the principles and standards for school mathematics in the highschool level as follows:

The students should;

-Understand meanings of operations and how they relate to one another.

-Compute fluently and make reasonable estimates.

-Represent and analyze mathematical situations and structure using algebraic symbols.

-Use mathematical models to represent and understand quantitative relationships.

-Understand the basic concepts of probability.

-Solve problems that arise in mathematics and in other contexts.

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-Apply and adapt a variety of appropriate strategies to solve problems.

-Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

-Analyze and evaluate the mathematical thinking and strategies of others.

-Organize and consolidate their mathematical thinking through communication.

New York State Learning Standard for Mathematics (NYSS, 2005) indicated the mathematics-resource guide with core curriculum as follows:

-Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

-Students use mathematical operations and relationships among them to understand mathematics.

-Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communication, and connecting mathematical information and relationships.

Permutations and combinations are composed of the basic concepts and specific concepts. The basic concepts are the conceptions of tree diagram, fundamental counting principle, and factorial notation (Papaieronymou, 2006; Senk & Thompson 2003; IPST, 2001). The specific concepts (especially in this study) are composed of 4 concepts; linear permutations, permutations of similar things, circular permutations, and combinations. These concepts in permutations and combinations are briefly described as follows:

Tree diagram

The tree diagram is useful to organize students' thinking. It is a visual way of making an organized list of combinations of options where the first level indicates the first choice to be made, and the branches indicate succeeding choices, given the first choice has been made. It shows the total possible ways in each sample space.

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Fundamental counting principle

The fundamental counting principle states that if a task involves a sequence of k choices, where n_1 is the number of ways the first stage or event can

occur and n_2 is the number of ways the second stage or event can occur after the first stage has occurred, and so on, then the total number of different ways the task can occur is: $(n_1)(n_2)(n_3)(n_4)\cdots(n_k)$. The fundamental counting principle is the guiding rule for finding the number of ways to accomplish *k* tasks.

When to use it: Each object is chosen from its own group.

How to use it: Total number of ways is equal to multiplying together the numbers of possibilities for each object.

Factorial notation

The factorial notation is a shorter way of writing down that you multiply a number with all the numbers less than it. For example, 10×9×8×7×6×5×4×3×2×1 = 10! = 3628800. When applied to a variable *n* instead of a number, it means *n*! (called "*n* factorial") = $(n)(n-1)(n-2)\cdots(3)(2)(1)$. One can use the factorial notation to write a formula for the number of permutations of *k* items from a group of *n* items (that is the number of combinations of *k* items, with no repeats, and where order matters). The fundamental counting principle gives us the answer; $(n)(n-1)(n-2)(n-3)\cdots(n-k+1)$. Rewriting this with the factorial notation: The number of permutations of *k* items from a group of *n* items is: $\frac{n!}{(n-k)!}$.

Permutations

A permutation means an arrangement of things. The word arrangement is used, if the order of things matters. Since the number of permutations is the number of ways you can arrange objects, it will always be a whole number. The denominator in the formula will always divide evenly into the numerator. The number of permutations of *r* objects from the total of *n* objects is equal to $P_{n,r}$. In addition, the permutations can be listed in an organized list, or on a tree diagram, and then counted.

When to use it: All objects are chosen from the same group, and the order matters, meaning that selecting objects in a different order will give you a different result.

Warning: Here are some situations where the order does matter:

- lining things up from left to right.
- scheduling things.

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- choosing a committee of people where the order you choose dictates what job they will have in the committee.

How to use it: Total number of ways = $P_{n,r} = \frac{n!}{(n-r)!}$

where n = the number of possible choices

and r = the number of things you choose

The permutation concept is composed of 3 sub concepts, i.e., linear permutations, permutations of similar things, and circular permutations, which are briefly described as follows:

A *linear permutation* is an arrangement of objects or people in a line.

A *permutation of similar things* is an arrangement of objects, some of which are identical. Permutations can be distinguishable if one cannot be obtained from another simply by rearranging the positions of the elements of the same kind.

A *circular permutation* is an arrangement of objects or people in a circle. There are two cases of circular-permutations: (a) If clockwise and counterclockwise orders are different, then number of circular-permutations is given by (n-1)!, (b) If clockwise and counterclockwise orders are taken as not different, then number of circular-permutations is given by $\frac{(n-1)!}{2!}$.

Combinations

A combination means a selection of things. The word selection is used, when the order of things has no importance. Another definition of combination is an arrangement of objects in which the order is not important. The number of combinations of n objects taken r at a time is written as $C_{n,r}$.

When to use it: All objects are chosen from the same group, and the order does not matter, meaning that selecting the same set of objects in a different order will give you the same result.

Warning: Here are some situations where the order does matter:

- choosing a committee of people where they all have the same job.
- choosing a group of objects, where you don't care what order they are in.

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How to use it: Total number of ways = $C_{n,r} = \frac{n!}{(n-r)!\cdot r!}$

where n = the number of possible choices

and r = number of things you choose

Permutations are similar to combinations but extend the requirements of combinations by considering order.

In a permutation and combination lesson, permutation and combination problems are suitable for students who are at an appropriate age to learn because some of them involve complicated conditions and many difficult-to-understand situations. Piaget and Inhelder (1951, cited in Jones, 2005) suggested that "Permutations and combinations are operations on operations. And for permutations, it is necessary to wait until students are 15 years old". Garfield and Ahlgren (1988) contended that before the teaching of probability, students must have an understanding of ratio and proportion. Students must be able to function at the formal operational level. They must have the necessary skills in dealing with abstractions. However, Fischbein and Grossman (1997, cited in Jones, 2005) showed that "Combinatorial problem solving capacity is not always reached, not even at the formal operations level, without specific teaching".

In a normal mathematics class, most teachers teach the concepts step by step, without repeated teaching of the previous concepts, and lack the necessary review of prior knowledge, concepts and computing skills. So some students have inadequate background knowledge and computing skill. Students have partial viewpoints and cannot understand the relationship among sections. They cannot link concepts together. It makes them unable to solve the tortuous problems. In addition, permutation and combination activities do not support students' understanding. Students cannot construct new knowledge and new conceptions by themselves. So they cannot associate the previous concepts with the new one. Moreover most students do not clearly understand the concepts.

Many research studies indicated that the teachers use few visual representations for teaching permutations and combinations and they take short time and provide few examples for each visual representation. So students are unaccustomed to using visual representations and they cannot select the correct visual

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representation for a problem. According to Novick and Hmelo (1994), having an appropriate problem representation does not guarantee that the problem can be solved and Novick showed that students are not always successful in finding correct representations for problems. Moreover, Yanze (2002) explained that students lack experience in problem solving. Teachers have to provide random experiments and representations of the real phenomena, and increase the number of repetitions.

The real-life problems can be used to develop conceptual understanding and help students link those conceptual understanding to the formulas. In addition they should also enhance students' ability in problem solving. Many research studies indicated that the instruction should be a process of extracting the suitable concept from a concrete or real-world situation with which students are familiar. The situations which teacher cautiously designed for using in mathematics teaching could enable students to experience authentic problem solving. Teachers should select situations which challenge students with new information for creating the new knowledge and students should discover the production of new knowledge using existing mathematical knowledge. Moreover, mathematical explanations or interesting results about realistic situations have a great impact on students and enhance students' learning (Johnson, 1991; Abramovich & Pieper, 1996; Nunokawa ,2005; Lee, 2006).

Because of the teachers' instruction and their activities affect students' errors in permutations and combinations; it is necessary for effective secondary mathematics teachers to have an awareness of the students' mathematical difficulties. Teachers need to understand and foster them through learning the concepts behind the mathematical procedures and they have to design and conduct effective mathematics lessons for facilitating mathematics learning. Teacher should search for effective and alternative learning approaches for helping students construct and understand concepts (Khoh, cited in Lee, 2006). Teachers need to recognize and confront common errors in students' probabilistic reasoning (Fischbein, 1987; Konold, 1991). It is important to make students aware of how conceptions can affect probabilistic judgments. Moreover, Fischbein (1975) and Jones (2005) have established that children are responsive to appropriate instruction on probability concepts.

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2.2 Students' Errors in Permutations and Combinations

The Institute for the Promotion of Teaching Science and Technology (IPST) provided highschool teachers with developing and training programs in order to improve teachers' instruction and point out students' misconceptions in mathematics including probability since 2007. Moreover, in world mathematics education movements, there are many projects which aim at improving, developing and reforming mathematics education, including probability, at the secondary level.

Many research papers pointed out that permutations and combinations, which are the important mathematics contents, are difficult for students to understand and their problems difficult to solve by indicating students' errors in permutations and combinations as follows (Riley, Greeno, & Heller, 1983; Novick & Hmelo, 1994; Batanero & Sanchez 1997, cited in Jones, 2005; Lee 2006).

1. Error of order: students are confused with the criteria for permutations and combinations. Permutations are the possible arrangements where the order chosen or the position arranged is important. Combinations are arrangements of elements without considering their order or positions. Most students often fail in analyzing the problems. They cannot classify whether a problem is permutative or combinative.

2. Error of conceptions: students' misconceptions in permutations and combinations get into students' problem solving. For example, the students indicate that the number of ways to draw a (6, 6) on a pair of dices is equal to the number of ways to draw a (5, 6) (Ben-Hur, 2006). Another example is that students do not consider the possibility of repeating the elements when it is possible, or students repeat the elements when there is no possibility to do so. Students are also confused about identical objects being distinguishable or different objects being indistinguishable. Moreover students exclude some elements to form the configurations; they only produced the permutations of the three letters A, B, and C when asked to find all the permutations of the four letters A, A, B, and C.

3. Error of computing: students are unable to solve problems because they have no idea how products can grow or decay in counting problems which is one of the contents in probability. Students try to solve the problem by listing nonsystematically using trial and error. In addition students lack computing skills. They do not know how to calculate the permutations or combinations.

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4. Error about formulas: students cannot remember the correct formula for a combinatorial operation that has been correctly identified or students can not remember the meaning of the values of parameters in a combinatorial formula.

5. Error of using visual representations: students lack appropriate schemas and have poor arithmetic skills. Students are not always successful in finding correct representations for problems. In permutation and combination problem solving, some weaker problem solvers choose many external visual representations which lead to a lower rate of success, and they often carry out correct arithmetic procedures on incorrect representations of the problems. Some students have faulty interpretations of the tree diagram, either producing an inadequate diagram or incorrectly interpreting the diagram produced.

These errors specified students' misconceptions in permutations and combinations and pointed out the reasons those students were not successful in permutation and combination problem solving. Moreover, the research showed that both school children and adults held intuitions about probability. These problematic intuitions may explain why probability seems to be difficult to learn (Kahneman, Slovic & Tversky, 1982; Konold, 1989; Papaieronymou, 2006). So teachers have to know students' nature of mistakes and the causes of students' errors. Teachers have to understand the thinking of students in permutations and combinations in order to prevent and eliminate those errors. They also have to know how to support students' further learning and to improve students' conceptual understanding and ability to solve the problems. Moreover, Pittman, Koellner & Brendefur (2007) said that since teachers are as likely to exhibit some of the same misconceptions as their students, there is a strong need for teachers to think deeply about the mathematics of the task before implementation.

2.3 Nontraditional Word Problems and Probability Learning

Probability has come to gain importance as a content area that students need to have experience with in order to be well-informed citizens since its study "can raise the level of sophistication at which a person interprets what he sees in ordinary

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life, in which theorems are scarce and uncertainty is everywhere" (Cambridge Conference on School Mathematics, 1963, p.70; as cited in Jones, 2004).

In 1983, the National Commission for Excellence in Education (NCEE, 1983) published *A Nation at Risk*, a report aimed at pointing out the immediate need for reform in education, with the suggestion that highschool graduates understand elementary probability and be able to apply it in everyday life.

Furthermore, in his publishing, *Teaching secondary school mathematics: A Resource Book*, Lee (2006) indicated that the techniques of probability can be used to describe the organization of subatomic particles, how genes are transmitted from parents to children, and the models of the AIDS epidemic. Probability models can be used to evaluate life assurance and annuities, to set the premiums and contributions fairly, and to estimate the risk of granting credits in a bank. In addition, probability influences forensic evidence in court cases. However, Lee's book contains no such problems.

However, many researchers used the nontraditional word problems to enhance students' understanding of probability. The examples of nontraditional word problems in several research papers and their implementation are described as follows:

Anno's Hat Tricks problem

Anno's Hat Tricks problem, by Nozaki (1985, cited in Bay-Williams & Martinie, 2004), is used to explore probability by asking students to figure out the probability that shadow-child's hat is red in the different hat scenarios in the book (e.g., one white and two reds). Students refigure the probability that shadow-child's hat is red after seeing what color another child is wearing. In class, the tree diagram is used to illustrate the possibilities for 2 children. Students have to discuss how the tree diagram can be used to find probabilities. In groups, students have to create a new tree diagram for the new situations (e.g., tree whites and two reds for shadow-child and other 2 children).

Bay-Williams & Martinie (2004) explained that after students have solved these problems, they can share their answers and strategies such as a tree diagram or a list of all the possible combinations. The students can explain how the tree diagram shows all the different possibilities.

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My little sister ate one hare problem

"My little sister ate one hare" is the problem in Bill Grossman's counting book (1996, cited in Bay-Williams & Martinie, 2004). The situation in this problem is the story of a brother who tells the story of his sister's diet. First, she eats one hare; next she eats two snakes, then tree ants and so on. Her eating pattern continues until she eats ten peas and then she will throw up all the things she has already eaten. In class, students do an experiment with selecting food items (on cards). After the experiment, students have to determine the theoretical probability. Moreover, Bay-Williams & Martinie explained that students can do similar experiments related to grouping of food items, for example, the probability of selecting four-legged animals.

The Phantom Tollbooth problem

"The Phantom Tollbooth" is a novel created by Norton Juster (1961and 1989, cited in Bay-Williams & Martinie, 2004). This novel contains many options for launching mathematical investigations. Bay-Williams & Martinie had the idea from this novel to create a permutation activity as follows. In the activity with funny stories, students have to write down a tree-word sentence, such as "Dogs eat bones." Students find the number of three-word sentences if the words can be in any order. Students should have some organized way of listing the word combinations. Then students have to repeat this process with four-word and five-word sentences. Students are challenged to search for a shorter process for finding the number of possible combinations than writing all the word arrangements. This activity introduces factorial symbol used for finding permutations.

In addition, Sriraman (2004) used problem-solving situations to help improve students' mathematical reasoning, abstraction, and generalization. He used a set of combination problems in his study. The set is composed of the dinner problem, the schoolgirl problem and the prisoner problem. He said that "A traditional highschool curriculum has very little emphasis in the area of combinatorics". He highly recommended problem-solving situations because they are easy to understand and students can success by trial-and-error strategies.

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A good problem can be the effective learning vehicle. It can develop students' learning more quickly than traditional instructions (Kahan & Wyberg, 2003).

2.4 Sport and Probability Learning

Nowadays, most textbooks and educational websites always created the sport problems/settings involving probability in the traditional way. For example:

- A soccer club will appoint a president and a vice-president. Eight people have volunteered for either of the two positions. In how many different ways can these positions be filled?

- A cricket team of 11 players is to be selected, in batting order, from 15. How many different arrangements are possible if there are no restrictions?

- The starting 5 in a basketball team is to be picked, in order, from the 10 players in the squad. In how many ways can this be done if John needs to play at number 5?

- A mixed netball team must have 3 women and 4 men in the side. If the squad has 6 women and 5 men wanting to play, how many different teams are possible?

- A rugby union squad has 12 forwards and 10 backs in training. A team consists of 8 forwards and 7 backs. How many different teams can be chosen from the squad?

- In an eight-team football conference, each team plays every other team exactly once. How many games must be played?

- A sportswriter makes a preseason guess of the top 5 football teams (in order) from among 40 major teams. How many different possibilities are there?

- Theoretically, how many possibilities are there for first, second, and third places in a marathon race with 1000 entries?

- A bag contains 4 white softballs, 2 pink softballs, 3 blue softballs, and 5 red softballs. You reach into the bag without looking and take out a ball. You replace it and then take out another ball at random. Find the probability that the first ball is red and the second ball is white.

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- A manager is choosing his infield from among 3 third-base players, 2 shortstops, 4 second-base players, and 5 first-base players. How many different ways can an infield are chosen?

- Ann, Cat, and Dream are among 15 players competing for 3 spots on a table-tennis team. Every player has an equal chance of making the team. Find the probability that all three will make the team.

- How many ways can a batting order be made for 10 players if you know the one player has already been designated to bat first and another to bat fourth?

- A golf bag pocket contains 4 pink golf balls, 2 white balls, 3 blue balls, and 1 red ball. You pull out one ball at a time, without replacing it. Find the probability that the first ball is red, the second ball is white, and the third ball is pink.

These sport problems/settings were some examples of a set of questions in a book that test students' knowledge or skill in fundamental counting principle and simple linear permutations and combinations. However, these sport problems/settings did not mention the sport scoring or sport rules in order to use to teach or support students to get deeper understanding and to derive the probability formulas.

From many research studies, the researchers tried to decrease the difficulties in teaching and learning of probability and tried to increase the conceptual understanding of permutations and combinations. In terms of using sports to facilitate learning of probability, most research was geared more toward a college-level statistics course rather than a highschool-level probability course (Albert, 2002; Cochran, 2005; Gelman & Nolan, 2002; Smeeton, 2003).

Greer (2001) studied probabilistic thinking, in particular its development in children, based on Fischbein (1975)'s works. He developed effective instruction in concepts central to modern models of scientific and social phenomena. He studied about the role of intuition in mathematical and scientific thinking, the development of probabilistic thinking, and the influence of instruction on that development. The results from the study showed that Fischbein's pioneering work can be built upon, in particular for improving the teaching of probability in schools.

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Ritson (1998) found that weak understanding of probability concepts imposed limitations on children's ability to make probabilistic judgments. Kahneman

and Tversky (2001) created the problems to enhance students' probabilistic judgments. One of those problems was a sport problem: "A game of squash can be played either to 9 or to 15 points. If you think you are a better player than your opponent, which game gives you a higher chance of winning, the short game, or the longer game? Suppose, instead, that you are the weaker player, which game gives you a higher chance now? The results indicated that the sport problem can led to good judgments.

The secondary students should explore situations by experimenting with and simulating probability models, construct sample spaces in the attempt to determine probabilities of "realistic situations", and appreciate the use of probability in the real world. Particular to grades 9-12, recommendations included the understanding of the difference between experimental and theoretical probabilities, theoretical and simulation techniques for computing probabilities, and interpreting discrete probability distributions (NCTM, 1989).

In terms of using sports to facilitate learning of probability, most research was geared more toward a college-level statistics course rather than a highschool-level probability course. For example:

Albert (2002) used baseball perspective to teach statistics and probability in an introductory statistics course. Topics in data analysis, including methods for one batch, comparison of batches, and relationships, are communicated using current and historical baseball data sets. Probability is introduced by describing and playing tabletop baseball games. Inference is taught by first making the distinction between a player's ability and his performance, and then describing how one can learn about a player's ability based on his season performance. Baseball issues such as the proper interpretation of situational and streaky data are used to illustrate statistical inference. Albert concluded that the course was fun for both the instructor and the students. The baseball situation provided an interesting context to learn about statistical thinking. Baseball material seemed to have a positive impact on the learning. In a student evaluation given at the end of the course, students overwhelmingly said that the course was useful. This comment doesn't mean that the students will use what they learned about baseball in their future work. Rather, it meant that the students could make sense of the statistical material since it was taught from a baseball perspective. In addition, he said that "The positive experience in this class suggests that we should encourage

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alternative models for teaching statistics. We should explore ways or contexts to engage students so they can make more sense of statistical thinking."

Besides, Cochran (2005) used of sports board games to introduce or illustrate a wide variety of probability concepts to introductory statistics students in an integrated manner. He demonstrated the use of a single game (Strat-O-Matic® Baseball) to introduce probability distributions, sample spaces, the laws of addition and multiplication of probabilities, independence, mutual exclusivity, randomization and independence, conditional probability, and Bayes' Theorem. Empirical and anecdotal evidence suggested that student comprehension and retention were enhanced by use of examples constructed from the simple and interesting contexts provided by a sports board game.

Gelman and Nolan (2002) derived a model, using trigonometry and the Normal distribution, for the probability that a golf putt is successful. They described a class activity in which it lead the students through the steps of examining the data, considering possible models, constructing a probability model and checking the fit. The models, of necessity, oversimplified a point which the class discussed at the end of the demonstration.

Smeeton (2003) used a football problem to teach probability. The issue of whether football teams have good or bad patches during the course of a season is of concern to many supporters. The interest can be harnessed to make the classroom teaching of probability more relevant.

In addition, the growth of sports betting has seen commercial application for the predictions and methods, and has led to the development of a subject 'Chance and Gaming', where probability is taught through applications in the gaming industry. Clarke (2003) used the web, www.swin.edu.au/sport, to create interest in statistics. The website was used by teachers as a source of motivation, data or practical problems. They are composed of sporting events based on statistical modeling and simulations such as the football, tennis, and cricket situations in order to raise students' interest. Clarke pointed out that the site received over 4,000 visits per week in 2002 and has raised interest in statistical methods in the general public and students.

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Besides, Schwertman, Mccready, and Howard (1991) suggested that the sports events and tournament competitions provide excellent opportunities for model

building and using basic statistical methodology in an interesting way. They used National Collegiate Athletic Association (NCAA) regional basketball tournament data to develop simple linear regression and logistic regression models using seed position for predicting the probability of each of the 16 seeds winning the regional tournament. The accuracy of these models was assessed by comparing the empirical probabilities not only to the predicted probabilities of winning the regional tournament but also the predicted probabilities of each seed winning each contest.

The previous data suggested that if students could learn mathematical contents from real-life problems or daily life situations, they would get more conceptual understanding, visualization and realization that are useful in mathematics learning.

2.5 Probabilistic Games and Probability Learning

The research studies starting with the work by Piaget and Inhelder (1951) first translated by Leake, Burrell, and Fishbein (1975), studied the development of the idea of chance in children. According to them, the concept of probability as a formal set of ideas develops only during the formal operational stage, which occurs at about twelve years of age. By that age, children can reason probabilistically about a variety of randomizing devices. In an experiment to demonstrate that children have an intuitive understanding of the law of large numbers and that intuitive thinking about chance events starts even before they are taught, they used a game with pointers which were stuck onto cards divided into various sectors and then spun. They found the children could predict that, in the long run, the pointer would fall onto every region marked on the card.

Pittman, Koellner, and Brendefur (2007) pointed out that “Twelve middle school teachers participated in a professional development focused on improving their content and pedagogical content knowledge of probability. A key part of the professional development was the planning, implementation and analysis of a rich probability task and the maze problem.”

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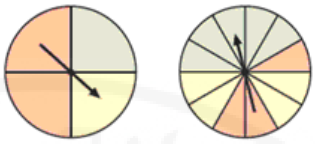
Because many research studies pointed out those games have good effects on students’ understanding, especially in mathematics, there are a lot of games in

mathematics instruction. The examples of probabilistic game settings (without playing these games) in the practice books are described as follows:

- Both spinners are spun, Find the probability that both spinners are red.



- You spin a spinner with 8 equal sectors, numbered 1 – 8. What is the probability of spinning a number that is neither odd nor greater than 6?
- You roll a pair of dice. Find the probability that both dices are even.
- A box contains 100 cards, numbered from 1 – 100. Two cards are taken at random from the box, one at a time, and not replaced. Find the probability that the first number is less than 40 and the second number is more than 80?
- You draw a card at random from a standard pack of playing cards. Find the probability that the card is a club or a jack.
- You select a marble from this jar without looking. You know $\frac{1}{5}$ of the marbles are red and $\frac{1}{4}$ are blue. What is the probability that you will select neither a red nor a blue marble?

There are a lot of probabilistic games used in secondary probability lessons. However, they are used to introduce simple probabilities. For example:

Dice game:

Instruction: Play this game with a partner. Use a pair of 6-sided dices.

1. Player A rolls first. If player A rolls a 7, player B wins the game. If not, player B rolls.

2. If player B rolls a 7 or an 11, player A wins. If not, it is player A's turn. Continue taking turns until there is a winner.

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3. Play the game several times. Do you think one player has the better chance of winning?

In addition, there are lots of online probabilistic games in several websites. For example:

In Between Waters website, www.betweenwaters.com/probab/, it is composed of 5 online probability games, i.e., coin flip, dice roll, key problem, Monty hall puzzle, and coin game. This website described about the probabilistic games that they are enjoyable while exploring the concepts of probability. Their program automatically performs the number of flips the player specified and updates the displayed results. The problems of the games are described as follows:

Coin Flip: A coin has two sides: heads and tails. When you flip it, what are the chances it will come up heads? What are the chances it will come up tails?

Dice Roll: A die has six sides, numbered one through six. When you roll the die, each side has an equal chance of coming up. When you roll two dice, the total shown can be anywhere from two to twelve. When you roll three dice, the totals can range from three to eighteen. What are the chances of each total occurring?

Key Problem: In an imaginary contest, seven people have the chance to win a car. Out of seven keys, one will start the car. The players take turns, each selecting a key and trying it in the car. When a player chooses the correct key, he or she wins the car and the contest is over.

Monty Hall Puzzle: A game show presents a contestant with a choice of three doors. Behind one door is a new car. Behind the others are goats. The host knows where the car is and has scripted the scenario in advance:

Coin Game: The Problem: You and your opponent each have a coin. In each round you each choose to show either the heads or tails side of the coin. If both coins show heads, your opponent pays you $3. If both show tails, she pays you $1. If they don't match, you pay her $2. Is this a fair game?

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In *Demos with positive impact family of projects* website *(http://mathdemos.gcsu.edu/mathdemos/plinko/*), Plinko, the most popular game of chance on the television game show "*The Price is Right*", is illustrated how mathematics is useful for predicting a strategy that gives the highest probability for a big win. In highschool level, the students use the Plinko board to motivate a study of probability and use the board and calculator simulations to find experimental probabilities. The students can explore the board in more detail. For example, counting the number of paths from each slot and use path counts to make predictions on likelihood a chip will fall into a particular slot. Smaller Plinko boards are useful to facilitate a discussion of tree diagrams and using tree diagrams to compute theoretical probabilities. A complete discussion of the big Plinko board is probably not appropriate for average classes at this level.

Besides, this probabilistic game is in the secondary practice book. In addition to many research studies, the researchers used games to enhance their students' understanding. The examples of probabilistic games and its implementations are described as follows:

Krulik, Rudnick & Milou (2002) presented the unusual ways of presenting usual topics in the middle school level. There are 3 probability lessons out of 29 lessons. The probability lessons consist of 2 games called A Game of Chance # 1 and A Game of Chance #2.

A Game of Chance #1

In this game, each pair of students has two different spinners. The first spinner has 3 numbers (4, 6, 8) and the other has 2 numbers (5, 9). They are going to spin the two spinners. Whoever has the spinner with the higher number scores one point. The winner is the person who has the higher point score after spinning the spinner a total of 20 times.

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This game can lead students to a discussion of the concept of a fair game. It presents the concept of sample space and tree diagram. Moreover it can explore probability in an enjoyable setting.

A Game of Chance #2

This lesson provides students with an opportunity to make some conjectures about probability when rolling a pair of dices. It can lead students to a discussion about sample space and the number of permutations when the conditions of the game change, such as changing the results from adding the two numbers that appear to subtracting them or increasing the number of dices.

In 2003, Baker (2003) used FreeCell to teach probability. He explained that he used FreeCell to stimulate interest in mathematical concepts and to examine combinatorics in a special topic class.

In 2004, Batanero, Godino & Roa (2004) used a probabilistic game in training teachers to teach probability. The game is based on the role of problem solving in the construction of mathematical knowledge. The authors said that the game reflects on the concepts of dependent experiments and conditional probability.

However, in terms of using probabilistic games to facilitate learning of probability, most research was geared more toward a middle-school-level rather than a high-school-level probability course.

In game problem instructions, students enjoy the gaming experiences and the experiences can provide students with opportunities to analyze difference solution strategies. Hence games can be the effective tools in teaching probabilistic thinking.

Educational Review

2.6 Inquiry-Based Learning and Teaching in Mathematics Education

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An inquiry-based learning and teaching means teaching and learning strategy wherein the teacher's role is being a facilitator and the learning unit focuses on central concepts. This strategy is a well known technique to promote students'

investigation and development of their concepts relevant to everyday life. Students are provided with the opportunity to explore and experience a confrontation to their own way of thinking. It emphasizes that the students should be able to authentically investigate problems, raise notions, interpret the results by themselves, and plead their own statements to show their conceptual understanding in the process of inquiry.

Chin, Lin, Chuang and Tuan (2007) indicated that a large-scale international achievement survey revealed that Taiwanese students were with high performance in mathematics but weakness in higher order thinking skills. The main reason might be most schools in Taiwan tend to remain a conservative pedagogy with behaviourist paradigms to teach mathematics. Besides, they suggested that the students should develop their mathematics knowledge through inquiry-based teaching. Because the inquiry-based learning and teaching provides a more meaningful and effective way for students to learn, the students could develop significantly better metacognitive capacities after receiving the inquiry-based mathematics teaching.

The inquiry-based teaching and learning is relied mainly on the pedagogical approach that leads the way to develop students' conceptual understanding. Its emphasis on the process of questioning, investigating, and problem solving is consistent with the process that the scientists do (Olson & Loucks-Horsley, 2000; Bybee, 2004; DeBore, 2004; Sandoval 2005). Moreover, this alternative approach provides a more hands-on, student-centered method and allows them to make connections with prior learning and personal experiences (Wang, Chin, Hsu & Lin, 2008).

As a result of mathematics reforms, mathematics teachers were encouraged to adopt different planning practices. Using the features of the teaching for understanding framework (Blythe, 1998; Wiske, 1998) and, where possible, planning cross-curricular and collaborative units were encouraged. This was followed by a need for teachers to closely match purposes with appropriate teaching and learning activities, with an emphasis on inquiry and reflective thinking. The inquiry can bring about increases in student mathematical outcomes and in their capacity to explain mathematical thinking. The emphasis on mathematical thinking in a situation where students were constructing their own knowledge appeared to be very powerful in

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assisting student understanding of the mathematical concepts involved (Brown, Wilson, & Fitzallen, 2007).

The teachers should provide the students with the experiences based inquiry in which they are able to work as young mathematicians or researchers. The students should have ample opportunities to utilize the inquiry cycle in carrying out their own mathematical investigations so that they are actively engaged in the construction of knowledge (Carter, 2004). Though inquiry-based teaching strategies typically engage students in investigations, it is not the physical activity that defines inquiry. Teaching through inquiry is distinguished by its emphasis on a questioning attitude, gathering data, reasoning from evidence, and communicating explanations that can be justified by available data (Haury, 2001). The inquiry-based teaching can promote a deeper conceptual understanding. One of the strong influences on teachers' enactment of inquiry-based teaching is the degree of coherence. The inquiry approach emphasizes students' ownership of their learning, their engagement with problems and open-ended investigations, and their learning to reason with real-world data (Falk & Drayton, 2000).

Inquiry-based mathematics teaching can be a useful guide for helping teachers to make the most of learning experiences for students. Therefore, we might be able to argue that the inquiry-based mathematics teaching method may serve as a catalytic metacognitive experience that informed students about what was for some an alternative conception of learning (Chin, Lin, Chuang, and Tuan, 2007).

In inquiry-based mathematics learning, the students were asked to personally construct their own understanding by posing questions and considering how investigations will proceed and how findings are analysed and communicated (Hinrichsen & Jarrett, 1999).

There is documented and compelling evidence that suggests providing students with participatory and inquiry-driven mathematical opportunities that highlight reasoning and problem-solving skills allows them to develop conceptual understandings of mathematical ideas that are often neglected by traditional instructional approaches (Hiebert & Lefevre, 1986; Hiebert & Carpenter, 1992; NCTM, 1989, 2000; Owens, 1993; Wenglinsky, 2002).

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In the inquiry-based mathematics learning, listening, discussion and explaining mathematical thinking are important characteristics of an inquiry classroom. The use of inquiry approach can develop students' mathematical thinking. (Brown, Wilson, & Fitzallen, 2007). The students have their own prior knowledge. They have to make sense of the world through an existing conceptual structure, so whatever knowledge is constructed by the students, this will be an interpretation of experience in terms of extant knowledge (Tobin & Tippins, 1993).

For most teachers, however, the shift in emphasis from traditional instructional approaches to inquiry is a difficult transition. The focus is shifted away from merely learning about science to doing it, with time set aside for the collection, discussion and analysis of data (Falk & Drayton, 2000).

Guided-inquiry is the instructional method that supports students' inquiry by guiding one or more steps to complete the task (Sandoval, 2005). In guided-inquiry, the teacher posts the questions and provides materials for the students to address the problem. Then the students devise their own solution and come up with the conclusion by themselves (Colburn, 2000). The g*uided-inquiry activity* is composed of a set of inquiry-based activities in which students perform investigations to answer the questions posted by the teacher. The main responsibility of the students is figuring out how to answer the given questions (NRC, 2000). The g*uided-inquiry* can help students make the transition from learning through a structured inquiry stage to an open inquiry level. Besides, it can develop students' conceptual understanding (White & Frederikson, 1998)

According to the National Research Council (NRC, 1996), the students at all grade levels and in every domain of science should have the opportunity to use scientific inquiry and develop the ability to think and act in ways associated with inquiry, including asking questions, planning and conducting investigations, using appropriate tools and techniques to gather data, thinking critically and logically about relationships between evidence and explanations, constructing and analyzing alternative explanations, and communicating scientific arguments.

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An inquiry-based teaching and learning has been advocated by many mathematical researchers in many features such as having the potential to provide for

a diversity of learners, in terms of engagement and challenge, and using interesting and authentic tasks (Carter, 2004; Diezmann, 2004; Heibert, 2003; Lovitt, 1999; Schoenfeld, 2002; Stein, Grover & Henningsen, 1996; Sullivan, 2001). In an inquiry mathematics classroom, students are actively engaged in the construction of knowledge (Carter, 2004). The inquiry approach engages students and suits a range of student capabilities. It also provides authentic opportunities for teamwork and co-construction of knowledge (Diezmann, 2004).

Teaching through inquiry can take many forms, with most descriptions of inquiry emphasizing investigations. According to the National Science Education Standards (NSES) and the National Science education Standards (NRC, 2005; Olson & Loucks-Horsley, 2000), inquiry-based teaching and learning is composed of five essential features: (i) the students are engaged in challenging questions, (ii) the students give priority to evidence, which allows them to develop and evaluate explanations that address scientifically oriented questions, (iii) the students formulate explanations from evidence, (iv) the students connect explanations to scientific knowledge, and (v) the students communicate and justify their proposed explanations. It shows that the inquiry-based learning and teaching are related to the students' problem solving.

2.7 Inquiry-Based Approach and Mathematical Problem Solving

In a mathematics class, the problem is the one important factor in students' achievement and attitude because it has an effect on mathematics teaching and learning. In an inquiry-based learning and teaching, problem solving is central to instructional procedure with minimal guidance. Because the inquiry-based learning is based on answering questions or solving problems, asking and posing questions are the heart of the inquiry approach to learning. **(**Southwell, 2004). It is the important role for the teachers to develop strong questions/problems that are applicable to the lessons. Moreover, the students receiving instruction with real-life problems achieve significantly higher on immediate and delayed posttests, and retain their knowledge longer (Fan & Zhu, 2007). Besides, the inquiry approach motivates and challenges

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students to solve an authentic problem in a way that may help students understand themselves and the natural world (Chiapetta, 1997; Sadeh & Zion, 2009).

The inquiry-based teaching should focus on: (a) comprehending the problem; (b) constructing connections between previous and new knowledge; (c) considering strategies appropriate for solving the problem; (d) reflecting on the processes and solution. Chin, Lin, Chuang and Tuan (2007) suggested that the focus in inquiry-based teaching was effective in promoting students' development. In their interviews, the students pointed out that they had more opportunities to try different approaches to a problem and the discussion can also inspire them with new ideas in the inquiry teaching.

Additionally, several researchers encourage using constructivist theory to teach mathematics in which students construct mathematical knowledge through exploring, reasoning, or problem solving (Draper, 2002; Grant, 1998; Kim, 2005; Noddings, 1993).

Researchers have documented the success of alternative approaches to teaching mathematics. For example, students performed better on achievement tests when teachers provided inquiry-driven, hands-on learning opportunities and focused on higher-order thinking skills (Wenglinsky, 2002). Likewise, mathematical manipulatives, often referred to as hands-on learning, can play an instrumental role in assisting students to develop a conceptual understanding of concepts and skills (Sowell, 1989). Researchers have documented that instruction that centers on the use of manipulatives produces greater mathematical gains and achievement as compared with instruction not using them (Fuson, 1992; Fuson & Briars, 1990; Suydam & Higgins, 1977; Wearne & Hiebert, 1988).

2.8 Theoretical Framework of This Study

An inquiry approach provides a more meaningful and effective way for students to learn. Students should be provided with the opportunity to explore and experience a confrontation to their own way of thinking. This strategy is a well known technique to promote students' investigation and development of their concepts relevant to everyday life. Students could develop significantly better metacognitive

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capacities after receiving the inquiry-based mathematics teaching (Chin, Lin, Chuang, and Tuan, 2007). Asking and posing questions are the heart of the inquiry approach to learning. So education in general is enhanced when the process of asking questions is encouraged. **(**Southwell, 2004)

Inquiry-based learning is based on answering questions or solving problems; the teacher has to develop strong questions/problems that are applicable to the lessons. Furthermore teachers should also provide certain kinds of experiences in which students are able to work as young mathematicians or researchers. Students should have ample opportunities to utilize the inquiry cycle in carrying out their own mathematical investigations when learning mathematics. The essential traits of inquiry in learners, according to some reports (e.g., AAAS, 1994; Hinrichsen & Jarrett, 1999), are as follows: connecting former knowledge and experiences with the problems, designing procedures (plans) to find solutions to the problems, investigating phenomena through conjecture, and constructing meanings through use of logic, evidence, and reflection (Chin, Lin, Chuang & Tuan, 2007). Inquiry-based mathematics teaching is a warranty for student metacognition development. In an inquiry mathematics classroom, students are actively engaged in the construction of knowledge (Carter, 2004). Listening, discussion and explaining mathematical thinking are important characteristics of an inquiry classroom as is a climate of cooperation between the teacher and students, creating a community of learners (Brown, Wilson & Fitzallen, 2007). In a mathematics class, the problem is the one important factor in students' achievement and attitude because it has an effect on mathematics teaching and learning. Students receiving instruction with real-life problems achieve significantly higher on immediate and delayed posttests, and retain their knowledge longer (Fan & Zhu, 2007).

Difficulties in teaching and learning of probability have been well documented. To partially overcome this problem, we let students tackle some probability questions from nontraditional word problems, popular sports, and probabilistic games before introducing the formulas. We put more emphasis on counting of possibilities to make the learning experience more realistic. Learning outcomes and retention by students from teaching the topics in an appropriate pedagogical sequence are encouraging.

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2.9 Summary of the Chapter

According to the literature review, an inquiry-based approach is one of the approaches that is foremost in learning and teaching mathematics. In learning mathematics, practical works or activities provide beneficial opportunities for students to develop critical thinking, to develop mathematical process skills, and to enhance achievement as well as to promote positive attitudes toward mathematics. In the next section, methodology and methods are discussed in order to frame the study.

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CHAPTER III
METHODOLOGY

Overview

This chapter describes the methodology and methods used to conduct the research to answer the research questions posed earlier. It begins with the participants involved in this research. Then the development of the instructional units used for conducting this research and the research methodology are described. Next, the research instruments used for the evaluation of the instructional units are specified. Finally, the data analyses are described.

3.1 Methodology

Study Design

The instructional units based on inquiry strategy were developed for enhancing highschool students' achievement in permutations and combinations. The instructions were designed in such a way that the teacher had opportunities to encourage students to inquire, search, investigate, and construct knowledge by themselves.

The development the instructional units in permutations and combinations:

1. Develop the four instructional units: traditional instructional unit, nontraditional word problem instructional unit, sport problem instructional unit, and probabilistic game instructional unit.

2. Consult with curriculum experts to improve the instructional units.

3. Revise the problems in each unit according to suggestions of the experts.

4. Pilot the instructional units with highschool students.

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5. Revise the traditional instructional unit, the nontraditional word problem instructional unit, and the probabilistic game instructional unit according to comments and suggestions of teachers and students.

3.2 Participants

This research involved 149 grade-eleven students aged between 16–17 years from four classes in a school in the rural area of the country. These students were enrolled in an applied probability course during the second semester of the academic year 2009. The details of the four classes were showed as follows:

Table 3.1 The details of the four classes of participants

Class	Boys	Girls	Total	The average previous-semester mathematics scores (%)
1	10	29	39	78
2	9	31	40	69
3	17	13	30	68
4	13	27	40	59

In each class, the mean from the latest mathematics score of each student was used to consider the method to select the control group and 3 treatment groups. From statistic test, the differences were significant, all 4 classes were ordered by their means. The best one was selected to be the control group and the next classes were treatment group I, the treatment group II and the treatment group III respectively. The control group received the traditional instructional unit. The three treatment groups received the nontraditional word problem instructional unit, sport problem instructional unit and probabilistic game instructional unit respectively ordered by their average previous-semester mathematics scores. The most motivating instructional unit, probabilistic game instructional unit, was used for the group who had the least average previous-semester mathematics scores and the next group

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received the sport problem instructional unit and the best one received the nontraditional instructional unit respectively.

3.3 Development of the Instructional Units for Highschool Students

Four different inquiry-based instructional units on permutations and combinations with different types of mathematical problems were developed. They are (i) traditional unit (TU), (ii) nontraditional word problem unit (NTU), (iii) sport problem unit (SU), and (iv) probabilistic game unit (GU). Four concepts were incorporated into each unit: linear permutation, permutation of similar things, circular permutation, and combination.

Traditional unit's contents and exercises were created by collecting exercises, examples, and practice problems from national standard textbooks for highschools, exercise books, and educational websites. The nontraditional, sport, and game problems in the respective instructional units were created by us. All units had the same number of sessions, including ones for drills and problem-solving. Details of the four instructional units are described below.

3.3.1 Development of the Traditional Instructional Unit

The traditional instructional unit was created in the form of worksheets. It contained the basic contents found in common textbooks and supplemented with similar problems from other sources.

3.3.2 Development of the Nontraditional Word Problem Instructional Unit

The emphasis of this unit was on real-life probability word problems designed to engage students' interest.

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Eight nontraditional word problems were created to teach possibility and probability for highschool students

Problem 1: Sentence-Maker Problem

Arrange all these English words into a sentence that has meaning.

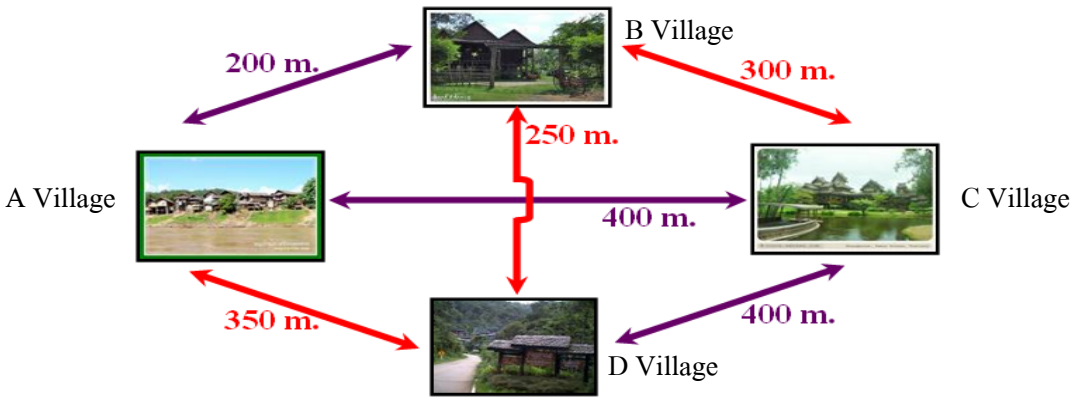
HOME WANT I TO GO DO

What is the probability of getting a meaningful sentence provided that the words are arranged randomly?

The problem is used to introduce linear permutation. It provides an enjoyable problem for students to solve, compared to a traditional one. (For example, find the number of possible arrangements of six different letters.) Students use their common sense to get the size of the sample space. Moreover, they should be able to derive the linear permutation formula when deleting or inserting words.

Problem 2: Travel Problem

Nobita and Doraemon tour a town composed of 4 small villages. The mutual plan is to travel all 4 villages. Each village can be visited only once and can be used as the starting point. The distances between villages are shown below:



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Find the probability of the routes having the minimum distance.

This authentic problem is used to learn linear permutation while being an enjoyable computing exercise.

Problem 3: Triplet-Code Problem

DNAs are composed of 4 bases, i.e., A, T, G, and C. Three-letter encoding from these four bases (A, T, G, and C) can yield an amino acid which is part of the proteins in a cell. Find the number of possible arrangements of three-letter encoding out of 4 bases (if each letter can be repeated).

This authentic problem is used to introduce permutation of similar things. It involves both mathematical knowledge and biological knowledge and help students realize the importance of mathematics in biology. Moreover, students are expected to derive the permutation of similar things formula.

Problem 4: Base Number and Probability

- *What are the possible base number systems (binary to base nine) for the number 1230?*
- *In each possible base number system n, what is the value of 1230_n in the decimal system?*

The students have to fill the answers in Table 3.2.

Table 3.2 *The value of 1230_n in each possible base number system*

The possible base number system	*The value of 1230_n*
2	
3	
:	
9	

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Next, the students have to list and write the answers in their worksheet to find out the answer by using tree diagram and counting principle of binary number system to base-four numeral system. Table 3.3 is the example of base-3 numeral system practice.

Table 3.3 *Example of base-3 numeral system practice*

Digits	*Tree diagram*	*Counting principle*
1 digit	*Ones*	*ones*
2 digits	*tens ones*	*tens ones*
3 digits	*hundreds tens ones*	*hundreds tens ones*
Total	_____ *numbers*	

To help students in finding the relationship between the answer from tree diagram and counting principle including the linear permutation formula, the students are provided the space to fill the answer by using the mathematics conjunction related to linear permutation formula.

1. Binary number system can create the number from1-digit number to 3-digit number = _____ *number.*

2. Base-3 numeral system can create the number from1-digit number to 3-digit number = _____ *number.*

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3. Base-4 numeral system can create the number from1-digit number to 3-digit number = _____ number.

.

9. Decimal number system can create the number from1-digit number to 3-digit number = _____ number.

The students revised the tree diagram concept and counting principle concept as well as base number system. In addition, they should be able to connect the prior knowledge (the tree diagram and counting principle) to the new knowledge (linear permutation) by using this problem.

Problem 5: Genome Problem

Nowadays, we know that an organism's genome contains the same genes with the same order. But in the past, many scientists found that, for example, the genome consists of A, B, C, D, and E genes in different orders:

A B C D E B C D E A C D E A B D E A B C E A B C D

What's wrong with their discovery?

Why do the 5 orders (above) comprise the same genome?

This authentic problem is used to introduce circular permutation. Students should realize the usefulness of mathematics in solving biological puzzles. Moreover, the problem should induce students' awareness of the difference and the relationship between linear permutation and circular permutation.

Problem 6: Palindrome Problem

Palindrome Problem I

One student in the class is randomly chosen.

1. What is the probability that he/she has a palindrome name?

2. What is the probability that he/she has a palindrome nickname?

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3. What is the probability that he/she has a palindrome name and a palindrome nickname?

4. What is the probability that he/she has a palindrome name or a palindrome nickname?

These problems are used to emphasize the differences between the words "and" and "or" in the word problems.

Palindrome Problem II

A whole number from 1 to 1,000,000 is randomly chosen. What is the probability that it is a palindrome?

Consider:

- *12321 is a palindrome.*
- *83 is not a palindrome.*
  83 + 38 = 121:
  121 is a palindrome (1-time adding).
- *152 is not a palindrome.*
  152 + 251 = 403:
  403 is not yet a palindrome.
  403+304 = 707:
  707 is a palindrome (2-time adding)

Palindrome Problem III

One of the students in class is randomly chosen.

1. What is the probability that his/her student number is a palindrome without reverse adding?

2. What is the probability that his/her student number is a palindrome with 1-time adding?

3. What is the probability that his/her student number is a palindrome with 2-time adding?

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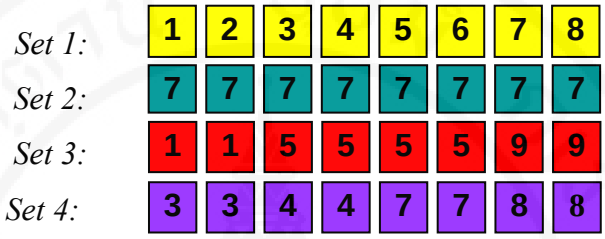
4. What is the probability that his/her student number is a palindrome with more than 2 times adding?

Palindrome Problem IV

Arrange the number cards in each set into palindrome numbers (1 card – 8 cards). Each set is composed of 8-number cards as follows:



How many palindrome numbers are there in each set?

The palindrome problems are used to teach linear permutation and permutation of similar things.

Problem 7: Grouping Problem

1. One class is composed of 50 students. How many possible ways can we divide all students into groups with the same number of students?

2. Four classes are composed of 360students altogether. How many possible ways can we divide all students into groups with the same number of students?

3. One school is composed of 1,260students. How many possible ways can we divide all students into groups with the same number of students?

This problem looks simple but students have to use high order thinking to solve it. The grouping problem was used to teach permutations and combinations.

Problem 8: Common-Fund (Share) Problem

In a common-fund (share) group, the periodic contribution is 100 dollars and bids must be integers only. Before opening the bids of 5 yet-to-win members, water is spilled on 2 bidding papers making it difficult to read

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the interest payments offered. If the 2 bidders are not present and we know only that the 3 remaining offers are 10, 20, and 30 dollars,

1. What is the probability of guessing the offered amounts correctly?

2. What is the probability that one of two bidders in question is the winner in this period?

This authentic problem is used not only to teach probability but also to provide students with a practice problem to analyze. Students have to analyze this problem so that the correct answer can be found by using the permutation or combination formulas.

Explanatory Note:

In a common-fund (share) group, each member contributes an equal amount of money periodically. Each period, yet-to-win members compete to bid for the whole amount by offering an interest payment. The highest bidder wins and is obliged to repay the interest in the remaining periods, in addition to the regular contribution. The last member thus gets all the interest payments in the last period.

3.3.3 Development of the Sport Problem Instructional Unit

The sport problem instructional unit emphasized on creating word problems on popular sports and sport situations. For example, the 4 x 100 sprint relay team problem was used to introduce the relationship and difference between linear permutation and combination concept, and the football (soccer) game results were used to help them recognize the relationship between permutation of different objects

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and permutation of identical objects. These authentic problems were expected to better enhance learning of the concepts when compared to the traditional problems. A problem about possibilities of lining up 4 different books on a bookshelf cannot be compared, in terms of students' interest, with the 4 x 100 relay team with possibilities of selecting the first to the fourth batons and the prospect of victory or defeat by a certain selection.

Six problems were designed for teaching possibility and probability via popular sports

The following problems should be taught in order.

Problem 1: *In a modern ping-pong match, the winner has to win 4 out of 7 games. One point is given to the player who wins each service. Each game is won by the player who first scores 11 points. If, instead of requiring a normal 2-point margin of victory, a game could end at the score of 11:10, what is the minimum point percentage of the total points scored by both players the winner needs to win a match? [Answer: 44/(44+73)]*

The problem is used to help students realize that it is possible to win a match by winning significantly less than half the total points. Students should realize that what is possible may not be probable. The next problem is about accumulated points in a soccer league:

Problem 2: *In the English Premier League, 3 points are awarded to the winner of each match, 0 points to the loser, and 1 point to both teams in case of a draw. Teams are ranked according to the total number of points accumulated. If a soccer team, e.g., Manchester United, has played 35 matches and obtained 81 points, what are the possible results that the team could have had at this stage of the competition (win, draw, loss)? [Answer: 5 possibilities (23–27 wins, the remaining points should come from draws)]*

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To solve the problem, students have to think about all the possible results without the help (or the burden) of formulas. Besides, such thinking would help them solve harder problems (e.g., problems 5 and 7) which require similar analyses.

After introducing the basic counting principle (the principle used to count the number of possible outcomes of a series of events), the researcher poses a 4 x 100 sprint relay team problem:

Problem 3: *If a national team coach can select only 4 runners for a 4 x 100 relay team and there are 5 runners (A, B, C, D, E) to choose from, how many different teams are possible (a) when order does not matter, and (b) when order does matter?*

For most students it should be very easy to find the solutions. The no-ordering answer (combination) is five: ABCD, ABCE, ABDE, ACDE, and BCDE. However the selected runners can be ordered 24 ways (4!). Thus the answer for having to select four runners from five candidates and order them (permutation) is $5 \cdot 4! = 120$ possibilities. This real problem helps students recognize the significant difference between permutations and combinations, which are formally introduced, together with the factorial notation, right after the problem.

Next, the following scoring sequence problem is used to help them understand permutation of identical objects (before formally introducing the formula):

Problem 4: *The final score of a soccer match was 3:2. Find all possible scoring sequences that may have happened during the match if either team can be the winner.*

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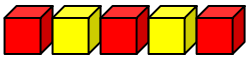
To help students solve the problem, different colored cubes are used to represent the teams as follows:

means “one goal scored by the home team”

means “one goal scored by the opposing team”

So that  represents the scoring sequence (home team: visiting team) 1:0, 1:1, 2:1, 2:2, 3:2. Figure 3.1 shows the 20 possible scoring sequences in achieving the final score of 3:2 regardless of who the victor is: home team or visiting team.

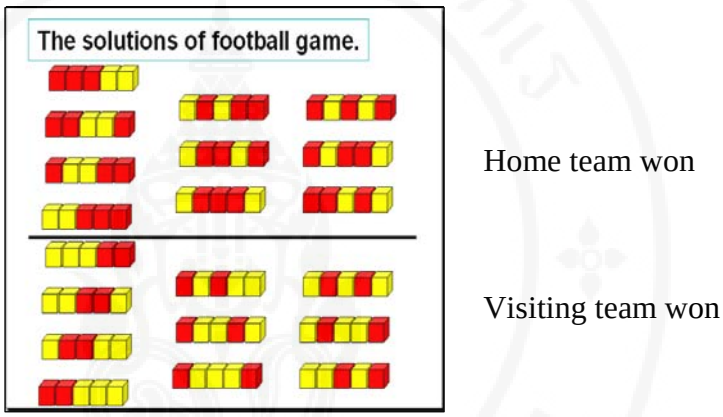
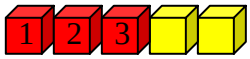


Figure 3.1: All the 20 possible sequences for the final score of 3:2 in soccer match

Once they find the answer, the following activity is used to help them recognize the relationship between permutation of different objects and permutation of identical objects.

1. Select a scoring sequence, e.g., the first one, and label the cubes:


2. Let students find the number of ways the cubes could be rearranged within a line. [Answer: 5!]
3. Label only the red cubes and repeat step 2. [Answer: 3!] Ask them why is the number of ways reduced or how are the answers related? [Answer: 3!2! ÷ 2!]
4. Label only the yellow cubes and repeat step 2. [Answer: 2!] Ask the same question. [Answer: 3!2! ÷ 3!]

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5. From figure 3.1, the number of possible sequences for the home team winning with a score of 3:2 is 10. Ask them how is it related to the answers from steps 1 and 2? [Answer: 5! ÷ (3!2!)]

From our experience, after the activity, students should be able to deduce that, if some objects are identical, the permutations of all the objects would be reduced by a certain factor, i.e., the permutations of the identical objects.

Students should also be reminded that a soccer match has two halves. If the teacher wishes to challenge some of the more enthusiastic students, he/she can pose a more difficult question as to the probability of having a scoreless draw in the first half for a certain final score (see appendix F).

Next is a group activity concerning an American football problem.

Problem 5: *In the National Football League:*

- *A touchdown (TD) is worth 6 points; the subsequent successful kick-conversion is given 1 point; whereas the subsequent successful touchdown-conversion carries 2 points.*
- *A field goal is awarded 3 points.*
- *A safety is worth 2 points.*

What are the possibilities for a team to score n *points* ($0 \leq n \leq 15$) *if different arrangements of the results are to be counted?*

Students are divided into groups of 5 persons. Each group has to fill out a worksheet similar to worksheet 1 (see Figure 3.2) for each score and the groups earn extra credits based on the accuracy of the answers. Once again different colored cubes are used to represent different methods of scoring. This activity combines the skills from problems 2 and 4. Besides it helps students practice using the permutation formula and encourages them to work with others.

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Worksheet 1: 9 possibilities to score 9 points in American Football

Possible Score (0-15)	Touch-downs (TD) (6)	TD + 1-point conversions (7)	TD + 2-point conversions (8)	Field goals (3)	Safeties (2)	The number of permutations
9	1	-	-	1	-	2
9	-	1	-	-	1	2
9	-	-	-	3	-	1
9	-	-	-	1	3	4

Figure 3.2 Sample Worksheet for the American Football problem

We end with a pair of Match Play problems in golf in problem 6.

Problem 6: *In a golf game called the Match Play, one of the two players can win, draw, or lose a hole for each of the 18 holes. The match can be won or lost before the 18th hole depending on the number of wins, draws, and losses during play and the number of holes remaining to be played. For example a 3-and-2 score means a 3-hole lead with 2 holes remaining and a 1-and-0 score means a 1-hole lead after the final hole. A drawn match is also possible. Q – If Tiger Woods played Phil Mickelson in a Match Play event, what possible scores can occur? List them all.*

If the students are unfamiliar with the golf game, they have to be provided with information such as examples of the possible scores (see Figures 3.2 and 3.3). Ready-made tables should be given to them to facilitate problem solving.

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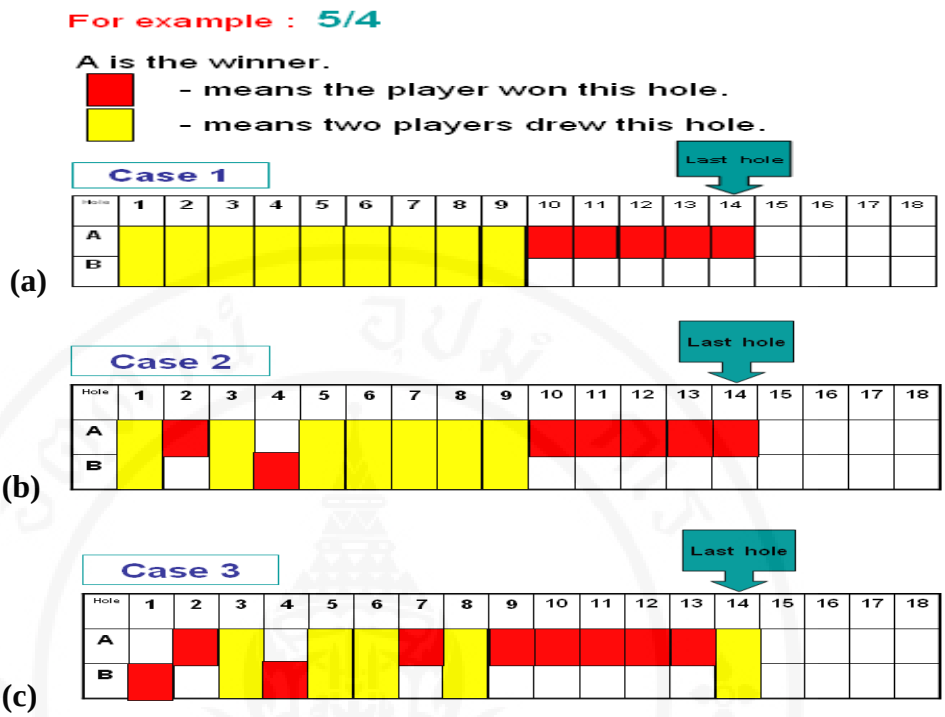

Figure 3.3 Three different examples for a 5-and-4 score

Figures 3.3(a), (b), and (c) are examples of a 5-and-4 score with golfer A the winner over B. In a 5-and-4 winning score, A had a 5-hole lead and there were 4 holes remaining, hence the 14th was the last hole played. Figures 3.4(a), (b), and (c) are examples of a 1-and-0 score with golfer A the winner over B. In a 1-and-0 score, A had a 1-hole lead and there were no remaining holes, hence the 18th was the final hole. Table 3.4 is given as a checklist of the possible and impossible scores. Even though it appears to comprise all scores, most students surely can eliminate and give some reason for the impossible scores. The answers are **highlighted**.

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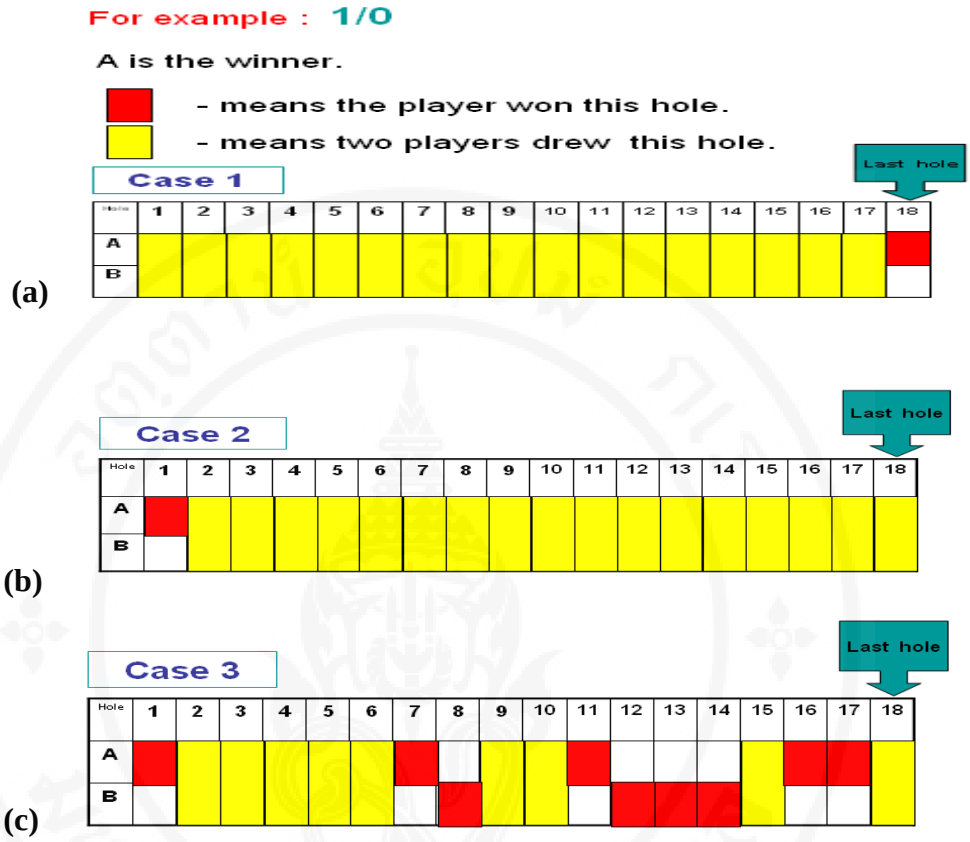

Figure 3.4 Three different examples for a 1-and-0 score

Table 3.4 Scores listed for students to select as possible or not in a Match Play competition

0/0	0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9	0/10	0/11	0/12	0/13	0/14	0/15	0/16	0/17	0/18
1/0	1/1	1/2	1/3	¼	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/17	1/18
2/0	2/1	2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9	2/10	2/11	2/12	2/13	2/14	2/15	2/16	2/17	2/18
3/0	3/1	3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9	3/10	3/11	3/12	3/13	3/14	3/15	3/16	3/17	3/18
4/0	4/1	4/2	4/3	4/4	4/5	4/6	4/7	4/8	4/9	4/10	4/11	4/12	4/13	4/14	4/15	4/16	4/17	4/18
5/0	5/1	5/2	5/3	5/4	5/5	5/6	5/7	5/8	5/9	5/10	5/11	5/12	5/13	5/14	5/15	5/16	5/17	5/18
6/0	6/1	6/2	6/3	6/4	6/5	6/6	6/7	6/8	6/9	6/10	6/11	6/12	6/13	6/14	6/15	6/16	6/17	6/18
7/0	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9	7/10	7/11	7/12	7/13	7/14	7/15	7/16	7/17	7/18
8/0	8/1	8/2	8/3	8/4	8/5	8/6	8/7	8/8	8/9	8/10	8/11	8/12	8/13	8/14	8/15	8/16	8/17	8/18
9/0	9/1	9/2	9/3	9/4	9/5	9/6	9/7	9/8	9/9	9/10	9/11	9/12	9/13	9/14	9/15	9/16	9/17	9/18
10/0	10/1	10/2	10/3	10/4	10/5	10/6	10/7	10/8	10/9	10/10	10/11	10/12	10/13	10/14	10/15	10/16	10/17	10/18
11/0	11/1	11/2	11/3	11/4	11/5	11/6	11/7	11/8	11/9	11/10	11/11	11/12	11/13	11/14	11/15	11/16	11/17	11/18
12/0	12/1	12/2	12/3	12/4	12/5	12/6	12/7	12/8	12/9	12/10	12/11	12/12	12/13	12/14	12/15	12/16	12/17	12/18
13/0	13/1	13/2	13/3	13/4	13/5	13/6	13/7	13/8	13/9	13/10	13/11	13/12	13/13	13/14	13/15	13/16	13/17	13/18
14/0	14/1	14/2	14/3	14/4	14/5	14/6	14/7	14/8	14/9	14/10	14/11	14/12	14/13	14/14	14/15	14/16	14/17	14/18
15/0	15/1	15/2	15/3	15/4	15/5	15/6	15/7	15/8	15/9	15/10	15/11	15/12	15/13	15/14	15/15	15/16	15/17	15/18
16/0	16/1	16/2	16/3	16/4	16/5	16/6	16/7	16/8	16/9	16/10	16/11	16/12	16/13	16/14	16/15	16/16	16/17	16/18
17/0	17/1	17/2	17/3	17/4	17/5	17/6	17/7	17/8	17/9	17/10	17/11	17/12	17/13	17/14	17/15	17/16	17/17	17/18
18/0	18/1	18/2	18/3	18/4	18/5	18/6	18/7	18/8	18/9	18/10	18/11	18/12	18/13	18/14	18/15	18/16	18/17	18/18

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Once students are familiar with the Match Play, we pose a problem that requires the skills from problems 2, 4, and 5, and the analysis from problem 6.

In terms of each hole's score (win, draw, or loss), how many ways could Tiger Woods win a match with a score of 8 and 7?

If there is a winner in a match play, the winner has to either win or draw the last played hole. In addition, if the difference between the number of leading holes and the number of remaining holes is 2, the winner has to win the last played hole. Thus when calculating the number of possibilities, we may have to restrict the result of the last played hole. With that in mind, let us look at the possibilities for an 8-and-7 score.

Table 3.5 *265 possibilities for an 8-and-7 score*

Case	*Win*	*Draw*	*Loss*	*Last Played Hole*	*Possibilities*
1	*8*	*3*	*-*	*-*	*165*
2	*9*	*1*	*1*	*W*	*90*
3	*9*	*1*	*1*	*D*	*10*

The possibilities are obtained from the permutation formula. In case 1, there is no restriction on the result of the last hole, thus the formula could be applied directly. In case 2, since the last hole has to be won, the calculation is reduced to 10 holes with 8 wins, 1 draw, and 1 loss. Similarly, the calculation in case 3 is reduced to 10 holes with 9 wins and 1 loss.

3.3.4 Development of the Probabilistic Game Instructional Unit

The probabilistic game instructional unit focused on new probabilistic games. Examples of the games were the icosahedron game to introduce probability concepts, and the number guessing game to teach linear permutation and permutation of similar things.

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Five games were designed for teaching possibility and probability

Game 1: Icosahedron Game

Three icosahedrons (20-face-dice) are composed of scores 1, 10, and 100 on each face as shown below:

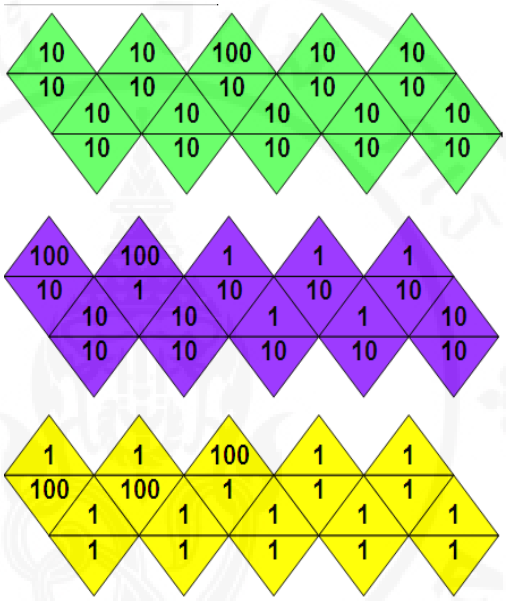

Figure 3.5 *Three icosahedrons (20-face-dice) composed of scores 1, 10, and 100 in icosahedron game*

Game rule:

1. Divide students into groups of three.

2. Each student selects 1 icosahedron and adds the scores from rolling the dice 10 times.

3. The winner is the student who gets the highest sum.

Question: Which icosahedron offers the best chance to win the game?

This game provides student opportunity to learn probability and statistics by using the following activities:

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Activity 1.1 Play 1 game → select the icosahedron number _____

No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*Total*
Score											
Results	*Get 1 - score _____ times = _____ %* *Get 10 - score _____ times = _____ %* *Get 100 - score _____ times: = _____ %*										

The competition result: _____

Activity 1.2 Play 10 games → select the icosahedron number _____

Round	*Rolling No.*										*Total*	*Result*
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*		
1												
2												
3												
:												
10												
Results	*Get 1 - score _____ times = _____ %* *Get 10 - score _____ times = _____ %* *Get 100 - score _____ times: = _____ %*											
	win _____ rounds, draw _____ rounds, loss _____ rounds											

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<u>Activity 1.3</u> Collect the playing statistics in activity 1.1 from all the students

Student no.	*Icosahedron no. 1 (times)*			*Results*	*Icosahedron no. 2 (times)*			*Results*	*Icosahedron no. 3 (times)*			*Results*
	1	*10*	*100*		*1*	*10*	*100*		*1*	*10*	*100*	
1												
2												
:												
20												
Total												
percent	*	*	*	*	*	*	*	*	*	*	*	*
Results	*Select icosahedron no. 1* *_____students* *= _____%* *win _____ students* *draw_____ students* *loss _____ students*				*Select icosahedron no. 2* *_____students* *= _____%* *win _____ students* *draw_____ students* *loss _____ students*				*Select icosahedron no. 3* *_____students* *= _____%* *win _____ students* *draw_____ students* *loss _____ students*			

Remark: * is computed from _____

<u>Activity 1.4</u> Collect the playing statistics in activity 1.2 from all the students (Use the table in activity 1.3)

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Game 2: Random Number Game

5 sets of 4-number cards are composed of the numbers as follows:

Set 1: 1 2 3 4

Set 2: 1 2 6 0

Set 3: 1 2 2 5

Set 4: 1 4 4 0

Set 5: 2 2 3 3

Game rule:

1. Divide students into groups of five.

2. Each student selects one set of 4-number cards. (five students randomly arrange the order of play)

3. Each student makes a 4-digit number by randomly picking one card at a time from the set of 4 numbers and placing it starting from the most and progress to the least significant digits. A zero cannot be at the thousands digit.

4. The student who gets the highest number is rewarded 1 point.

5. The winner of this game is the student who has the highest points after playing 24 times (with the same set). In case of a tie, those who tie have to continue playing until the winner is found.

Question: Which is the set of 4 numbers that offers the best chance of winning this game?

The linear permutation and permutation of similar things concepts are taught by using this game as well as the statistics concept. This game is composed of 2 activities as described below.

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<u>Activity 2.1</u> Fill the ✔ symbol according to the competition results.

No.	*Game no. 1*					*Game no. 2*					*Game no. 3*				
	Select the set no._____					*Select the set no._____*					*Select the set no._____*				
	Result (the winner set no.)					*Result (the winner set no.)*					*Result (the winner set no.)*				
	1	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*	*1*	*2*	*3*	*4*	*5*
1															
2															
:															
24															
Win (times)															
Conclude	*the winner set no._____*					*the winner set no._____*					*the winner set no._____*				

<u>Activity 2.2</u> Collect the playing statistics in activity 2.1 from all the students.

Game no.	*Total win (games)*				
	Set of numbers no. 1	*Set of numbers no. 2*	*Set of numbers no. 3*	*Set of numbers no. 4*	*Set of numbers no. 5*
1					
2					
3					
:					
30					
Total					
Results	*Set of numbers no. (descending ordering)* _____, _____,_____,_____,_____				

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Game 3: Number Guessing Game

5 sets of 4-number cards compose of the number as follows:

Set 1: 1 2 3 4

Set 2: 1 2 6 0

Set 3: 1 2 2 5

Set 4: 1 4 4 0

Set 5: 2 2 3 3

Game rule:

1. Divide students into groups of five.

2. Each student selects one set of 4-number cards. (Five students draw lots to arrange the starter and descending orders)

3. Each student creates a secret 4-digit number from the set: zero cannot be at the thousands digit.

4. Each student can select one of the others to guess his/her secret number. The guesser gets one point if the guess is correct.

5. If the first guess is wrong, the owner has to reveal the thousands digit. Afterwards, the guesser just makes another guess until it is correct.

6. Continue playing until all secret numbers are exposed. .

Question: Which set of 4 numbers is the most difficult to guess the secret number?

This problem is used to review the linear permutation and permutation of similar things concepts with new conditions to win the game. Students would be trained to solve more complicated problems and the answer would lead them to find the linear permutation and permutation of similar things formulas.

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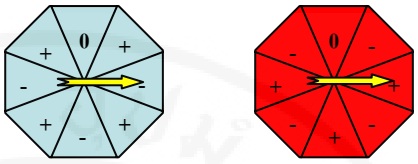
Game 4: Wheel Games

Wheel Game I

This game is composed of 2 probability wheels as shown below:



After spinning the 2 wheels, the product of the 2 wheels are computed.

Question: Which event has the highest probability?

Choice 1: the product is negative.

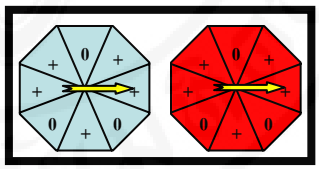
Choice 2: the product is zero.

Choice 3: the product is positive.

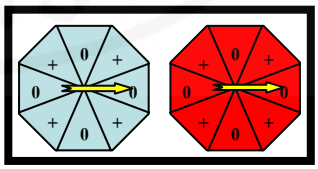
Wheel Game II

This game is composed of 3 sets of 2 probability wheels as follows:

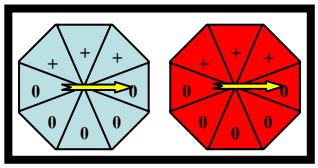
Set 1



Set 2



Set 3



After spinning both wheels in each set, divide the left wheel by the right wheel.

Question: Which set yields the highest probability that the result is equal to zero?

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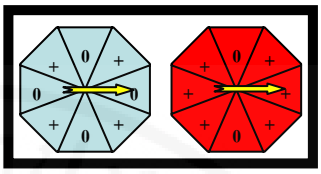
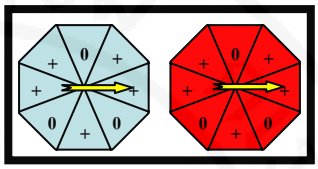
Wheel Game III

This game is composed of 3 sets of 2 probability wheels having the same total number of zeros as shown below:

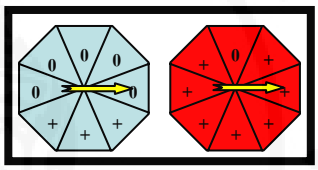
Set 1



Set 2



Set 3

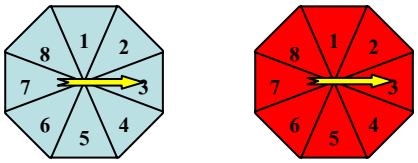


After spinning both wheels in each set, the product of the 2 wheels are computed.

Question: Which set yields the highest probability that the result is equal to zero?

Wheel Game IV

This game is composed of 2 number wheels as follows:



Four events are proposed:

Event 1: 2 number wheels give the same number.

Event 2: 2 number wheels give different numbers.

Event 3: The sum of 2 number wheels equals 8 or 10.

Event 4: The sum of 2 number wheels is more than12.

Question: If every event gives the same reward, which event yields the highest probability to get the reward?

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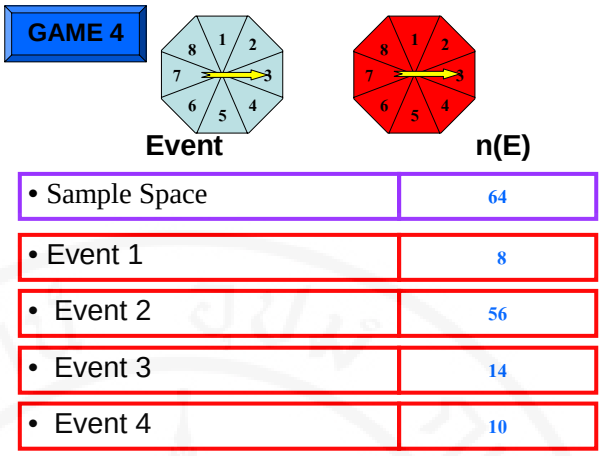

Figure 3.6 *The number of possibilities of each event including that of the sample space.*

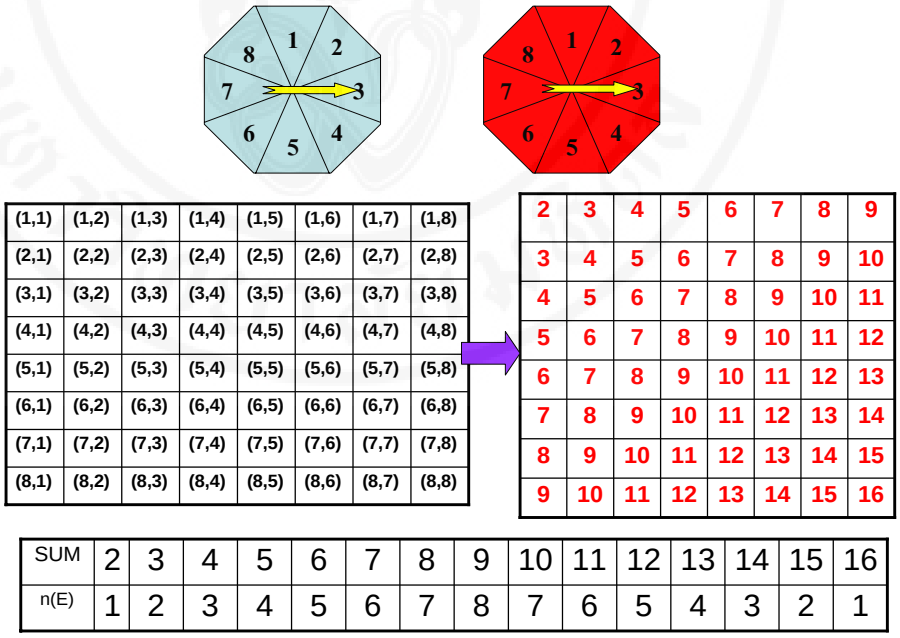

Figure 3.7 All possible results from spinning 2 number wheels.

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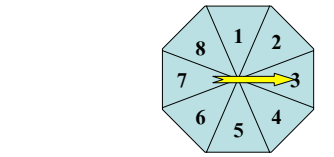

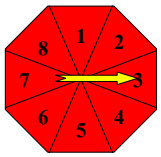

SUM	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
n(E)	1	2	3	4	5	6	7	8	7	6	5	4	3	2	1

- *The sum of 2 number wheels equals 8 or 10*

  = 7+7 = 14 Events

- *The sum of 2 number wheels is more than12*

  = 4 + 3 + 2 + 1 = 10 Events

Figure 3.8 The numbers of two events from spinning 2 number wheels.

This probabilistic game is used to teach linear permutations. In addition to increasing students' understanding and thinking, the students are asked by next question.

Q: How to make these 4 events to have the same reward (fair game)?

Examples of the answer: change the numbers in the number wheels; change some event(s); change the amount of reward; and increase/ decrease the sector size (area) of each number in the number wheels.

For the next three games (in Game 5) which are similar, students had to familiarize themselves with the game rules and the way they are played. Thus five student representatives were trained before demonstration to the class and each would assist classmates in playing the games later. A document containing the rules and examples of how each game is played was also provided for reference. The complexity increased as they went from game one to three.

In each five-card game, two students played opposing each other in each round. Five cards were selected from a stack of nine cards (number 1 to 9) and shown to both players before put face down and shuffled. The players were each given two of

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the face-down cards and allowed to look at their own numbers close to their chest. Thus there would be one face-down card as an unknown to both players.

Each pair of players used rock-paper-scissors to decide which one would initiate the first round of play. The initiator could decide whether to play, which involved guessing the opponent's card or cards, or to pass, which would force the opponent to be the guesser. After this initial move, the players alternated in deciding who should be the guesser. Scores were awarded to the ones who guessed correctly. After each play, five cards were drawn anew from the same but reshuffled stack of nine cards.

Game 5: The Five-card games

This game is composed of 5 different number cards. The example is shown below:

The Rule for Game I

The guesser had to guess the sum of the opponent's numbers and call out whether they were equal, higher or less than the guesser's sum. If the initiator decided to be the guesser and the guess was correct, one point was given to the guesser and one point was deducted from the opponent. If the guess was wrong, both got zero. If the deciding player allowed the opponent to guess and the guess was correct, the opponent would get one point whereas the decider got zero. In this latter case, if the opponent got it wrong, both players got zero. The player that reached a total of five points would win the game.

Question:

1. With which pair of numbers should the initiator select to play? Explain.

2. With which pair of numbers should the initiator select to pass? Explain.

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Figure 3.9 and figure 3.10 are used to demonstrate the game as follows.

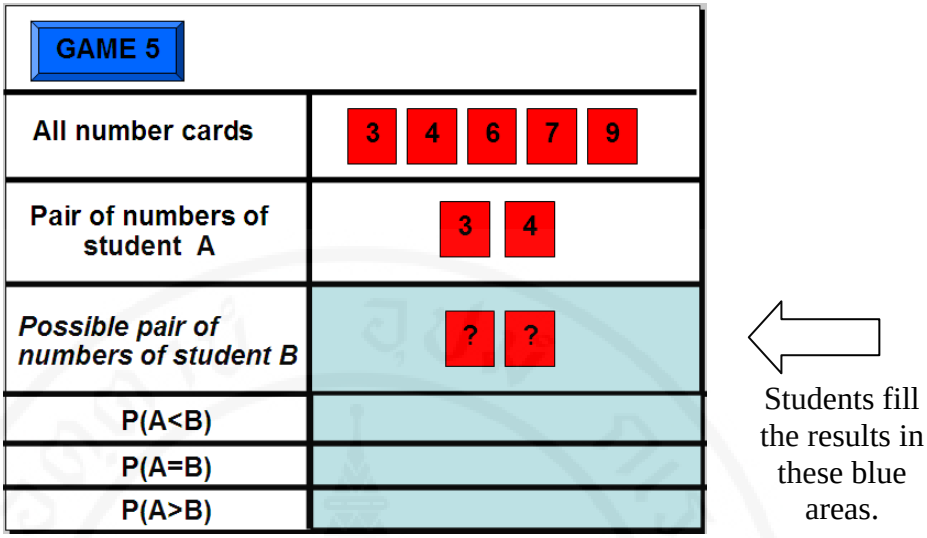

Figure 3.9 An illustration of five-card game I.

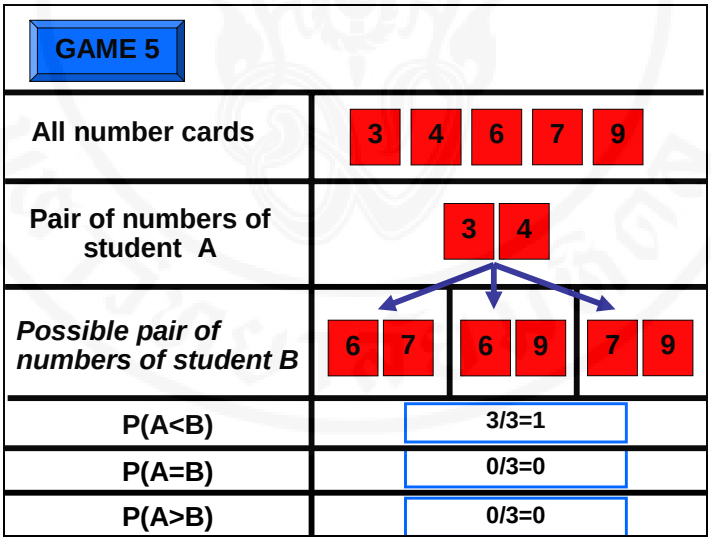

Figure 3.10 An illustration of five-card game I.

This game is composed of activity 5 that collects the game results in order to give the data for students to recheck and find the way to win the game by confirming the best strategy for each pair of numbers.

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Activity 5 Collect the game results into the following table.

Player	*Strategy*		*Results*	*Players' number cards*	
	Play	*Pass*	*A : B*	*A*	*B*
A					
B					
:					
A					
B					

This game is used to introduce linear permutations and combinations.

The Rule for Game II

This was the only game where both players had to guess in each round. The deciding player initiated the play by electing to be the first or the second guesser in the first round of play. Both players then took turns being the first guesser. The first guesser guessed the relative face value of one of his/her own cards to one of the opponent's cards. Both cards were then turned face up. If the guess was correct, one point would be given to him. If not, one point would be awarded to the opponent. The second guesser then compared the remaining cards. The winner had to score up to 20 points first.

Question:

1. Can you guess the pair of opponent's numbers?

2. What is your strategy to win this game? Explain.

The students can revise their strategy in game playing and they can understand the probability concept and realize that the concept is useful for their real life.

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This game is not only used to introduce probability but also used to teach linear permutations and combinations. Figure 3.11 is used to demonstrate the game as follows.

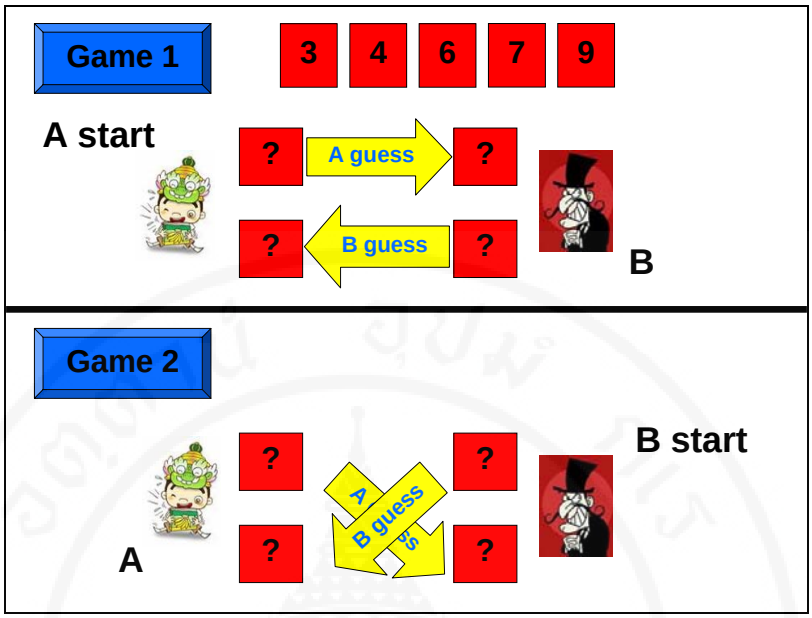

Figure 3.11 An illustration of number card game II.

The Rule for Game III

The guesser had to pair one's own cards to those of the opponent before guessing the relative value of each pair of cards. Before the game, the pair had to choose among these three ways of playing the game:

(i) Guessing both pairs of cards before showing the face values.

(ii) Showing the face value after guessing the first pair of cards.

(iii) Informing the relative value after guessing the first pair of cards.

If the guess was correct, the guesser was awarded one point. Otherwise, the opponent got the point. The winner had to reach 10 points first.

Question:

1. What is your strategy?

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2. What is the pair of numbers of the opponent which you can win? Explain.

During playing this game, the students' discussion of their strategies is useful to enhance students' conceptual understanding. This problem enables students to develop their knowledge and their validation.

3.4 Pilot Study

The pilot study was first conducted on sport problem instructional unit with grade twelve students. The first trial was on 40 grade twelve students aged between 17-18 years in a school in the city area of the country in second semester of the academic year 2008. These students had already learned an applied probability course. The second trial was on two classes of grade eleven students aged between 16-17 years in a school in the city area of the country in first semester of the academic year 2009. These students had not been taken highschool probability courses before the intervention. One class was control group (39 students) while another was a treatment group (43 students). The traditional problems were used for control group while the sport problems were used for the treatment group with the same teaching periods (5 hours). Both groups received the same pretest and posttest in which all questions were traditional problems.

The students were assessed for their learning outcomes by using various instruments, i.e., classroom observation, written work, pretest, and posttest. Pretest and posttest are parallel tests and were the same tests of this study. The quantitative data was classified and analyzed by means presented in forms of table while the qualitative data was presented in narrative text.

The pilot study on nontraditional word problem instructional unit and probabilistic game instructional unit was performed after obtaining the successful results from trial on sport problem instructional unit. The participants in this study were only from six volunteer highschool students in the city area of the country. This pilot study was conducted on round table in school area after the school hours for 15 hours in one week. The students expressed their willingness to collaborate in all activities including doing the tests. The assessment of these students was done by

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qualitative method. The students' speech, students' statements, students' remarks, students' comments, their learning behaviors, and group discussions were used in permutations and combinations with the tests for evaluating the effectiveness and a weakness of the instructional units before being used in subsequent studies.

3.5 Implementation of Instructional Units on highschool students

An eight-week intervention was implemented in four classes of grade eleven students enrolled in a Science-Mathematics program. The classes were assigned to be the control group (receiving traditional problem instructional unit-TU) and the three treatment groups which were given nontraditional word problem instructional unit (NTU), sport problem instructional unit (SU), and probabilistic game instructional unit (GU). The average percentage scores of TU, NTU, SU, and GU from the previous-semester mathematics exams were 78, 69, 68 and 59, respectively.

Table 3.6 The details of the use of four instructional units for four groups

Group	Instructional Unit	Abbreviation	N	Average previous semester mathematics scores
Control Group	Traditional	TU	39	78%
Treatment Group I	Nontraditional word problem	NTU	40	69%
Treatment Group II	Sport Problem	SU	30	68%
Treatment Group III	Probabilistic Game	GU	40	59%

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All four classes of students were taught by the same researcher using the inquiry approach, according to the teaching sequence in Table 3.7. All students were supposed to learn the same concepts in permutations and combinations, and they took

the same tests. During the learning activity, the students worked collaboratively in groups of 5 to share ideas, and discuss and derive the mathematical formulas from the given problems. After going through each concept, the teacher debriefed the students during which the latter were asked to reflect on what they had learned.

Table 3.7 The teaching sequence of the instructional units (4 periods per week)

Week	Concept	Research instrument
-	Examine students' prior knowledge	Pretest questions
1	Basic contents on the tree diagram and the counting principle	-
2	Linear permutation	Students' reflection I
3		
4	Permutation of similar thing	Students' reflection II
5	Circular permutation	Students' reflection III
6	Combination	Students' reflection IV
7	Evaluate students' concepts	Posttest questions Questionnaire
8	Evaluate students' overall comprehension	Comprehensive test

3.6 Research Instruments

Students' achievement was assessed by the pretest, the posttest, and the comprehensive test. Students' reflection was also used for evaluating students' understanding.

1. Pretest and Posttest

The pretest and the posttest contained parallel problems to examine the conceptual understanding. They consisted of 20 items: 5 items for each of the four concepts. The pretest scores were used to assess students' prior knowledge in the four classes whereas the posttest scores were used to follow students' achievement after the intervention.

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2. Comprehensive test

The comprehensive test was somewhat parallel to those of the pretest and the posttest but contained more complicated analytical problems. The students had to apply concepts learned and used the problem-solving skills to find the answers to these problems. The 20-item problems (5 items for each concept) were designed by two in-service mathematics teachers with more than 25 years of teaching experience; they were responsible for the class. The comprehensive test was administered one week after the posttest.

3. Retention test

The retention test contained parallel questions to those of the comprehensive test. The test was given 2 months after the comprehensive test.

4. Students' reflection

Students' reflection was used to provide in-depth data on students' understanding of permutations and combinations. After going through each concept, they were asked to write what they knew, what they did not understand, factors affecting their learning, and new mathematical problems gathered from other sources.

5. Questionnaire

The questionnaire was administered to the students at the end of the unit. It was composed of students' demographics, and students' attitudes toward their instructional units and self-assessment of understanding. The students were asked to rate the 7-point-scale questionnaire (3 = most strongly agree, 2 = strongly agree, 1 = agree, 0 = neutral, -1 = disagree, -2 = strongly disagree, and -3 = most strongly disagree), and write comments/suggestions in the space provided.

3.7 Data Analysis

The data in this study were analyzed by using SPSS program. The determination of the data analysis is shown as follows:

1. The means $(\bar{x})$, the standard deviations (S.D.), and the standard errors of the means (S.E.) for the pretest, the posttest, the comprehensive test, and the retention test of students' achievement on the four instructional units.

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2. The paired-samples t test at the significance level of 0.05 for the comparison of the mean scores between the pretest and the posttest in each of the four groups, and the comparison of the mean scores between the comprehensive test and the retention test in each of the four groups.

3. The independent-samples t test at the significance level of 0.05 for the comparisons of the mean scores among the pretests in the four groups and among the posttests in the four groups.

4. ANOVA at the significance level of 0.05 for the comparisons of the mean differences in the pretest mean scores, the posttest mean scores, and the comprehensive test mean scores among the four groups.

3.8 Summary of the Chapter

This chapter has described the process of developing of the four instructional units on permutations and combinations concepts. The four instructional units were differed in the nature of problem used in each unit; they were traditional problem, nontraditional word problem, sport problem and probabilistic game problem. These instructional units were piloted with the highschool students from the school in the city. Next the implementation these instructional units after revision were then implemented on highschool students from school in the rural area. Finally, the research instruments and data analysis were described.

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CHAPTER IV
RESULTS

4.1 Results from Pilot Study

4.1.1 Pilot study on sport problem instructional unit

At the beginning of the class, the students were evaluated the prior knowledge background by performing pre-test (20 items). After the lesson, students did post-test with the parallel set of problems as those of the pre-test.

The results from trial on one group of the students are shown in Table 4.1 showed that the posttest scores were significantly higher than the pretest scores. It should relied that this group of students already learned “probability” in their first semester. Table 4.2 shows student’s pretest score and posttest score in each of the four concepts. The value of pretest scores was lowest in combination and permutation of similar things concepts. However, after intervention the posttest scores increased in the four concepts. The highest normalized gain was observed in linear permutation. In addition, 50% of the students had 100% correct answers in posttest.

Table 4.1 The pretest and posttest mean scores from one class of grade 12 students (n=40).

Test (20 items)	Mean	S.D.	t-test
Pretest	7.94	4.40	11.197*
Posttest	17.51	3.14	

* significant difference at 0.05 level

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Table 4.2 Student's pretest and posttest mean scores in four concepts.

Concepts	Mean scores (maximum point = 5)		Normalized gain
	Pretest	Posttest	
Linear permutation	3.30	4.86	0.9176
Permutation of similar things	0.92	4.49	0.8750
Circular permutation	2.92	4.55	0.7837
Combination	0.80	3.61	0.6690

The finding from questionnaire, from the 7-point rating scale [-3,3], showed that the students were highly satisfied with sport problem instructional units (mean = 1.89). They also agreed that the unit helped them develop conceptual understanding in permutations and combinations (mean = 1.92). In addition, the students also gave comments on attitudes as shown in the following excerpts:

"I have more understanding in permutations and combinations. Although I already studied in this contents but I can not remember the permutation and combination formula, and I cannot solve their simple problems. But now I can recall the formula from my head and also can solve the novel problems in sport or even non sport problems."

"The sport problems help me review the knowledge, altogether with enjoyable and meaningful learning."

"I know the new method for solving the problems."

"The activities are enjoyable and encourage good teamwork."

However, a few students suggested that they should be given more time for learning in this instruction.

The investigation on effectiveness of the sport problem was repeated with another two classes of students who had never been studied probability. Students from two classes showed significantly differences between the pretest scores and posttest scores (Table 4.3). The results from data analysis indicated no significant difference between two groups in the posttest mean scores (t = 0.215, p-value = 0.831) while in

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the pretest mean scores, it was significant difference between two groups (t = 3.061, p-value = 0.004). The pretest mean scores of the students in this study were much lower than those of the first group. Moreover, the mean of the control group was higher than those of the treatment group. After the intervention, the mean score of the control group increased from 2.3 in the pretest to 14.1 in the posttest and those of the treatment group increased from zero to 14.27. The results in Table 4.4 show the pretest and posttest scores in each concept. The posttest scores were higher significant than those of the pretest. There were significant differences in the pretest mean scores in the two concepts, i.e., linear permutation and permutation of similar things while there was no significant differences in the posttest mean scores between two groups in every concept. The amount of increase was approximately the same for both groups. Lower percentage of improvement was observed in the concepts on combination and permutation of similar things.

Table 4.3 Pretest and posttest mean scores and standard deviation of two groups of students.

Group	N	Mean ± SE (maximum point = 20)	
		Pretest	Posttest
Control Group	39	2.31 ± 0.75	14.10 ± 1.13
Treatment group	43	0 ± 0	14.27 ± 0.73

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Table 4.4 The mean scores of student's pretest score and posttest score in each concept.

Concepts	Mean (maximum point = 5)					
	Pretest mean score			Posttest mean score		
	Control group	Treatment group	t	Control group	Treatment group	t
Linear permutation	1.03	0	3.13*	4.23	4.53	0.83
Permutation of similar things	1.03	0	3.13*	2.69	2.56	0.24
Circular permutation	0	0	-***	4.49	4.77	0.95
Combination	0.26	0	1.43	2.69	2.56	0.24

* significant difference at 0.05 level

*** cannot be computed because the standard deviation of both groups are zero.

In addition, the results from the 7-point rating scale [-3,3] of the treatment group illustrated that the students were highly satisfied with sport problem instructional units (mean = 1.97). They can develop conceptual understanding in permutations and combinations through the sport problem instructional unit (mean = 2.08). Moreover, the students also gave comments on their attitudes as shown in the following excerpts:

"Even though, I do not like to play sports. Your surprisingly, it works to learn probability by using lesson via sport problems."

"These fun activities which introduced the probability contents can help us derive the probability formula."

"I am very happy because I differentiate between permutation problem and combination problem."

"The problems are so cool!"

In addition, then students wish to learn mathematics through nontraditional problems.

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4.1.2 Pilot study on nontraditional word problem and probabilistic game instructional units

The pilot study on nontraditional word problem and probabilistic game instructional units are conducted with 6 volunteer students. The students' discussion during the activities, students' behaviors during doing the test, and students' comments were collected to be used in improving the problems/games in these two instructional units. The interview during the instruction was used to probe the understanding in each problem/game from each student.

In the pretest, the students cannot solve the problems. The finding from observation and interview indicated the student understood the problem, but they could not solve those problems. Some students used trial and error method to list the answer but they still could not find the whole correct answers. Some students attempted for short time and then gave up. After that the reminder students stopped to do the pretest.

The teaching-learning process of the instruction started with the same concepts in both nontraditional word problem and probabilistic game. All students always voted to select the starting problem/game. They discussed all the time during the problem solving. The students, however, collaborated to solve the problem/game and find the weakness of the problem/game together. During the course of pilot study, the researcher adjusted the problem according to the students' comments. The main issue to be correct was unclear statements in nontraditional word problems and unclear rules of the probabilistic games. The students said that both problems/games of the instructional units are appropriate for engaging and motivating them to learn in the topics of permutations and combinations.

After completing the intervention from both the instructional units, the students did the posttest. Since they were allowed to work in team to analyze and solve the problems together. Hence, all of them can find the whole correct answers. They then gave comments that the problems were now clearly understood and so easy to solve when compared to the pretest. They expressed that the nontraditional word problem instructional unit and probabilistic game instructional unit can be used to enhance students' understanding in permutations and combinations Moreover, they

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commented that nontraditional word problems and probabilistic games were more enjoyable and more interesting than the normal mathematics problems.

4.2 Effects of the Four Instructional Units on Students' Achievement

4.2.1 Finding from Pretest and Posttest

At the beginning of the class, students were evaluated for their background knowledge by a pretest (20 problems). Each of the four groups then participated in one of the four inquiry-based learning units: traditional word problems (TU), nontraditional word problems (NTU), sport problems (SU) and probabilistic games (GU). After the lesson, the students did the posttest containing parallel questions to that in the pretest. Table 4.5 shows the means and normalized gains (Hake, 1998) of students' pretest and posttest scores.

Table 4.5 Pretest and posttest mean scores, and normalized gains of students in the four learning units.

Group	N	Mean ± SE		t	<g>
		Pretest (20 pts)	Posttest (20pts)		
TU	39	5.64 ± 0.59	18.50 ± 0.48	21.13*	0.8955
NTU	40	3.44 ± 0.65	18.51 ± 0.36	21.45*	0.9100
SU	30	3.57 ± 0.71	18.00 ± 0.56	13.25*	0.8783
GU	40	3.69 ± 0.70	15.63 ± 0.89	10.98*	0.7321

* significant difference at 0.05 level

Results from ANOVA indicated no significant difference in the pretest mean scores among the four groups ($F = 2.515$, p-value = 0.061). They were all lower than 30% of the full score. The posttest mean score of each of the four groups was significantly higher than the pretest one (p-value < 0.001). ANOVA of differences among the posttest scores of the four groups indicated no significant difference among the TU, NTU, and SU groups ($F = 0.5490$, p-value = 0.001). However the GU group

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had the lowest posttest mean score, as compared to the other three groups. This was in agreement with the results from the percentage gains (normalized gains), albeit at a high level ($\langle g \rangle \geq 0.7$) for all four groups, which also indicated that the GU group gained the least.

Then we distinguished the permutation and combination scores according to four concepts: linear permutation, permutation of similar things, circular permutation, and combination to analyze further the results from using the instructional units. Table 3 shows the pretest and posttest scores, and the normalized gains of students for each concept.

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Table 4.6 Pretest and posttest mean scores, and normalized gains for each concept

Concept	Group	Mean ± SE		t	<g>
		Pretest (5 pts)	Posttest (5 pts)		
Linear permutation	TU	3.21 ± 0.39	4.88 ± 0.13	2.49*	0.9330
	NTU	1.75 ± 0.38	4.52 ± 0.23	6.87*	0.8523
	SU	1.90 ± 0.54	4.33 ± 0.32	5.84*	0.7839
	GU	1.25 ± 0.35	3.75 ± 0.35	4.18*	0.6667
Permutation of similar things	TU	0 ± 0	4.50 ± 0.16	26.45*	0.9000
	NTU	0.06 ± 0.06	4.11 ± 0.25	16.61*	0.8198
	SU	0 ± 0	4.17 ± 0.23	17.11*	0.8340
	GU	0.30 ± 0.19	3.50 ± 0.31	9.96*	0.6809
Circular permutation	TU	1.79 ± 0.39	4.75 ± 0.17	7.42*	0.9221
	NTU	1.00 ± 0.32	4.88 ± 0.12	11.42*	0.9700
	SU	0.95 ± 0.44	5.00 ± 0	9.05*	1.0000
	GU	2.00 ± 0.39	4.25 ± 0.29	5.24*	0.7500
Combination	TU	0.64 ± 0.27	4.38 ± 0.26	9.54*	0.8578
	NTU	0.63 ± 0.26	5.00 ± 0	20.20*	1.0000
	SU	0.71 ± 0.39	4.50 ± 0.28	7.66*	0.8835
	GU	0.13 ±0.13	4.13 ± 0.30	12.42*	0.8214

* significant difference at 0.05 level

For concepts of linear permutation and permutation of similar things, all the instructional units, except the probabilistic games (GU), gave similar outcomes. As shown in Table 4.6, normalized gains of TU, NTU, and SU were high (0.78-0.93) relative to that of GU (0.67). For other two concepts, circular permutations and combinations, all units gave similar results of high normalized gains.

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4.2.2 Finding from Comprehensive test

Table 4.7 Comprehensive test scores for each concept

Concept	Group	Comprehensive test (5 pts)
Linear permutation	TU	3.95 ± 0.16
	NTU	4.34 ± 0.09
	SU	3.54 ± 0.18
	GU	3.88 ± 0.15
Permutation of similar things	TU	3.80 ± 0.15
	NTU	4.06 ± 0.10
	SU	2.81 ± 0.15
	GU	3.86 ± 0.15
Circular permutation	TU	4.73 ± 0.08
	NTU	4.67 ± 0.11
	SU	4.35 ± 0.18
	GU	4.55 ± 0.13
Combination	TU	4.06 ± 0.14
	NTU	4.19 ± 0.14
	SU	3.19 ± 0.17
	GU	3.74 ± 0.17

Table 4.7 shows the results of the comprehensive test for each concept. The students were expected to apply the concepts learned from the given instructional units (either TU, NTU, SU, or GU) in solving and analyzing the more complex problems in the comprehensive test. Results from ANOVA showed no significant difference in the mean scores among the four groups for the concept of "circular permutation" ($F = 2.011$, p-value = 0.114). However, for the other three concepts (linear

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permutation, permutation of similar things, and combination), the lowest achievement was observed for the SU group. There was also no significant difference in the mean scores among the TU, NTU, and GU groups. It should be noted that, the lowest mean scores was achieved by the SU group for the four concepts and the highest mean scores was achieved by the NTU group for all concepts besides that of circular permutation for which the mean score of NTU was similar to that of TU. The results suggested that learning by NTU gave the best scores, followed by TU, GU, and SU.

Nevertheless, the results from the posttest (conceptual test) and the comprehensive test were slightly different. In the posttest, the lowest mean score was found in the GU group, whereas in the comprehensive test, SU came last.

Table 4.8 Comparison of mean scores between comprehensive test and retention test

Group	Comprehensive test (20 pts)	Retention test (20 pts)	t	Percentage reduction
TU	16.55 ± 0.40	15.17 ± 0.72	2.739*	8.34
NTU	16.96 ± 0.33	14.83 ± 0.60	4.010*	12.56
SU	13.89 ± 0.54	10.69 ± 0.93	3.891*	23.04
GU	16.03 ± 0.49	11.41 ± 0.84	6.232*	28.82

* significant difference at 0.05 level

To investigate whether the students in the four instructional units were able to retain their knowledge, the retention test containing parallel questions (to those of the comprehensive test) were administered two months later. The results in Table 4.8 show that the mean scores for the retention test were significantly lower than those of the comprehensive test. However, the percentage reduction was different in each group. They were 8.3, 12.6, 23, and 28.8 in TU, NTU, SU, and GU, respectively.

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4.3 Effects of the Four Instructional Units on Students' Perception

4.3.1 Finding from Questionnaire

Results in Table 4.9 showed students' satisfaction with the instructional units, and attitudes toward the increase in their understanding as probed by the self-assessment questionnaire.

Table 4.9 Scores of students' satisfaction and self-assessment understanding

Questionnaire [-3,3]*	**TU**	**NTU**	**SU**	**GU**
Satisfaction	2.34	1.90	1.68	1.89
Self-assessment understanding	2.04	1.88	1.23	1.36

* Max = 3.00

The results from the 7-point rating scale [-3,3] illustrated that all groups of students were highly satisfied with their instructional units. The highest satisfaction was seen in the TU group while the lowest was in the SU group which corresponded to the lowest score for the comprehensive test. They agreed that the unit helped them develop conceptual understanding on permutations and combinations. The students also gave comments on attitudes as shown in the following excerpts:

"*The unit is very interesting and thus helped learning*"

"*The novel problems from nontraditional word problem unit motivated my interest more than the traditional problems.*"

"*Mathematical knowledge gained from this unit can be exploited to solve real-life problems.*"

"*Group discussion helped promote thinking skill and problem-solving skill.*"

"*The given problems enabled the low achievers to solve them faster than the high achievers.*"

"*The nontraditional mathematics problems challenged my capability.*"

"*I want to learn this way in every other semester.*"

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"Learning in the new instructional unit was enjoyable and encouraged good relationship in the group."

However, a few students suggested that they should be given more details and time in the instructional unit.

"The teacher should give more details of unfamiliar sports including their rules."

"The teacher should give more time for thinking and solving the problems."

"Too many worksheets and too much homework to do, we need more time."

4.3.2 Finding from Students' Reflection

The students reflected on the knowledge gained as shown in the following excerpts:

"We now can derive the formulas by ourselves through enjoyable group work learning in a nontraditional word problems/sport problems/probabilistic games."

"First, I wonder how we could derive the formula for permutation of similar things through the football problems. Now, not only I know how to derive the formula but also apply the knowledge to solve other problems."

The students described the process of their thinking as shown in the following excerpts:

"During playing the guessing games on numbers, we tried several methods to win the game. First I wondered why my friends were fighting for certain set of numbers, then I realized that they were going for the best set to win the game."

"Knowledge on permutation provided students more opportunities to win the game. In addition to my prior knowledge I got new knowledge from discussion with peers to find the best way to win the game. I preferred to have just clues, not the answers from friends or teacher because it made me proud."

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The students voiced that the problems given in the units were appropriate for learning in each concept:

"The palindrome problem gave us the permutation formula and thus helped us to solve this problem faster and made us beat the other groups."

"The genome problem was suitable for introducing circular-permutation concept."

"The nontraditional word problems are appropriate for analyzing of complicated problems."

"Learning through solving several sport problems makes me know various techniques for solving problems."

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CHAPTER V
DISCUSSION

Overview

This chapter aims to present the interpretation of the research findings and discuss them in relation to other research studies. The preliminary results are discussed. The effects of the newly created instructional units, i.e., the nontraditional word problem instructional unit, the sport problem instructional unit, and the probabilistic game instructional unit, on students' achievement are given.

5.1 Discussion of Pilot Study

5.1.1 Pilot Study on Sport Problem Instructional Unit

The main finding of this pilot study was the success in implementing the sport problem instructional unit. This inquiry-based learning unit enhanced highschool students' understanding of permutations and combinations, as evidenced from posttest mean scores. This finding support the work of Crawford (2000) that teachers need to be changed from the traditional way of teaching and the inquiry based teaching is the appropriate one. This finding is consistent with the suggestion of Olson & Loucks-Horsley (2000), Bybee (2004), DeBore (2004), and Sandoval (2005), the inquiry can be an effective approach to help students learn the concepts and develop their learning skills as well as develop their understanding. Moreover, the students also had positive attitudes toward the sport problem instructional unit as evidenced from the questionnaire.

The pilot study was conducted twice on sport problem instructional unit in a school in the city area which most students were rather well off. In the first trial, 40 students (grade twelve) who had already taken a two-semester course in probability were taught by using the sport problem instructional unit for 5 hours. Their average score for the pretest was 39.71% while the average post test score improved to

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87.57%. Surprising was their rather-low pretest mean score because they had just finished the applied probability course within 2 months and the national test was approaching. Moreover, the pretest of this pilot study was rather simple. It indicated that more than half of those students could not remember the lesson and they had unclear understanding. From reviewing the students' old worksheets, all the problems were traditional, similar to the problems in the pretest. It showed that traditional problems cannot enhance students' understanding of permutations and combinations. Even though the posttest mean score was highly improved but it could not prove that they would still remember the lesson and have clear conceptual understanding enough to score high in this posttest again two months later. Unfortunately, the researcher did not test for the retention directly because those students graduated, but those who took the college admission test stated that they could apply the knowledge learned from this lesson (not from the conventional lesson they had learned previously) to the related questions in the examination. So the sport problem instructional unit taught to those students in only 5 hours had the high effect of enhancing students' conceptual understanding and problem solving.

In the second trial, the pilot study was conducted for a class of grade eleven students who had not taken any probability course. Although students in this group (43 students) could do none of the pretest problems, their average post test score was 72.09%. It showed that the students had no prior knowledge in permutations and combinations and they did not attempt to find the answer by using trial and error method which could solve some problems in the pretest. Even though the posttest scores of these students were obviously lower than the posttest scores of those of students in the first trial but it was rather high compared with their prior knowledge and given the time (only 5 hours) spent to learn all four concepts in permutations and combinations for the first time. In addition, the tests were also administered to a control group (39 students). The average pretest score for the control group was 11.54%, indicating that they were better students on the average. The average posttest score of this group was 70.51%, about the same as the treatment group. The increases in average pre-to-post test scores of both the control group and the treatment group were 58.97% and 72.09% respectively. It indicated that the students who learned from the sport problem instructional unit performed better than those who learned from the

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traditional instructional unit. Moreover, the results from the data analysis indicated that there were significant differences in the pretest mean scores in the linear permutation and permutation of similar things concepts while there was no significant differences in every concept in the posttest mean scores between the two groups. It might be shown that the sport problem instructional unit could better enhance the concepts of linear permutation and permutation of similar things. Like other studies, as reported by Kahneman and Tversky (2001), they revealed that the sport problems can enhance students' probabilistic judgments. Albert (2002) used baseball perspective to teach statistics and probability in an introductory statistics course. The baseball situation provided an interesting context to learn about statistical thinking and developed positive impact on the learning. Besides, Brown, Wilson and Fitzallen (2007) indicated that the teachers had to recognize the importance of providing students opportunities to work on authentic tasks and engage in problem solving

The same posttest was administered again two months after both groups had taken a conventional introductory course in probability. This time the average score of the treatment group was 60.72% while the average score of the control group was 45.83%. These scores indicated that this teaching approach was effective and resulted in a longer retention of the ability to solve probability problems.

In both cases, the lesson spanned five one-hour periods. Since students in the first trial had already taken a course in probability, the researcher had more time to ask them other combination and permutation problems, both the sport related and unrelated ones. Most students were able to solve a majority of the problems and explain their thoughts clearly. In addition, they could explain, in their own words, to those who could not solve the problems and they were able to draw analogies to similar problems which were particularly helpful. They were actually surprised at their peers' lack of understanding of some topics. They no longer needed instructional media as an aid in solving problems. From the observation and questionnaire, it was evident that they enjoyed the learning experience and were proud of their resourcefulness.

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5.1.2 Pilot Study on Nontraditional Word Problem Instructional Unit and Probabilistic Game Instructional Unit

This pilot study was conducted to six highschool students on a round table in the school area after the school hours for 15 hours in one week. The pretest and posttest were used to evaluate the effectiveness and weakness of the instructional units. These students had no score in the pretest. They could not solve the problems even though the problems were clear for them. It showed that not only did they have no prior knowledge in permutations and combinations but from observing they also made little effort to find the solutions. They indicated that the problems looked so hard while they would rather spend the remaining time doing the activities and they did not want the others having to wait for them. This might be the one of the disadvantages of group learning. This finding is different from the suggestion of Hawley and Valli (1999) that they suggested that collaborative problem solving was an important factor for successful professional learning.

Because the students expressed their willingness to learn from the 2 newly created instructional units, they enjoyed learning and were eager to do all the activities. Similar to Xiao, Yu, and Yan (2009), they pointed out that the positive feelings of enjoyment and success, and negative feelings of frustration and anxiety have an affect on mathematics learning. In addition, the students said that they preferred the learning activities on which they could reflect, and in which they could explain and justify their own ideas to others. The students enjoyed these instructional units because they had the opportunities to plan and participate in various activities. Their enhanced conceptual understanding of permutations and combinations, including the technical skills, arose from their having to construct their own knowledge while interacting with their friends. They could communicate their findings and discuss with the group. They could also derive the formulas by themselves with few suggestions. Their knowledge level was enhanced by their ability to investigate various problems that might affect the instructional units. Hence it showed that the students' interests and group collaboration had highly positive effects on students' learning and students' achievement, as evidenced from students' posttest scores which reached 100%. These findings confirmed by Grouws and Cebulla (2000), they pointed out that the quality of the implementation of a teaching practice also greatly influences

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its impact on student learning. Using small groups of students to work on activities, problems and assignments can increase student mathematics achievement. In addition, Middleton and Spanias (1999) indicated that success in mathematics is a powerful influence on the motivation (students' willingness) to achieve. As indicated by Dickinson and Butt (1989), students will find a task more enjoyable when they have moderately high probability of success as compared to one with a lower chance of success.

5.2 Effects of the Four Instructional Units on Students' Achievement

The students in all four classrooms, despite the difference in mathematical scores from examinations in the previous semester (TU > NTU, SU > GU), showed no significant difference in problem-solving ability on permutations and combinations, as evidenced from the pretest. Based on only their pretest scores, the scores from the subsequent posttest and comprehensive test would thus reflect the effectiveness of the instructional units. The post-over-pre-test normalized gains (Table 4.5), which should measure the conceptual understanding, showed the NTU group to be slightly superior to TU, followed by SU and GU. The ability to apply concepts acquired to solve more complicated problems as probed by the comprehensive test indicated that the NTU group performed the best, followed by TU, GU and SU. We thus concluded that the nontraditional word problems were sufficiently engaging and stimulating to make the NTU group outperform all others. The NTU word problems are relevant to everyday life and yet the intrinsic concepts are simple and familiar enough to be grasped quickly.

The lowest score of GU on the posttest and of SU on the comprehensive test suggested that students might have difficulties in conceptualization from sport and game problems. However, it might be possible that teaching with SU could provide students with enough concepts to do the posttest but not for the more complex problems in the comprehensive test. Surprising to us was that many students found the sport and game problems to involve more complicated and unfamiliar concepts even for a popular sport like soccer. Previously, we carried out three pilot studies in three classrooms for sport problem instructional unit and a small group for probabilistic

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game instructional unit of a city-based school whose students were more well-off and sophisticated. The students were responding so enthusiastically to the SU and GU problems that there were huge percentage gains. Thus, the caveat learned from this study on the rural students is that one should not extrapolate a study result too far.

The results showed that the four developed instructional units, however, could similarly lead to better understanding of the circular permutation concept, as evidenced by both the posttest and the comprehensive test. For the concepts of linear permutation and permutation of similar things, all units except GU (data from the posttest) and SU (data from the comprehensive test) increased students' achievement. For the combination concept, all units except SU (data from the comprehensive test) also enhanced students' achievement. The results suggested that even though SU and GU seemed to be less effective than TU and NTU in promoting students' learning, they still worked well in certain concepts on probability.

The fact that students' retention test scores decreased two months after the intervention, in the descending order of TU, NTU, SU, and GU is not surprising. The percentage reduction was not that high, ranging from 10% in TU to 28% in GU. This is corroborated by the mathematical scores in the last semester of the four classes, which was in the same descending order, i.e., 78% in TU to 59% in GU. The use of parallel questions in the retention test ruled out the possibility that the students might remember the test, thus resulting in high scores.

If only the previous scores in mathematics of the students in each group were taken into account, it could be claimed that the use of nontraditional word problems seems to be the most effective one. It brought the low achievers to the level of high achievers, as evidenced from the similar normalized gain. Moreover, SU and GU were also effective in promoting students' learning, albeit not to the same level as that of NTU. All these successes might be due to integrating the inquiry approach to the instructional units. The inquiry-based teaching has long been known to improve students learning in many aspects including higher-order thinking skill and problem-solving skills (Carter, 2004; Chin, Lin, Chuang, & Tuan, 2007; Fan & Zhu, 2007). During the learning activity, the students worked in groups of four in a collaborative manner to solve the problem and derive the mathematical formula. After the exposure to each concept, the teacher helped in the debriefing before students reflected on the

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knowledge learned. All these collaborative and inquiry approaches contributed to the success in the implementation of the developed instructional units. The students' knowledge gained was evidenced by the posttest and comprehensive test as well as from students written documents about their reflection.

The reason for NTU enhancing students' achievement in every concept was because nontraditional word problems were relevant to real situations. This made it easy to understand and enabled them to experience authentic problem solving. The results are similar to those of Bay-Williams and Martinie (2004) that after solving the nontraditional word problems, the students were able to share their answers and strategies with various representations as well as explain the formula derivation. Problem-solving situations help improve students' mathematical reasoning, abstraction, and generalization (Sriraman, 2004). Moreover, Kahan and Wyberg (2003) suggested that a good problem can be the effective learning vehicle, because it helps develop students' learning more quickly than traditional instructions.

In this study sport problems can enhance students' achievement in problems involving circular permutation, and to a lesser extent in linear permutation. Certain students who were keen on sports tried to find the formulas by themselves during sport problem solving. Although more time was spent to understand the concepts when compared to the nontraditional word problems, students who could derive the formulas seemed to have high ability to solve other problems as well. This is because studying mathematical contents from real-life problems or daily life situations would lead to better conceptual understanding and perception that are useful in learning mathematics. Our results are in agreement with those of Albert (2002) that used baseball problems in the instruction of introductory statistics course. Probability was introduced by describing and playing tabletop baseball games. Students with backgrounds in various sports often are better able to understand the statistical concepts, as they are set within the familiar context of sports. Some students who had no interest in sports did not make any effort to solve the problem by themselves. They only wanted to know the formulas for the problems. Hence they did not understand the concept or formula derivation which affected their achievement and their retention. Use of examples constructed from the simple and interesting contexts provided by

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sports board game might give better achievement and retention as suggested by Cochran (2005).

Games should be the effective tools in teaching probabilistic thinking as several research reports (Baker, 2003; Krulik, Rudnick, & Milou, 2002) suggested that games could be used to stimulate interest in mathematical concepts and could lead students to a discussion of the concept of a fair game. Surprisingly, the probabilistic game unit was the least effective tool in enhancing students' learning, compared to the others. In the probabilistic games instructional unit, students enjoyed the gaming experiences which provided students with opportunities to analyze different solution strategies by using mathematics knowledge to win the games. Although games provided students with an opportunity to make some conjectures about probability when playing the game, some students in this study only wanted to win the games by guessing the answers without putting effort to use mathematics thinking. Thus the probabilistic games in this study did not seem to help them to get more understanding in permutations and combinations as expected. It should be noted that this group of students belonged to the low achiever group. Results from several research works suggested that games involve concepts of probability and games are effective means of reinforcing mathematical methods (Amit, 1998; Batanero, Godino, & Roa, 2004). The concepts of probability and statistics may be developed through experiments in simple situations such as rolling a pair of dice or tossing a coin, as well as by solving real-life problems.

5.3 Effects of the Four Instructional Units on Students' Perception

The finding from the questionnaire and students' reflection showed that the students were highly satisfied with the instructional units. They agreed strongly that the instructional units based on the inquiry approach could help them to develop their conceptual understanding of permutations and combinations. Results from this study agreed with those of Kahle, Meece and Scantlebury (2000), Hofstein and Lunetta (2003), and Colburn (2008) that the students were impressed by and enjoyed the instruction based on the inquiry approach and they considered the activities helpful in motivating and facilitating learning. In congruence with several other studies, an

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inquiry-based teaching could increase student achievement, narrowed the gap between high and low achieving students, and the students were more interested in what they had to learn.

5.4 Implications

In this research study, an inquiry-based learning and teaching were appropriately used to frame the instructional units for better teaching and learning in mathematics classrooms in highschool. The results that the newly developed instructional units enhanced students' conceptualization and promoted students' attitude toward mathematics should enable the teachers to adopt this inquiry strategy to use in teaching various mathematics topics. The guideline for teachers is to provide authentic problems/activities, with teacher-guided inquiry in the beginning to a more open inquiry in the subsequent steps on designing and conducting the instructional units. Students should be encouraged to actively involve in the learning activities. This research results should be a guideline for teachers or educators in finding a proper way to help students develop their achievement and conceptual understanding correctly or even construct their own knowledge. Based on the successful of the implementation of the instructional units on nontraditional word problem, sport problems, and probabilistic games, it is important to note that not only the highscool mathematics teachers that should use the instructional units in permutations and combinations, but also other middle-school teachers and college teachers should try to use these instructional units to enhance their students' achievement after adjusting the problems/games in those instructional units. Moreover, for the students to develop their knowledge, they should be encouraged to work collaboratively so that they can share and discuss ideas with groups. Teachers' role should be as a facilitator who engages and motivates students to learn, provides learning materials, encourages students to share ideas and discuss, and gives suggestions and reflection to the students. Moreover, the teacher should focus on providing students (1) to learn the mathematics content that is relevant to students' everyday life, (2) to express their ideas to the teacher and friends in the classroom, (3) to involve in classroom planning

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and management, (4) to share and discuss with class, and (5) to participate in the interesting and fascinating problems' solving.

5.5 Limitations of this Study

Although the findings in this study suggested that the four inquiry-based instructional units promoted students' achievement in permutations and combinations, there were still many drawbacks in this study. The results should be reconfirmed with other groups of students in different school settings. The participants in this study were from four classes of different achievements ranging from high grade in the first classroom to the rather low in the fourth. Even though there was no overall significant difference in pretest scores between the four classrooms, we could discern the differences in the enthusiasm as well as interest and willingness to learn in the lower achieving groups. Another concern was the students in this study were from the rural area and had less opportunity to explore sports or games in depth as students in the city.

5.6 Recommendations

5.6.1 Recommendations for Further Study

1. The results should be reconfirmed with other groups of students in different school settings.

2. For further study, all participants should have the same prior knowledge and achievement in mathematics.

3. For group discussion, it is necessary to have the teacher or trained students to help to facilitate all students in each group during the activities.

4. The instructional materials/medias should be provided before the game class session and students' comments are useful for the implementation.

5. The teacher has to survey students' interest in the sports before using the sport problem instructional unit.

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6. The teacher has to increase the knowledge about the other subjects which involve the problems in non-traditional word problems.

7. Further studies should be tried out with groups of students known to be accustomed to sophisticated sports or games in order to prove the effectiveness of the sport problem or game problem units.

5.6.2 Recommendations for Further Development

Based on the research findings, it could be suggested that the Thai mathematics curriculum for highschool students should be revised by placing more authentic problems and probabilistic games in the probability textbook. Concerning the students' comments on instructional units, teachers should give more time for them to think and discuss, to find the answers and solutions, to do the exercises, and to play the games. To solve these problems, the teachers could increase the time or might revise or change the problem conditions/settings or game rules in order to decrease their complexity and fit for students' capabilities.

Moreover, a variety of mathematical problems in the instructional units in this study should help the teacher to choose the problems that are most appropriate to student interest for better learning outcomes. It is recommended that these alternative inquiry-based instructional units should be incorporated as part of teaching and learning in permutations and combinations in several highschool mathematics classes.

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CHAPTER VI
CONCLUSION

6.1 Development of the Newly Created Instructional Units in Permutations and Combinations

This study was composed of four instructional units in permutations and combinations. One was the traditional instructional units while the others were the newly-created instructional units consisting of three instructional units:

6.1.1 Nontraditional Word Problem Instructional Units

This unit is composed of eight newly-created real-life probability word problems, i.e., the sentence maker problem, the travel problem, the genome problem, the base number problem, the triplet code problem, the palindrome problem, the grouping problem, and the share group problem.

6.1.2 Sport Problem Instructional Unit

This unit is composed of six newly created word problems on popular sports and sport situations, i.e., the ping-pong problem, the soccer problem, the 4 x 100 sprint relay team problem, the football score problem, the American football problem, and the golf problem.

6.1.3 Probabilistic Game Instructional Unit

This unit composes of five newly probabilistic games, i.e., the icosahedron game, the random number game, the number guessing game, the wheel game, and the five number card game.

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6.2 Effects of the Four Instructional Units

The rural students were engaged based on inquiry learning and teaching in all units to achieve high learning gains. Among the problems, the nontraditional real-life ones seemed to enable students to perform better in the posttest, comprehensive test, and retention test than did those of the probabilistic game instructional unit and the sport problem instructional unit, and were close with a slight edge on the traditional instructional unit's problems. Students need more relevant real-life problems to stimulate them to learn and to retain the knowledge. Having to go through problems requiring slow accounting of events also encourages students to feel that probability is not all mathematical formalism: they can figure things out slowly for now and for later. Based on formalism alone, some may feel that they will never be able to figure out the probability of anything or to deduce for themselves alternatives that produce certain number of occurrences.

One good lesson learned from this study is that just because certain problems (sport and game) make some students (city-based and well-off, in this case) learn about probability better dose not mean that they are good for other (rural and less well-off, in this case) students as well. One always has to keep in mind students' prior knowledge and their interest. These rural students have less exposure to international games and may not watch or play games and sports, even the familiar ones, with the same interest and angle as do the city students. This is in spite of the fact that cable televisions with tens of channels are common and some are devoted to sports only.

6.3 The Outcomes of this Study

The results of the study are described as follows:

6.3.1 Nontraditional word problems, sport problems, and probabilistic games can be used to enhance students' achievement in permutations and combinations.

6.3.2 Nontraditional word problems, sport problems, and probabilistic games can be the efficient tools for teaching permutations and combinations.

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6.3.3 Nontraditional word problems were the most appropriate tools to support students' understanding in permutations and combinations and efficiently

foster students to solve the permutation and combination problems compared to sport problems and probabilistic games.

6.3.4 The nontraditional word problem instructional unit was the best instructional unit at enhancing students' achievement, followed by the traditional instructional unit, the sport problem instructional unit, and the probabilistic game instructional unit respectively.

6.3.5 Most of the students had positive attitude toward the instructional units in permutations and combinations, as evidenced from the students' reflection and the questionnaire.

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APPENDICES

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APPENDIX A

PRETEST
ON PERMUTATIONS AND COMBINATIONS

Instruction: Find the correct answer.

1. In how many ways can six people line up at a single counter to order food at McDonalds?

_____

_____

_____

2. There are 12 people on the committee at the local softball club. In how many ways can a president and a secretary be chosen from this committee?

_____

_____

_____

3. In how many different ways can a starting line up of 5 students be selected from a group of 12 students?

_____

_____

_____

4. Three students out of 10 will be chosen to speak at a school assembly where order is important. How many different outcomes are possible?

_____

_____

_____

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5. A test is composed of 6 right or wrong questions. How many possible answers can be written for all 6 questions?

_____

_____

_____

6. How many different arrangements of 8 letters can be made from the word PARALLEL?

_____

_____

_____

7. A 5-letter word is to be made from 3 As and 2 Bs. How many different arrangements can be made?

_____

_____

_____

8. How many eight-digit numbers can be formed from these digits; 1, 1, 2, 2, 3, 3, 3, 4?

_____

_____

_____

9. How many different arrangements of 7 tables can be made from 4 black and 3 white tables?

_____

_____

_____

10. Dang enters a maze, and he has a compass. To prevent himself from going round in circles he decides that he will only travel south or west and never north or east. The maze is shown below. How many different ways can Dang travel from A to B?

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_____

_____

_____

11. How many ways can four people sit around a table?

_____

_____

_____

12. How many different arrangements are possible for a family of 7 seated in a circle?

_____

_____

_____

13. At a dinner party for 10 people, all the guests were seated at a circular table. How many different arrangements were possible?

_____

_____

_____

14. How many different arrangements were possible to thread 8 different jewels on a necklace?

_____

_____

_____

15. How many different arrangements were possible to thread 12 different keys on a key ring?

_____

_____

_____

16. A construction crew has ten members. A team of four must be chosen for a particular job. In how many ways can the team be chosen?

_____

_____

_____

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17. How many ways can a subcommittee of 3 people be selected from a committee of 7 people?

_____

_____

_____

18. In how many different selections of four tapes can be made by a consumer choosing from among a collection of eight different tapes?

_____

_____

_____

19. Jan is a tour guide. In how many ways can she choose 3 museums to visit from the 8 museums in a city?

_____

_____

_____

20. To win the lottery, one must correctly select 6 numbers from a collection of 50 numbers (one through 50). The order in which the selection is made does not matter. How many different selections are possible?

_____

_____

_____

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APPENDIX B

POSTTEST
ON PERMUTATIONS AND COMBINATIONS

Instruction: Find the correct answer.

1. In how many ways can eight people line up at a single counter to order food at KFC?

_____

_____

_____

2. A club elects a president, a vice-president, and a secretary-treasurer. How many sets of officers are possible if there are 15 members and any member can be elected to each position? No person can hold more than one position.

_____

_____

_____

3. A softball team has 14 players on its roster. There are 9 distinct positions in which these players can be placed. How many lineups can be fielded?

_____

_____

_____

4. Three students out of twelve will be chosen to speak at a school assembly where order is important. How many different outcomes are possible?

_____

_____

_____

5. A test is composed of 5 multiple choice problems (4 choices). How many possible answers can be written for all 5 problems?

_____

_____

_____

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6. How many different ways can all letters of the word 'MATHEMATICS' be arranged?

_____

_____

_____

7. A 7-letter word is to be made from 3 As and 4 Bs. How many different arrangements can be made?

_____

_____

_____

8. How many ten-digit numbers can be formed from these digits; 1, 1, 1, 1, 2, 2, 3, 3, 3, 4?

_____

_____

_____

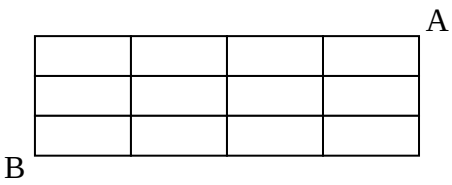
9. How many different arrangements of 9 tables can be made using 4 black, 3 red and 2 blue tables?

_____

_____

_____

10. Note enters a maze, and she has a compass. To prevent herself from going round in circles she decides that she will only travel south or west and never north or east. The maze is shown below. How many different ways can Note travel from A to B?



_____

_____

_____

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11. How many ways can seven people sit around a table?

12. How many different arrangements are possible for a family of 10 seated in a circle?

13. At a dinner party for six people all the guests were seated at a circular table. How many different arrangements were possible?

14. How many different arrangements were possible to thread 12 different jewels on a necklace?

15. How many different arrangements were possible to thread 9 different keys on a key ring?

16. A construction crew has eleven members. A team of four must be chosen for a particular job. In how many ways can the team be chosen?

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17. How many ways can a subcommittee of 4 people be selected from a committee of 9 people?

_____

_____

_____

18. A test is administered with 15 questions. Students are allowed to answer any ten. How many choices of ten questions are there?

_____

_____

_____

19. Jen is a tour guide. In how many ways can she choose 9 temples to visit from the 12 temples in a city?

_____

_____

_____

20. To win the lottery, one must correctly select 5 numbers from a collection of 100 numbers (one through 100). The order in which the selection is made does not matter. How many different selections are possible?

_____

_____

_____

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APPENDIX C

COMPREHENSIVE TEST
ON PERMUTATIONS AND COMBINATIONS

Instruction: Find the correct answer.

1. A teddy bear's wardrobe consists of 3 different hats, 4 different shirts and 2 different trousers. How many different outfits can the teddy bear wear?

_____

_____

_____

2. How many 4-digit even numbers can be made from the numbers 1, 3, 5, 7, 9 and 0 if the numbers can be repeated?

_____

_____

_____

3. How many words (by which we mean strings of letters) of three distinct letters can be formed from the letters *a, b, c, d, e, f, g*?

_____

_____

_____

4. How many permutations of the letters in the word NORMAL are there if the letters M and N appear side by side?

_____

_____

_____

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5. How many permutations of the letters in the word NORMAL are there if the letters M and N appear apart?

_____

_____

_____

6. How many three-letter patterns can be formed from the letters of the word *differentiate*?

_____

_____

_____

7. James has a briefcase that has a seven-digit security code. He remembers that the first number in the code was 6 and that the others were two 3s, and four 7s but forgets the order of the last 6 digits. How many different trials must he make to be sure of unlocking the briefcase?

_____

_____

_____

8. June has a briefcase that has two 4-digit locks. She remembers that she used the digits 1, 1, 2 and 2 on the left lock and 3, 4, 4 and 4 on the right lock, but can not remember the order. What is the maximum number of trials she would need to make before she has opened both the left lock and the right lock?

_____

_____

_____

9. Ten points are marked on a page and no three of these points are in a straight line. How many triangles can be drawn joining these points?

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_____

_____

_____

10. A nursery school teacher has collected a picture from each of the 12 children in the class. She wants to hang the pictures in four rows of three. How many different arrangements are possible?

_____

_____

_____

11. During an activity at school, 8 children (including Dang and Dam) are asked to sit in a circle. How many ways can the children be arranged if Dang and Dam appear side by side?

_____

_____

_____

12. During an activity at school, 8 children (including Dang and Dam) are asked to sit in a circle. How many ways can the children be arranged if Dang and Dam appear apart?

_____

_____

_____

13. How many different arrangements are possible if, from a group of 7 people, 4 are to be seated at a round table?

_____

_____

_____

14. There are 2 brothers among a group of 15 persons. In how many ways can the group be arranged around a circle so that there is exactly one person between the two brothers?

_____

_____

_____

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15. A committee consisting of 3 men and 4 women is to be chosen from 7 men and 9 women. In how many ways can this be done?

_____

_____

_____

16. One or two letters are to be chosen from 6 letters A, B, C, D, E and F. In how many ways can this be done?

_____

_____

_____

17. Udom is able to choose his work outfits from the following items of clothing: 4 jackets, 3 shirts, 7 ties, 4 pairs of trousers, 2 pairs of socks and 5 pairs of shoes. How many different outfits are possible if he wears one of each of the above items?

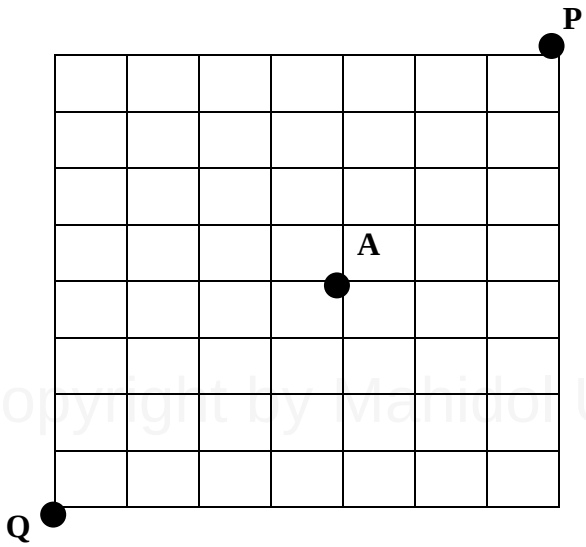
_____

_____

_____

Use this following situation for problems 18-20

Suda enters a maze, and she has a compass. To prevent herself from going round in circles she decides that she will only travel south or west and never north or east. The maze is shown below.



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18. How many different ways can Suda travel from P to Q?

_____

_____

_____

19. How many different ways can Suda travel from P to Q if she has to pass through A?

_____

_____

_____

20. How many different ways can Suda travel from P to Q if she does not have to pass through A?

_____

_____

_____

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APPENDIX D

RETENTION TEST
ON PERMUTATIONS AND COMBINATIONS

Instruction: Find the correct answer.

1. A new computer system comes with a choice of 3 keyboards, 2 different monitors and 2 different mouse attachments. With these choices, how many different arrangements are possible?

_____

_____

_____

2. How many 5-digit even numbers can be made from the numbers 1, 3, 5, 7, 9 and 0 if the numbers can be repeated?

_____

_____

_____

3. How many ways could an arrangement of 5 letters be chosen from A, B, C, D, E, F, G and H without repetition?

_____

_____

_____

4. How many permutations of the letters in the word COUNTER are there if the letters C and N appear side by side?

_____

_____

_____

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5. How many permutations of the letters in the word COUNTER are there if the letters C and N appear apart?

_____

_____

_____

6. How many three-letter patterns can be formed from the letters of the word *Mathematics*?

_____

_____

_____

7. James has a briefcase that has a 10-digit security code. He remembers that the first number in the code was 9 and that the others were three 3s, and six 7s but forgets the order of the last 9 digits. How many different trials must he make to be sure of unlocking the briefcase?

_____

_____

_____

8. June has a briefcase that has two 4-digit locks. She remembers that she used the digits 1, 1, 1 and 2 on the left lock and 3, 4, 4 and 4 on the right lock, but can not remember the order. What is the maximum number of trials she would need to make before she has opened both the left lock and the right lock?

_____

_____

_____

9. Twelve points are marked on a page and no three of these points are in a straight line. How many triangles can be drawn joining these points?

_____

_____

_____

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10. A nursery school teacher has collected a picture from each of the 8 children in the class. She wants to hang the pictures in two rows of four. How many different arrangements are possible?

_____

_____

_____

11. During an activity at school, 10 children (including Dang and Dam) are asked to sit in a circle. How many ways can the children be arranged if Dang and Dam appear side by side?

_____

_____

_____

12. During an activity at school, 10 children (including Dang and Dam) are asked to sit in a circle. How many ways can the children be arranged if Dang and Dam appear apart?

_____

_____

_____

13. How many different arrangements are possible if, from a group of 8 people, 5 are to be seated at a round table?

_____

_____

_____

14. There are 2 brothers among a group of 12 persons. In how many ways can the group be arranged around a circle so that there is exactly one person between the two brothers?

_____

_____

_____

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15. A committee consisting of 2 men and 5 women is to be chosen from 6 men and 8 women. In how many ways can this be done?

_____

_____

_____

16. Two or three letters are to be chosen from the letters A, B, C, D, E, F and G. In how many ways can this be done?

_____

_____

_____

17. Udom is able to choose his work outfits from the following items of clothing: 3 jackets, 7 shirts, 6 ties, 5 pairs of trousers, 7 pairs of socks and 3 pairs of shoes. How many different outfits are possible if he wears one of each of the above items?

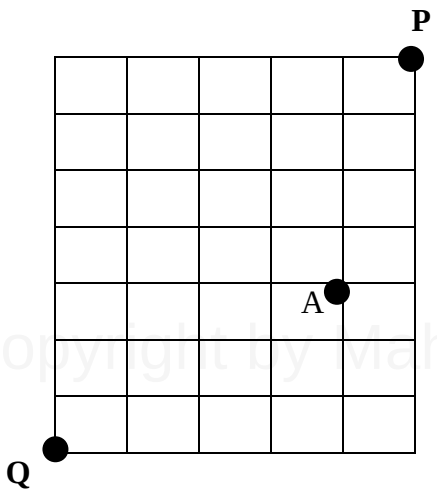
_____

_____

_____

Use this following situation for problems 18-20

Manee enters a maze, and she has a compass. To prevent herself from going round in circles she decides that she will only travel south or west and never north or east. The maze is shown below.



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18. How many different ways can Manee travel from P to Q?

_____

_____

_____

19. How many different ways can Manee travel from P to Q if she has to pass through A?

_____

_____

_____

20. How many different ways can Manee travel from P to Q if she does not have to pass through A?

_____

_____

_____

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APPENDIX E

QUESTIONNAIRE

Objective:

The objective of this questionnaire is to investigate the instructional units in permutations and combinations. Your answers will be kept in secret. It will be used for research study that does not have any effects on individual or school.

Part I: Personal Data

Instruction: Fill the answer corresponding to your opinion.

1. Sex ○ male ○ female

2. The latest mathematics grade: ……

3. The most favorite mathematics concept: ……

Reason: ……..…

4. The most disliked mathematics concept: … ……

Reason: ……..…

5. The most favorite subject: ……..……

Reason: ……..…

6. The most disliked subject: ……

Reason: ……..

7. The mathematics teacher's characteristics: …..……

……..……..…

8. The most favorite type of learning: ○ alone ○ Group (____ students)

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Part II: Students' attitudes toward the instructional unit

Instruction: Tick (x) the level which best describes your feeling about teaching and learning of the instructional unit in permutations and combinations and give the reason. The criterion of each level is:

- 3 means most strongly disagree

-2 means strongly disagree

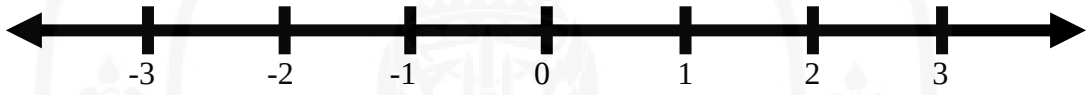
-1 means disagree

0 means neutral

1 means agree

2 means strongly agree

3 means most strongly agree



The reason: ....……

……

Part III: Students' self-assessment of understanding of permutations and combinations through the instructional unit

Instruction: Tick (x) the level which best represents your understanding in permutations and combinations through the instructional unit and give the reason. The criterion of each level is:

- 3 means most strongly unclear understanding

-2 means strongly unclear understanding

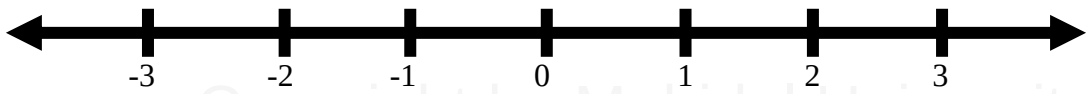
-1 means unclear understanding

0 means neutral

1 means clear understanding

2 means strongly clear understanding

3 means most strongly clear understanding



The reason: ....……

……

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Part IV: Students' comments or suggestions on the instructional unit

Instruction: Briefly answer the following questions:

1. What are the advantages of the instructional unit in permutations and combinations?

……

……

……

2. What are the disadvantages of the instructional unit in permutations and combinations?

……

……

……

3. What are your comments or suggestions?

……

……

……

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APPENDIX F

SCORELESS FIRST HALF PROBLEM

Let m:n denote the final score of a soccer match whose details (of the scoring moments) are not known. If we were asked to estimate the probability of a scoreless draw in the first half, how would we go about finding it?

Without any information about the match, it is reasonable to assume that each score could happen at anytime during the match with equal probability. Imagine a line segment representing the timeline of the match and a point on the line representing a moment of scoring. In effect, there would be $m + n$ points uniformly and randomly placed along the line. A scoreless first half means that all the points lie in a certain, predetermined half of the line. The probability of a point lies in a particular half is ½. Since the positions of the points are stochastically independent, the probability of a scoreless first half is $(½)^{m+n}$.

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BIOGRAPHY

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